# INDUSTRIAL ELECTRICITY

## DIRECT-CURRENT PRACTICE

By WILLIAM H. TIMBIE

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Principles of Electrical Engineering

# INDUSTRIAL ELECTRICITY

## VOLUME ONE

## DIRECT-CURRENT PRACTICE


BY

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## PREFACE TO THE SECOND EDITION

In this edition the subject matter on Direct-Current Practice has been brought up-to-date. A brief discussion of the modern concept of the composition of matter and of the electron theory has been included. The general scope of the text, however, has not been altered.

Fifteen years experience in using the original text has shown it to be advisable to devote more time to a standard treatment of the magnetic circuit, and a more complete study of generator and motor performance. It was also thought wise to postpone the treatment of electronic devices until the student has become familiar with the elements of alternating currents, and to omit the original first chapter, which was devoted to the production and use of power in general. The space and time thus gained has been devoted to the development of a more thorough and more teachable treatment of those principles which are directly applicable to direct-current practice.

Mr. Frank G. Willson, Head of the Department of Applied Electricity at Wentworth Institute, Boston, Massachusetts, has borne the brunt of the work of revision, and to him belongs the credit for the production of a more teachable text.

Grateful acknowledgment is also made to Mr. George M. Willmarth and Mr. Frederick L. Tedford, both of Wentworth Institute, for many criticisms and suggestions which have aided materially in bringing the text abreast with modern practice.

W. H. Timbie.

Cambridge, Massachusetts
*January*, 1939.

## PREFACE TO THE FIRST EDITION

This book was written to meet the demand from technical schools and electrical workers for a text covering the important principles of electrical science as applied to modern industry. The purpose is to explain how direct-current electricity is generated, transmitted and used, and to afford an adequate foundation for further study in the application of alternating-current electricity to modern practice. It is written in plain English and employs no mathematics beyond simple algebra.

The features of this book which are especially desirable are:

(1) It presents in a thorough manner the most important principles of electrical science in language which the student can understand.

(2) It applies these principles to present-day practice in the various branches of the electrical industry.

(3) It takes up fully and rationally the magnetic circuit, a most important feature of modern electrical engineering.

(4) It presents in a simplified but practical form the operating characteristics of direct-current generators and motors, including the parallel operation of generators.

(5) It includes sufficient material on the electrostatic circuit and thermionic emission to give a student an appreciation of the underlying principles of such problems as the stresses in insulators, the action of vacuum tubes, the phenomena of arcs, sparks and corona.

(6) Over one thousand typical examples and practical problems based on engineering data have been added for the purpose of driving home the principles and illustrating their use.

Grateful acknowledgment is extended to Mr. A. L. Jordan, of the Polytechnic High School, San Francisco, California; to Mr. F. G. Willson and Mr. G. M. Willmarth of Wentworth Institute, Boston, Massachusetts, for their many valuable suggestions and criticisms of the text; and to Mr. K. L. Wildes and Mr. J. W. Voelcker for reading proof and checking problems.

W. H. T.

CAMBRIDGE, MASSACHUSETTS
*August*, 1924.

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# INDUSTRIAL ELECTRICITY

## CHAPTER I

## FUNDAMENTAL IDEAS

### OHM'S LAW

Electricity in motion transfers energy, lights lamps, drives motors, refines metals, raises electric heating devices to high temperature, energizes the telephone, radio, and other electrical appliances. Electricity at rest is important to us mainly in relation to high voltage effects.

**1. Nature of Electricity.** The precise nature of electricity is not known. It is not at present an available source of energy. It **cannot be made nor destroyed.** It can be said to exist in the universe about us.

The attraction between a dry glass rod, rubbed with a silk handkerchief, and small bits of cotton or paper is an experiment most of us have seen. We say the glass rod has received **a charge** of electricity. Benjamin Franklin called the charge on the glass rod rubbed with silk a **positive charge**, and the charge on the silk, a **negative charge.** So for years, it has been customary to speak of **positive and negative** charges of electricity. In the experiment above, the silk and the glass rod have opposite charges and attract each other. Like charges repel each other. These effects are commonly said to be due to static electricity.

It has long been an accepted truth that all tangible substances, which we call matter, are composed of almost countless minute particles called molecules, and these in turn are composed of still more minute particles called atoms. The atom can be said to be the smallest component part to which any substance, as such, can be divided and still keep its identity. This idea of the composition of matter is of almost common knowledge today.

It is further known that there is non-material space between the molecules in which they have some sort of vibrating or oscillating movement among themselves.

There are as many different kinds of molecules as there are different substances. The number is almost inconceivably large. However, there are at the present time only ninety-one known different kinds of atoms, corresponding to the ninety-one chemical elements so far discovered. Chemical research indicates that there must be one or possibly more elements or atoms, but these have not yet been identified.

The exact nature and structure of the atom is not yet thoroughly understood. However, scientific research has established the fact that electricity exists in the atom in the form of minute charges.

At the present time,* it is generally believed that atoms are composed of a nucleus, consisting of positively charged particles called **protons,** and uncharged particles called **neutrons,** together with negatively charged particles called **electrons,** which revolve or move about the nucleus in more or less irregular paths or orbits. Each of these particles has a spin or rotation about its own axis, and the nucleus itself has a resulting spin.

The protons and neutrons in the nucleus are very closely associated or **bound** together, and the nucleus has a positive charge. In a normal atom, the negative charge on the free moving or orbital electrons exactly neutralizes the positive charge on the nucleus, so that the atom itself has no electrical charge. The negative charge on each electron in an atom is the same as the positive charge on each proton. There are no electrons in the nucleus. Also, since each electron has a negative charge, all electrons exert a repelling force on one another.

The discussion above implies that all matter is fundamentally composed principally of positive and negative charges of electricity.

As an illustration — an ordinary drop of water contains millions of molecules. Each molecule is just as definitely water as the whole drop. Each molecule is composed of two atoms of hydrogen and one atom of oxygen. None of these atoms is water, but consists of one of the component parts of water. Each of these atoms is composed of **protons, neutrons,** and **electrons,** which are neither water, nor hydrogen, nor oxygen, but principally **charges of electricity.**

The structure of each of the atoms of the various elements is not the same. The atom of the element "lithium," for instance, contains 7 protons and 4 neutrons in the nucleus, together with 3 free-moving or orbital electrons as indicated in Fig. 1–1. In some

* June, 1938.

way not easily explained, the 4 neutrons neutralize the charges of 4 of the positive protons in the nucleus, leaving it positively charged, while the 3 free-moving electrons neutralize the positive charges of the other 3 protons in the nucleus, so that the atom itself has no charge.

In the atoms of metals, there are extremely large numbers of these free-moving electrons, and when an electrical pressure is applied they are torn from the atom and move through the metal. It is the **movement of these electrons** which constitutes **an electric current.**

In metals such as copper or aluminum, these **free-moving electrons are easily torn from their atoms by a moderate voltage,** and these metals are called **conductors.** In glass, rubber, porcelain,

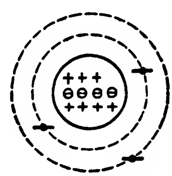


Fig. 1–1. Representation of the atom of "lithium," composed of a nucleus consisting of seven protons and four neutrons, together with three additional "free" electrons which circulate about the nucleus and make up the complete atom. These three electrons balance the excess of protons in the nucleus, so that the atom itself has no charge.

etc., the **electrons are torn from their atoms with great difficulty,** and these substances are therefore called **insulators.**

**2. The Flow of Electricity. An Electric Circuit.** In the main, we are going to study electricity as it **moves,** that is, **flows** and does work. Our purpose is to become familiar with the laws governing the effects and the applications of electricity, rather than with its nature. We do not know what it **is,** but we may know many of the things it will **do.**

It has been stated in Art. 1 that the movement of the electrons through the metal is an electric current. This movement can be likened to the flow of water through a pipe packed with sand. Therefore, throughout our study, we must remember that we are concerned always with something flowing in, or along, or through a conductor, and not with something stored up in a tank. We are concerned not with quantities of electricity, but with **currents or flow of electricity.** The sooner we become familiar with the idea of flow, the sooner will we get a real grasp of the subject.

We speak of an electric circuit, by which we mean a path for the flow of electricity. Similarly, a pipe line is a path, or circuit, for the flow of water.

Let us consider Fig. 2*a*–1, which represents a water circuit with a water pump forcing a current of water through a closed system

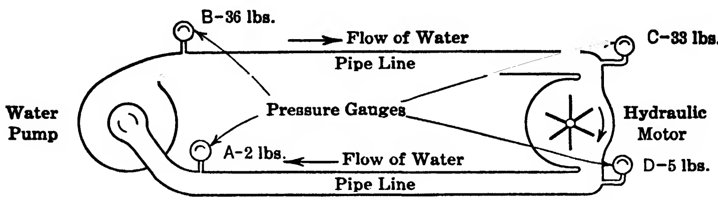


FIG. 2a–1. Hydraulic or water circuit with a pump and a hydraulic motor.

of pipes to run a hydraulic motor. We are interested here in: (1) the flow of water in the system; (2) the water pressure developed by the mechanically driven pump; (3) the opposition offered by the hydraulic motor and the pipe line to the flow of the water.

In many ways this figure is similar to Fig. 2*b*–1 which represents an electric circuit with a mechanically driven electric generator

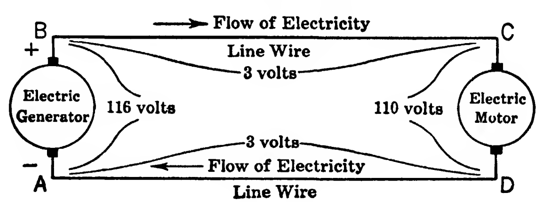


FIG. 2b–1. Electric circuit with an electric generator and a motor.

sending electricity through a motor. Note that in both cases we have **closed** circuits, and that the water or the electricity flows back to the pump or to the generator. Also, here we have to consider three things.

(*a*) **Current** — flow of electricity along a conductor or through a circuit.

(*b*) **Pressure** — that which causes the electricity or the current to flow.

(*c*) **Resistance** — that which regulates or opposes the flow of current.

**3.** (*a*) **Current. Rate of Flow. Ampere.** In the first place, where water is flowing through a pipe we never ask, " How much

water is there in the pipe?" but rather, "How much water is **flowing through** the pipe?" That is, "How much water is flowing through the pipe in a given time?" The answer, for instance, would never be "Five gallons," but "Five gallons per second." We are interested, not in the quantity, **but in the quantity that flows through in a second — rate of flow.**

Similarly, we never ask "How much electricity is there **in** that wire?" but rather, "How much current is **flowing along** that wire?" By which question we mean "How much electricity flows along that wire per second?" The answer could not be "A certain number of electrons," but it might be "A certain number of electrons per second." We are not interested in the quantity but in **the quantity that passes or flows in a second — rate of flow.**

Now an electron is just as definite a quantity of electricity as a drop is a definite quantity of water. Drops are too small to be convenient in measuring water, so we use the gallon. For the same reason, the electron is such a small quantity of electricity that we use a much larger value called the **coulomb,** and this is the unit in which the **quantity** of electricity is measured.

It has been calculated that there are about

6,300,000,000,000,000,000 electrons in a coulomb.

The number is usually written $6.3 \times 10^{18}$.

The answer to the question, "How much electricity flows along that wire?" might now be "So many **coulombs per second.**"

But we are fortunate in not having to use the term "coulombs per second" when we speak of the **quantity** of electricity that **flows per second.** We call this **coulomb per second** an **ampere.** Instead of saying there is a current of so many electrons per second, or so many coulombs per second, we say, so many amperes. Note that one ampere, one coulomb per second, and 6.3 million, million, million electrons per second all mean the same thing, current flow. We generally use the term, **ampere,** because it is so much more convenient.

Thus an ordinary 25-watt lamp when operating on a 110-volt circuit takes about 0.25 ampere. That is, there is 0.25 ampere flowing through it all the time it is glowing. A flatiron takes about 5 amperes. Street arc lamps may take from 4 to 10 amperes according to the size.

The ampere is an international unit of current flow and is legally defined as that rate of flow of a steady current which deposits 4.025 grams of silver per hour from a standard solution of silver nitrate.

**4.** (*b*) **Pressure** (that which causes the current to flow). **Volt.** We know that **pressure** causes a water current to flow in a pipe line. We say that water is supplied to the houses in a town under, or at, a pressure of 30 **pounds per square inch.**

Similarly, **electric pressure** causes an electric current to flow in an electric circuit. We say that electricity is supplied to lights in our house at a pressure, usually of 115 volts. This pressure will send about 0.25 **ampere** through a 25-watt incandescent lamp, just as 30 **pounds per square inch** will send about 0.25 **gallon per second** through our water pipe. The **volts** (pressure) cause the amperes to flow very much as the **pounds per square inch** (pressure) cause the gallons per second to flow.

Note that **volts** do not flow in the circuit any more than **pounds per square inch** flow in a water circuit. It is the **amperes** or the **gallons per second** that flow in the circuit.

We can produce a water pressure by a mechanically driven pump at a pumping station, or by a standpipe or a reservoir on a hill. We can produce an electrical pressure in several ways — either by a battery cell, a storage battery, or by a mechanically driven electric generator.

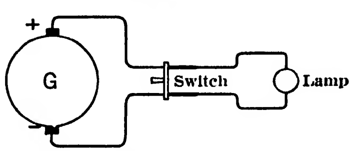


FIG. 3a–1. Electric generator developing a pressure which sends current through the lamp.

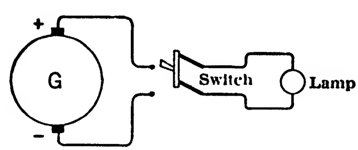


Fig. 3b–1. The switch in the circuit is open, and although the generator is developing a pressure, it cannot send current through the lamp.

It is possible to have a pressure without a flow of current. We know that when the valve in the pipe line is turned off, or closed, there is still pressure in the pipe, but no current flows.

Now in Fig. 3*a*–1, the electric generator produces or "generates" a pressure as long as the generator is running, which causes a current to flow through the lamp. In Fig. 3*b*–1, there is a switch in the circuit which is thrown off (open) and although the generator is running and producing a pressure, it cannot cause current to flow, as the electrons will not cross the open gaps of the switch.

There is no current in the circuit when the switch is thrown off (open). When the switch is thrown on (closed), current will flow (electrons will move), and the size of the current (number of amperes) will depend upon how many lamps or other devices are connected in the circuit. Note that "turning on," "throwing on," or "closing" a switch is like **turning on,** or **opening** a valve in a water pipe; and that "turning off," "throwing off," or "opening" a switch is like **closing** the valve in the pipe.

The volt has been legally defined as $\frac{1}{1.434}$ (= 0.697) of the pressure of a Standard Clark Battery Cell under specified conditions.

**5.** (*c*) **Resistance** (that which regulates the current). **Ohm.** When electrons move through a wire, the filament of a lamp, or any other conductor, they are in continual collision with the molecules of the material, and are thus hindered or opposed in their progress through the circuit. It is this opposition or **resistance** to the movement of the electrons which constitutes the **resistance** of the circuit. It is somewhat similar to the friction which acts against the flow of water in a pipe.

If we connect a flatiron in a circuit of 115-volts pressure, a current of about 5 amperes will flow through the heating element of the iron. If, however, we place a tungsten lamp in the same circuit, only about 0.25 ampere will flow through the filament. In the latter case the electrons have more collisions with the molecules of the filament and encounter more opposition. Therefore, the same pressure, 115 volts, cannot force as many electrons per second (amperes) through the filament as through the flatiron. We explain this by saying the **resistance** of the lamp filament is greater than the **resistance** of the heating element in the iron.

**Resistance** may, then, be defined as the property of a circuit which limits, or opposes, or resists the flow of electricity through it.

When a pressure of 1 volt can force 1 ampere of **current** through a wire we say the resistance of the wire is 1 **ohm.** If 1 volt can force only 0.5 ampere through the wire we say the resistance is 2 ohms. In order to force 1 ampere through 2 ohms resistance, it would be necessary to apply 2 volts pressure.

This agrees with what we know about the flow of water through pipes. If the pipe is small and rough, we know it offers great resistance to the flow of water through it, and a high pressure is necessary to force much current through it.

It is unfortunate that we have no unit for resistance in a water circuit.

Similarly, if a wire is small and ill-suited for carrying an electric current, we find its resistance is large (high) and that great pressure (volts) is needed to force much current (amperes) through it.

The **ohm** is a standard international unit of resistance, and is legally defined as the resistance of a column of mercury 106.3 centimeters long and one square millimeter in cross-sectional area, at the temperature of melting ice (0° Centigrade or 32° Fahrenheit).

The Greek letter, omega, $\Omega$, is often used as an abbreviation for the word "ohm." That is, 10 ohms may be written $10^{\Omega}$.

**6. Comparison of Hydraulic and Electrical Units.** The following table will aid in fixing the meaning of the three terms discussed above. It should be studied until one is thoroughly familiar with the meaning of the terms **amperes, volts,** and **ohms.**

| UNITS OF | WATER | ELECTRICITY |
|---|---|---|
| QUANTITY | GALLON | COULOMB |
| CURRENT<br>Quantity per sec<br>Rate of flow | GALLON per sec | AMPERE<br>(Coulomb per sec) |
| PRESSURE | POUNDS per square inch | VOLT |
| RESISTANCE | No Unit | OHM |

**7. Resistors. Rheostats.** When we wish to limit, or control, the current in an electric circuit, we connect in the circuit a conductor having the desired resistance, called a resistor. This is shown diagrammatically in Fig. 4–1. It is generally made of iron, nickel alloy, or German silver. Copper and aluminum are rarely used as resistors because they are such good conductors; their resistance is too "low." If the resistor is to carry a small current, it is generally made of wire or ribbon arranged in the form of a coil, or in zigzag sections, and mounted on an insulating frame. In the laboratory, incandescent lamps are often used as resistors.

FIG. 4–1. Conventional diagram of a resistor for use in an electric circuit.

Figures 5–1 and 6–1 are good illustrations of commercial **resistors** for small currents. If the **resistor** is to carry large currents

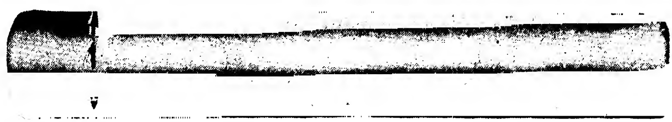

Fig. 5–1. A resistor constructed of a coil of resistance wire wound on an insulating tube and covered with insulating enamel. *General Electric Co.*

it is usually made in the form of cast grids of iron or iron alloy, and is illustrated in Figs. 7–1 and 8–1.

A **rheostat** is a resistor in which the amount of the resistance

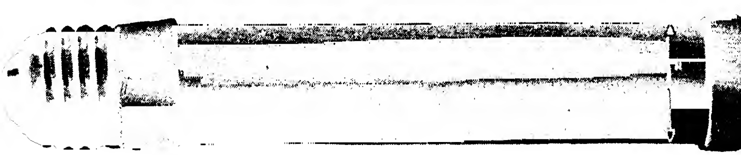

Fig. 6–1. A resistor mounted for attachment to a screw receptacle. *General Electric Co.*

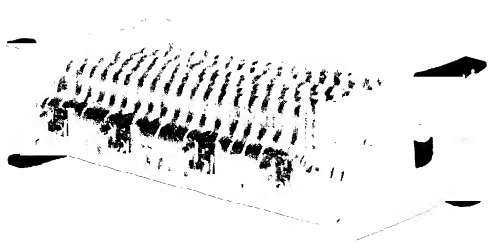

Fig. 7–1. Grid resistor for larger currents. *General Electric Co.*

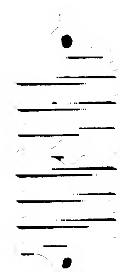

Fig. 8–1. One of the cast units used in the assembly of a grid resistor. *General Electric Co.*

can be varied in order to change the value of the current flowing in a circuit. It is shown diagrammatically in Figs. 9*a*–1 and 9*b*–1, and is illustrated in Figs. 10–1 and 11–1. It is constructed of sets of resistors which can be inserted ("cut in"), or taken out ("cut out"), of the circuit by mechanical means. There are many applications of rheostats in electric circuits. They

may be used as theatre dimmers; to control the starting current and speed of motors, and the voltage of generators, etc.

It is important not to allow **too much current** to flow through a rheostat. There is always a definite maximum current any rheostat is constructed to carry, and if this value is exceeded, the heat generated may ruin it. Many rheostats have this maximum current rating given on the name plate. Always look for it.

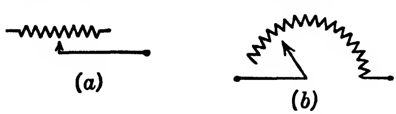


FIG. 9-1. Diagrams of rheostats used to change the value of the current in an electric circuit.

**8. Potential. Difference of Potential. Pressure or Voltage Relations in a Circuit.** In Art. 4, it has been shown that there must be an **electric pressure** to force current to flow. We can also say that there must be a difference in pressure, or a **difference in potential** between two points in a circuit to force current to flow

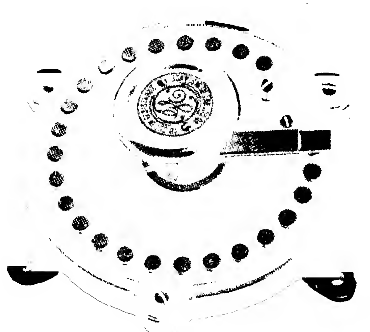

FIG. 10-1. A field rheostat. *General Electric Co.*

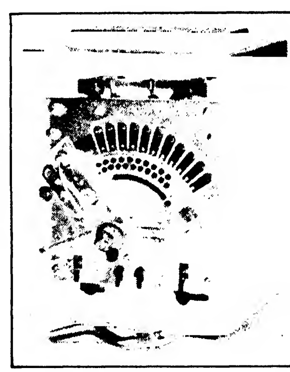

FIG. 11-1. A motor starting rheostat. *Westinghouse Elec. Mfg. Co.*

from one point to the other. This idea is shown in Fig. 12*a*-1 where two storage tanks, I and II, are filled to the same level. All valves are closed. The same pressure will be exerted at valves *B* and *C*, or the **potential** of the water in the two tanks is the same. If valve *A* is opened no current will flow from one tank to the other. There is no difference in pressure, or **no difference of potential** between the two tanks. Now suppose tank II is only half full as in Fig. 12*b*-1. The pressure at valve *B* will be different from that at valve *C*, and if valve *A* is opened, current will flow from tank I to II. There is a difference of pressure or a **difference**

**of potential** between the water in the two tanks, and the water in tank I is at the **higher potential.**

Similarly, consider an electric battery, Fig. 13–1, developing a pressure of 2 volts between its terminals. We mark one terminal (+) to indicate that its potential is higher than the other marked

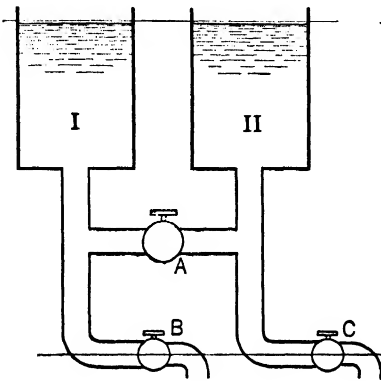


FIG. 12a–1. Water in two storage tanks at the same potential.

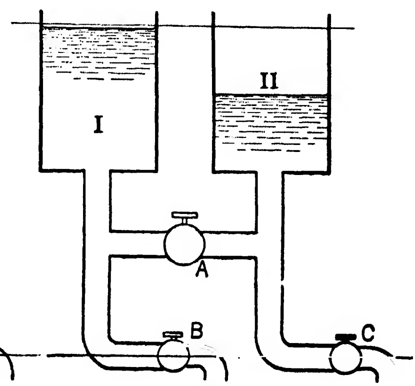


FIG. 12b–1. Water in tank I at a higher potential than that in tank II.

(−). There is 2 volts **difference of potential** between the two terminals, and if we connected a wire between these two terminals a current would flow. It is customary to consider the current as flowing from the plus (+) terminal to the minus (−) terminal, from the higher potential to the lower, just as in the water analogy in Fig. 12*b*–1.

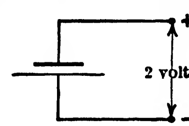


FIG. 13–1. A battery with two volts difference in potential between its terminals.

This idea can be further shown by referring to Fig. 2*a*–1. The pressure gauge *A* might read 2 pounds to the square inch, gauge *B*, 36 pounds, *C*, 33 pounds, and *D*, 5 pounds. The action of the rotary pump raises the pressure to 36 pounds — 34 pounds above that at gauge *A*. As the water is forced along the pipe, it loses 3 pounds pressure (36 − 33 = 3) by the time it reaches gauge *B* due to the friction of the pipe. As the water is forced through the motor it loses 28 pounds pressure (33 − 5 = 28), and as it flows back from *D* to *A* it loses 3 pounds pressure (5 − 2 = 3). Difference in pressure can be calculated in pounds per square inch. Note that the difference in the pressure readings between gauges *B* and *A* (34 pounds), is just equal to the **sum** of the differ-

ences in pressure in the remainder of the circuit, 3 + 28 + 3 = 34 pounds.

Similarly, in Fig. 2*b*–1, the action of the generator might raise the electrical pressure (voltage) of the plus (+) terminal *B*, 116 volts above that of the minus (−) terminal *A*. The point *C* might be 3 volts lower in potential than *B* (just as the pressure gauge *C* in Fig. 2*b*–1 is 3 pounds less than at gauge *B*) due to the resistance of the wire between the points *B* and *C*. Point *D* might be 110 volts lower in potential than point *C*, giving 110 volts **difference in potential** across the motor. And point *A* might be 3 volts lower in potential than *D*. These **potential differences** can be read by a voltmeter. Note again that the **sum** of the **differences in potential** in the wires and the motor are just equal to the difference in potential between the two terminals of the generator, that is, 3 + 110 + 3 = 116 volts.

Another important fact is that in both the closed water system and in the electric circuit, the **gallons per second** and the **amperes** are not used up. They continue to flow in the circuit. It is the pressure that is used — the **pounds per square inch and the volts.**

**9. Direction of the Flow of Current.** As has been already mentioned, it has always been customary to think of an electric current as flowing from a higher to a lower potential. That is, from the (+) terminal of a battery or generator through the circuit to the (−) terminal. It is absolutely true that there must be a **difference in potential** between the two terminals to cause current flow. It really is immaterial as to **which terminal** is at the **higher potential,** as long as there is a **difference** between the two. And accordingly, in many appliances, it makes little difference as to **which way** the current flows, just so long as there **is** a current flow.

In the light of recent theory, this newly discovered electron, a negative charge of electricity, is believed to move in a circuit from the negative (−) terminal, through the motor or lamp filament and toward the positive (+) terminal of the generator. This does not need to upset our ideas of current flow. The actual direction of the movement of electrons is vital to us, mainly, when we consider the action of vacuum tubes.

To be in agreement with **common or conventional practice,** we will continue to consider electric current as flowing through the connected circuit from the positive (+) to the negative (−) terminal, from the higher to the lower potential.

**10. Measurement of Current and Pressure.** When we wish to measure the **flow of water** in a pipe, we insert a flow meter, as in Fig. 14–1. The current of water flows through the pipe and

through the meter causing it to indicate the number of gallons per second which pass through it. Note that we must open up the pipe and insert the meter so that the current we wish to measure

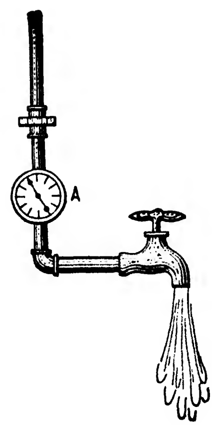


FIG. 14–1. The water meter, A, is inserted into the pipe line to measure the flow of water through the pipe.

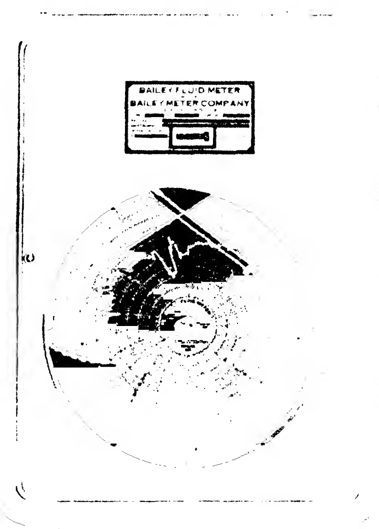

FIG. 15–1. A practical flow meter for measuring the current of steam, water or air. *Bailey Meter Co.*

flows directly through the instrument. Figure 15–1 shows a Bailey recording flow meter of modern type.

Similarly, when we wish to measure the electric current in a circuit, we insert a current meter into the circuit so that the current we wish to measure flows **through** the meter. Such a meter is called an **ammeter.** Figure 16–1 represents an ammeter, $A$, inserted in the circuit to measure the current flowing **through** the resistor, $R$. Note that all the current which flows through the resistor and wires must flow through the ammeter. An ammeter must, therefore, have very little resistance so as not to oppose the current. Such an instrument is very delicate, besides being expensive, and must be handled carefully. Note that the in-

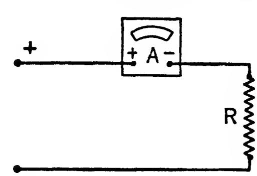


FIG. 16–1. The ammeter, A, is inserted in the line and measures the current through the line and resistor R.

strument is connected so that the current enters it at the (+) terminal and leaves at the (−) terminal.

When we wish to measure the **pressure** in a water pipe, we tap a pressure gauge to the pipe, as *A* in Fig. 17–1. Note that **no current flows through** the gauge. It is merely tapped to the pipe at the point at which we wish to know the pressure, so that pressure in the pipe can make it indicate. We do not disturb the pipe line or the current flowing through it.

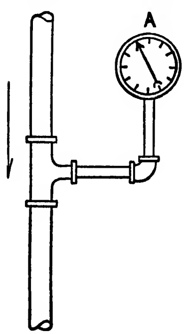


FIG. 17–1. The pressure gauge, A, is attached to the pipe and measures the water pressure at that point in the pipe.

Similarly, when we wish to measure the electric pressure (volts) forcing a current through, say, a resistor, we do not disturb the circuit or the current flowing through it. We just **tap** the terminals of a **voltmeter** to, or across, the circuit. We do not ordinarily measure electrical pressure at one point in an electric circuit, but, instead, we measure the **difference in potential** between two points in a circuit. This the voltmeter does, as indicated in Fig. 18–1. It measures the **difference in potential** between the two terminals of the resistor, or the pressure (volts) forcing current through the resistor. The (+) terminal of the voltmeter is connected to the (+) terminal of the resistor and the (−) terminal, to the (−) terminal of the resistor.

Note that the current flowing through the resistor **does not** flow through the voltmeter, and that the pressure across the resistor is the pressure across the voltmeter. Since the voltmeter is not designed to register current, it has large, or "high," resistance. A comparison of Figs. 16–1 and 18–1 clearly shows the difference in the method of connecting an ammeter and a voltmeter. An **ammeter** is **inserted into** the circuit and a **voltmeter** is **tapped on** the circuit. Great care must be used **not** to **tap** an ammeter **across** a circuit by mistake, since it has very low resistance and will be instantly ruined by a large rush of current.

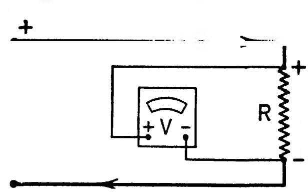


FIG. 18–1. The voltmeter, V, is attached to the terminals of the resistor R, and measures the pressure across the resistor.

**11. Ohm's Law.** We have seen that it requires pressure to force a current of electricity through a circuit, just as it requires pressure to force a current of water through a water system. We have seen, also, that resistance offers opposition to the flow of electricity, just as the friction in a pipe offers opposition to the flow of water.

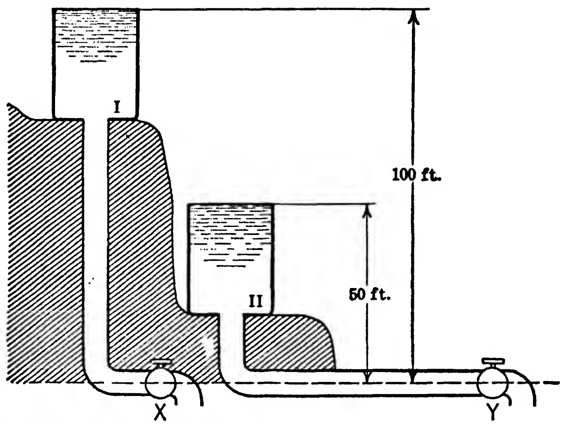


FIG. 19–1. Pipes of equal size and length are connected to the two tanks, I and II. Twice as much water will flow from tank I as from II, because tank I exerts twice the pressure at its outlet valve as does tank II.

Consider Fig. 19–1 which represents two tanks, I and II, each connected to pipe lines of the same size and length. Valves $X$ and $Y$ are at the same elevation. Since the water in tank I is twice the height of that in II, the pressure at valve $X$ will be twice that at $Y$. Since the two pipes have the same area and length, they offer the same opposition (or resistance) to the flow of current. When the valves are opened, twice as much water (gallons per second) will flow from I as from II.

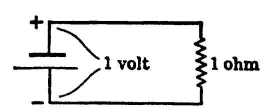


FIG. 20–1. A battery connected to a circuit of 1 ohm resistance.

Similarly, in Fig. 20–1, the battery cell, developing 1 volt at its terminals when 1 ohm is connected in the circuit, will force 1 ampere to flow in the circuit, for when 1 volt can force 1 ampere through a circuit the resistance is 1 ohm (Art. 5). If the battery developed 2 volts at its terminals it would send twice as much current or 2 amperes through the same circuit. Note that in both cases **doubling the pressure, doubles the current flowing.**

Now consider Fig. 21–1, in which two pipes are connected to a single tank. Pipe II is half the size (cross-sectional area) of pipe I. There is the **same** pressure at valves *M* and *N*, but if the valves are opened only about half as much current will flow from pipe II, because the opposition it offers (its resistance) to the flow of water, is about twice as great as that of pipe I. Similarly, in Fig. 20–1, if we connect 2 ohms in the circuit when the battery has 1 volt at its terminals, only 0.5 ampere will flow. Note again the similarity of the two cases. **Doubling** the **resistance halves** the **current flowing.**

This relation can be stated in general terms as follows: The current flowing in a circuit is **proportional** to the pressure (voltage)

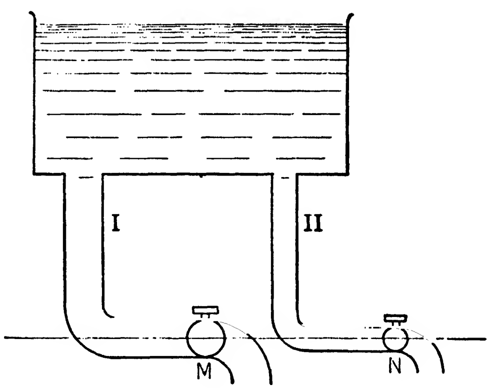


FIG. 21–1. Two pipes of different size connected to the same tank. Pressure on each outlet valve is the same. If valves are open, less current flows from pipe II than from pipe I.

on the circuit, and **inversely proportional** to the resistance (ohms).

If we connected 4 ohms in circuit with the 1 volt battery, only 0.25 ampere would flow. Note that by dividing the 1 volt by the 4 ohms we obtain the current 0.25 ampere.

Suppose a battery produced 8 volts at its terminals when the resistance of the circuit were 2 ohms. Note again that we find the current, 4 amperes, by dividing the voltage (8 volts) by the resistance (2 ohms).

**The current which an electric pressure forces through a resistance is equal to the pressure divided by the resistance.** In fact, this relation is stated as one of the great general laws of the electric circuit, and is called **Ohm's Law,** after the man who first stated it.

## OHM'S LAW

This may be written briefly:

$$\text{Current} = \frac{\text{pressure}}{\text{resistance}} \quad \text{or} \quad \text{amperes} = \frac{\text{volts}}{\text{ohms}}$$

or

$$I = \frac{E}{R} \tag{1}$$

in which

$I$ = Current in **amperes**
$E$ = Pressure in **volts**
$R$ = Resistance in **ohms.**

**Example 1.** How much current will flow through a lamp having 88 ohms resistance, when it is connected across 110-volt mains?

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}} \quad \text{or} \quad I = \frac{E}{R} = \frac{110}{88} = 1.25 \text{ amperes.}$$

Equation (1) can be transposed and written:

$$E = IR. \tag{2}$$

That is, the voltage across a circuit equals the current in amperes multiplied by the resistance of the circuit in ohms. This voltage is often called the $IR$, drop, or the resistance drop.

**Example 2.** What pressure will be required to force 1.25 amperes through a lamp having 88 ohms resistance?

$$\text{Volts} = \text{amperes} \times \text{ohms} \quad \text{or} \quad E = IR = 1.25 \times 88 = 110 \text{ volts.}$$

Ohm's law (equation 1) can also be transposed and written:

$$R = \frac{E}{I}. \tag{3}$$

That is, the resistance of a circuit equals the pressure on the circuit in volts, divided by the current in the circuit in amperes.

**Example 3.** What resistance must a lamp have if it takes 1.25 amperes when connected to 110-volt mains?

$$\text{Ohms} = \frac{\text{volts}}{\text{amperes}} \quad \text{or} \quad R = \frac{E}{I} = \frac{110}{1.25} = 88 \text{ ohms.}$$

It is particularly important to learn these **three** forms of expressing Ohm's law. Compare the values in the three examples above. If any two of the values are known, the third can be determined. The following chart will assist in fixing these relations.

| To Find | Form of Equation |
|---|---|
| Current in Amperes or $I$ | $\text{current} = \dfrac{\text{pressure}}{\text{resistance}}$ <br> $\text{amperes} = \dfrac{\text{volts}}{\text{ohms}}$ <br> $I = \dfrac{E}{R}$ (1) |
| Pressure in Volts or $E$ | $\text{pressure} = \text{current} \times \text{resistance}$ <br> $\text{volts} = \text{amperes} \times \text{ohms}$ <br> $E = IR$ (2) |
| Resistance in Ohms or $R$ | $\text{resistance} = \dfrac{\text{pressure}}{\text{current}}$ <br> $\text{ohms} = \dfrac{\text{volts}}{\text{amperes}}$ <br> $R = \dfrac{E}{I}$ (3) |

**Prob. 1-1.** How much current can 60 volts force through 5 ohms?

**Prob. 2-1.** What current flows when 32 volts acts across 0.25 ohm?

**Prob. 3-1.** A 20-ohm resistor is put across a 440-volt circuit. What will be the current in the resistor?

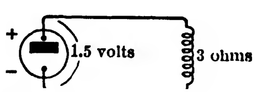


Fig. 22-1. A dry cell connected to a coil of wire having 3 ohms resistance.

**Prob. 4-1.** A dry cell, Fig. 22-1, has a terminal voltage of 1.5 volts when a 3-ohm coil of wire is connected across its terminals. What current flows in the coil?

**Prob. 5-1.** The resistance of a certain tungsten lamp when cold is 20 ohms. It is placed on a circuit of 115 volts. What current flows through the lamp the instant the switch is snapped on?

**Prob. 6-1.** When the filament of the tungsten lamp in Prob. 5 is heated to a white heat, the resistance rises to 400 ohms. What steady current now flows through the lamp?

**Prob. 7-1.** An electric flatiron has a resistance of 22 ohms. What current flows when the flatiron is put across a 115-volt circuit?

**Prob. 8-1.** What voltage will force 12 amperes through 17 ohms?

**Prob. 9-1.** What pressure is needed to force 1.56 amperes through an incandescent lamp having 80 ohms resistance?

**Prob. 10-1.** What pressure will be required to force a current of 0.075 ampere through a 1000-ohm resistor?

**Prob. 11–1.** An electric bell has a resistance of 350 ohms and will not ring with a current of less than 0.02 ampere. What is the smallest pressure that will ring the bell?

**Prob. 12–1.** The current required to operate a telephone receiver is 0.006 ampere. What voltage must be impressed across the receiver if it has 95 ohms resistance?

**Prob. 13–1.** An incandescent lamp has 220 ohms resistance when glowing, and requires 0.5 ampere. What is the pressure of the circuit on which it should operate?

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**Prob. 14–1.** An automobile windshield defroster, Fig. 23–1, takes 4.8 amperes from the 6-volt battery circuit. What is the resistance of the defroster?

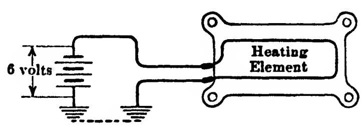



FIG. 23–1. A windshield defroster operated from a 6-volt battery.

**Prob. 15–1.** Through what resistance will 140 volts force 11 amperes?

**Prob. 16–1.** An electric soldering iron takes 3.2 amperes when used on a 110-volt circuit. What is the resistance of the iron?

**Prob. 17–1.** What resistance must a 115-volt incandescent lamp have in order to take 0.435 ampere?

**Prob. 18–1.** The voltage across a trolley-car heater is 600 volts. The current is 7.5 amperes. What is the resistance?

**Prob. 19–1.** What current is forced through a resistance of 3.5 ohms by a pressure of 60 volts?

**Prob. 20–1.** Through what resistance will a pressure of 60 volts force a current of 8 amperes?

**Prob. 21–1.** What voltage will force a current of 16 amperes through a resistance of 4.5 ohms?

**Prob. 22–1.** What voltage will force a current of 0.03 ampere through a resistance of 2000 ohms?

**Prob. 23–1.** Through what resistance will 90 volts force 41 amperes?

**Prob. 24–1.** What current flows when 25 volts acts across 0.09 ohm?

**Prob. 25–1.** Tha pressure generated by a dynamo is 220 volts. The resistance of the electric circuit is 42 ohms. What current flows through the circuit?

**Prob. 26–1.** The voltage on a certain circuit is 5 volts. If the current is 700 amperes, what is the resistance?

**12. Measurement of Resistance. Voltmeter and Ammeter Method.** It has been seen from equation (3) that, if both the voltage across a piece of electrical apparatus and the current flowing through it are known, the resistance can be found. Thus to find the resistance of the resistor $R$ in Fig. 24–1, an ammeter is

inserted **in** the circuit and a voltmeter is tapped **across** its terminals. The voltmeter reading divided by the ammeter reading is the resistance, if the current is steady while the readings are being taken. It is wise to take the two readings simultaneously if accurate results are to be obtained.

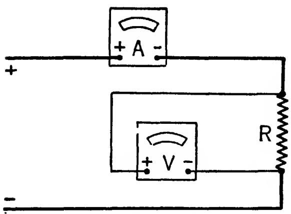



FIG. 24-1. The reading of the voltmeter divided by the reading of the ammeter equals the resistance of the resistor, R.

This is the simplest method of obtaining the resistance of any piece of electrical apparatus. See Chap. V for further discussion.

**13. Types of Circuits.** There are three general types of electrical circuits, depending upon how the resistors, or pieces of apparatus, are connected in the circuit.

(1) **Series Circuits.** When the resistors are connected in **tandem** or **end to end** they are said to be in series, as shown in Fig. 25-1.

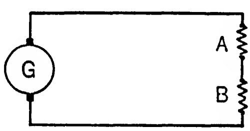



FIG. 25-1. The resistors A and B are connected in series.

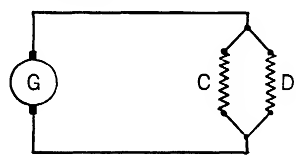



FIG. 26-1. The resistors C and D are connected in parallel.

(2) **Parallel Circuits.** When the resistors are connected **side by side**, that is, when one terminal of each of the resistors is connected to one common point in the circuit, and the other terminal of each resistor to another common point, so that the current

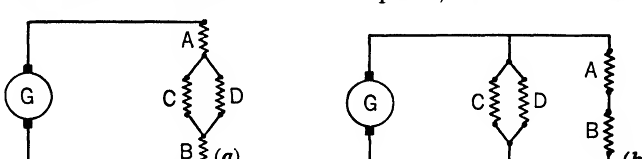



FIG. 27-1. Series-parallel circuits. (a) A parallel combination of resistors C and D is connected in series with resistors A and B. (b) The parallel combination of resistors C and D is connected in parallel with the series combination of A and B.

divides between them, they are said to be in parallel. **Multiple** or shunt are other names for the same combination. In Fig. 26-1, the resistors *C* and *D* are in parallel.

(3) **Series-Parallel Circuits.** The two combinations in (1) and (2) above may exist in the same circuit, as in Fig. 27*a*–1, where the parallel combination of resistors $C$ and $D$ is in series with $A$ and $B$. In Fig. 27*b*–1 the parallel combination of $C$ and $D$ is in parallel with the series combination of resistors $A$ and $B$.

**14. Series Circuit. Current.** If we join four pipes, $A$, $B$, $C$ and $D$, of unequal diameters in series, Fig. 28–1, and force a current of water through them, the water cannot go in at $M$ in any greater quantity per second than it comes out at $N$. There must be the same current (gallons per second) flowing through each pipe no matter what its size, because no more or less can go through one than through all the others. Similarly, we may join in a series several pieces of electrical apparatus, as in Fig. 29–1, a resistor ($A$) of 10 ohms, a resistor ($B$) of 50 ohms and a resistor ($C$) of 200 ohms. Although the resistance of resistor ($A$) is much lower than that of resistors ($B$) or ($C$), still no greater current can

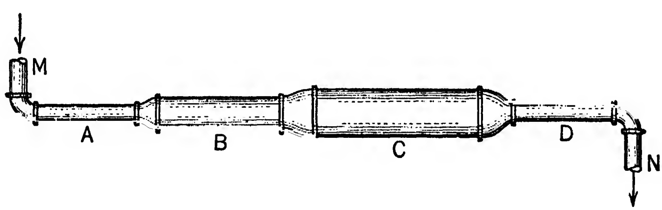



FIG. 28–1. No more water can flow through the large pipe C, than can enter at M, or flow out at N. The flow of water in all the pipes is the same.

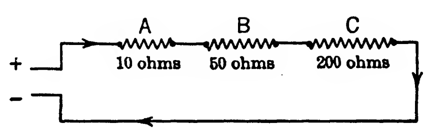



FIG. 29–1. No more current can flow through the small resistor of 10 ohms, than can flow through the large 50-ohm and 200-ohm resistors.

flow through it than through the other parts. No more current can enter one end of the electric circuit than can leave at the other end, any more than in the case of the pipe line. The first fact to be noted then, is that:

**In a series circuit the current is the same in all parts, no matter what the resistance of the several parts may be.**

This may be written:

$$I_{\text{total}} = I_1 = I_2 = I_3 \tag{4}$$

where $I_1$, $I_2$, and $I_3$ are the currents in the several parts.

**15. Series Circuit. Resistance.** When current is forced through the pipe line in Fig. 28–1, it flows against the friction exerted by all the pipes. Each section of the pipe exerts a **part** of the total opposition to the flow of the water current.

Similarly, in Fig. 29–1, the resistor $A$, the resistor $B$ and the resistor $C$, each offer a **part** of the total resistance to the flow of the current. The total resistance to the current flow is the sum of the resistances of all the parts, or is $10 + 50 + 200 = 260$ ohms.

The second fact, therefore, to be noted about a series circuit is that:

**The combined resistance of a series circuit is the sum of the separate resistances.**

This may be written:

$$R_{\text{total}} = R_1 + R_2 + R_3 \tag{5}$$

where $R_1$, $R_2$, and $R_3$ are the resistances of the separate parts.

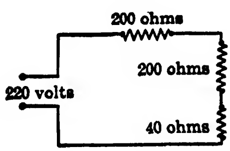



FIG. 30–1. The resistance of a series arrangement of resistors is the sum of the separate resistances.

**Example 4.** (*a*) What is the total resistance of the circuit in Fig. 30–1?

(*b*) What current flows in the circuit?

**Solution.** (*a*) $200 + 200 + 40 = 440$ ohms.

(*b*) $I = \dfrac{E}{R_{\text{total}}} = \dfrac{E}{R_1 + R_2 + R_3}$

$= \dfrac{220}{440} = .5$ amperes.

**Prob. 27–1.** (*a*) What is the total resistance of the circuit in Fig. 31–1? (*b*) What current flows in the circuit if the generator maintains a pressure of 125 volts?

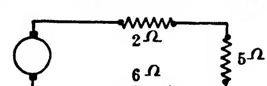



FIG. 31–1. The three resistors are in series.

**Prob. 28–1.** (*a*) If eight 5-ohm resistors are connected in series in a circuit, what will be the total resistance?

(*b*) What will the voltage on the circuit have to be to force 3 amperes through each resistor?

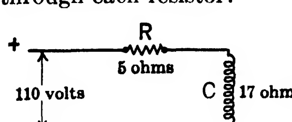



FIG. 32–1. The coil C is in series with resistor R.

**Prob. 29–1.** Three identical resistors are connected in series to a 240-volt line. If 10 amperes flows in the circuit what is the resistance of each resistor?

**16. Series Circuit. Voltage.** Suppose that a resistor ($R$) of 5 ohms resistance and a coil ($C$) of 17 ohms resistance are joined in series, as in

Fig. 32–1, across a 110-volt line. We have seen that the total resistance of the combination will be 5 + 17 or 22 ohms. The current flowing through the circuit is found as usual by Ohm's law.

$$I = \frac{E}{R} = \frac{110}{22} = 5 \text{ amperes.}$$

Since this is a series circuit, the same current flows in each part of it. Thus there are 5 amperes flowing through the resistor $R$ and 5 amperes flowing through the coil $C$.

Also we are able by Ohm's law to find the voltage required to force the 5 amperes through the 5-ohm resistor, since volts = amperes × ohms. Thus, for the resistor,

$$\text{Voltage} = IR = 5 \times 5 = 25 \text{ volts.}$$

There are required, then, 25 volts to force the 5 amperes through the 5-ohm resistor.

In the same way we may find the voltage required to force the 5 amperes through the 17-ohm coil. Because, for the coil, Ohm's law is still true, so

$$\text{Voltage} = IR = 5 \times 17 = 85 \text{ volts.}$$

There are required, then, 85 volts to force the current of 5 amperes through the 17-ohm coil. We found that 25 volts are required to force the current through the 5-ohm resistor.

We know that to force the current through both requires 110 volts, since they are on a 110-volt line.

Note that the 110 volts required to force the current through the two pieces exactly equals the sum of the 25 volts across the resistor and the 85 volts across the coil; that is, 25 + 85 = 110 volts. It is always true in a series circuit that, if we add up the voltages across all the pieces in series, the sum will exactly equal the voltage across the series combination.

The third fact then to be noted about a series circuit is that:

**The voltage across the pieces in series equals the sum of the voltages across the separate pieces.**

This can be written:

$$E_{\text{total}} = E_1 + E_2 + E_3 \qquad (6)$$

where $E_1$, $E_2$, and $E_3$ are the voltages over the several parts of the circuit.

**Example 5.** In the series circuit of Fig. 33–1, 3 amperes are flowing.
(*a*) What is the voltage across *AB*?
(*b*) What is the voltage across each resistor?

**Solution.** (*a*) The total resistance of the circuit = $10 + 40 + 15 = 65$ ohms. Voltage to force 3 amperes through 65 ohms = $3 \times 65 = 195$ volts. Answer to (*a*) then is: Voltage across the circuit is 195 volts.

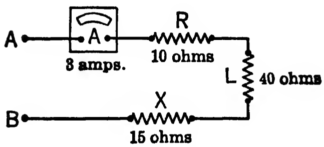



FIG. 33–1. The sum of the voltages across resistors R, L and X is equal to the total voltage across the circuit, or to the voltage across AB.

(*b*) Volts to force 3 amperes through *R* (10 ohms) = $3 \times 10 = 30$ volts.

Volts to force 3 amperes through *L* (40 ohms) = $3 \times 40 = 120$ volts.

Volts to force 3 amperes through *X* (15 ohms) = $3 \times 15 = 45$ volts.

**Check.** Volts to force 3 amperes through $R + L + X = 30 + 120 + 45 = 195$ volts.

This answer checks with the volts found in part (*a*) necessary to force 3 amperes through the series combination.

**17. Series Circuit. Current, Resistance and Voltage.** The three facts which should be learned with regard to a series circuit may be tabulated as follows:

SERIES COMBINATION

**CURRENT through series combination is SAME as current through each separate part.**

**RESISTANCE of series combination is the SUM of the resistances of the separate parts.**

**VOLTAGE across series combination is the SUM of voltages across the separate parts.**

or

$$\mathbf{I}_{\text{total}} = \mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I}_3.$$
$$\mathbf{R}_{\text{total}} = \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3.$$
$$\mathbf{E}_{\text{total}} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.$$

**Prob. 30–1.** A circuit consists of three resistors of 200 ohms, 150 ohms and 50 ohms, respectively, connected in series. If 1.25 amperes flows in the circuit:
(*a*) What is the current through each resistor?
(*b*) What is the voltage across each resistor?
(*c*) What is the total voltage on the circuit?

**Prob. 31–1.** There are seven arc lamps in series, Fig. 34–1, each requiring 6.5 amperes. If each lamp has a virtual resistance of 13 ohms:
(*a*) What is the voltage across each lamp?
(*b*) What voltage must the dynamo develop for the circuit?

**Prob. 32–1.** Motor $M$, in Fig. 35–1, requires a current of 20 amperes and a pressure of 220 volts across its terminals. The line wires have a resistance of 0.25 ohm each. What pressure must the generator develop at its terminals?

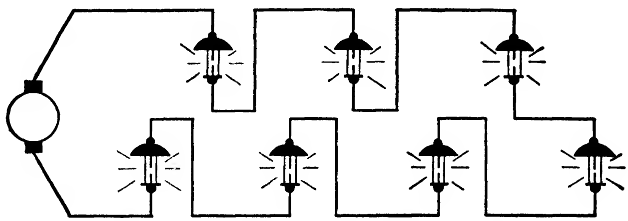


FIG. 34–1. Seven arc lamps in series.

**18. Application of Ohm's Law.** It should be noted that in solving Example 5, Fig. 33–1, Ohm's law was used in ($a$) to find the voltage necessary to force the current through the whole circuit of the three resistances, $R$, $L$ and $X$. First, the total resistance was found by adding $10 + 40 + 15 = 65$ ohms. Then, to find the total voltage necessary, we said:

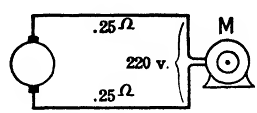



FIG. 35–1. The motor is in series with the line wires.

(Total) voltage = (total) current × (total) resistance
(Total) voltage $= 3 \times 65 = 195$ volts.

Note that to find the **total** voltage it was necessary to use the **total** current and **total** resistance.

But later when we wished to find the voltage necessary to force the current through the 10-ohm coil **only,** we used Ohm's law again. But this time, since we wanted the voltage across the 10-ohm coil **only,** we used the resistance of the 10 ohms **only,** and not the total resistance of the circuit. We also had to use the current through the 10-ohm coil **only.** Thus we said:

Voltage (across 10-ohm coil) = current (through 10-ohm coil) × resistance (of 10-ohm coil) $= 3 \times 10 = 30$ volts.

Note that to find the voltage across the **10-ohm coil** we used the current through the **10-ohm coil** and the resistance of the **10-ohm coil.**

Thus it can be seen that Ohm's law may be applied either to the whole of a circuit or to any part of a circuit. But if it is applied to the **whole** circuit, the voltage, resistance and current must be the voltage, resistance and current of the **whole circuit**

and not merely of a **part**. But if the law is applied to a **part** of a circuit, the voltage, current and resistance of **just that part**, must be used and no more or less.

This is of vital importance in applying Ohm's law. Many mistakes are made in the use of this simple law, just because we fail to be careful to use the voltage, current and resistance of the **same** part of the circuit.

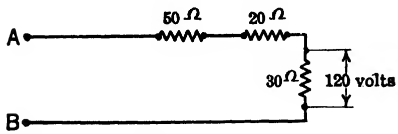



FIG. 36–1. The current through the 30-ohm resistor equals $\frac{120}{30}$ or 4 amperes.

Note the use of this rule in the following:

**Example 6.** In Fig. 36–1 a resistor of 50 ohms, a resistor of 20 ohms, and another resistor of 30 ohms are connected in series across the points *A* and *B*. The 30-ohm resistor has a pressure of 120 volts across it.

Find:

(*a*) Current through each resistor.
(*b*) Voltage across each resistor.
(*c*) Voltage across *AB*.

**Solution.**

We are able to find the current through the 30-ohm resistor because we know **both** the resistance and pressure.

$$I_{30} = \frac{E_{30}}{R_{30}}, \quad \text{or}$$

$$\text{Current (through 30-ohm resistor)} = \frac{\text{voltage (across 30-ohm resistor)}}{\text{resistance (of 30-ohm resistor)}} = \frac{120}{30} = 4 \text{ amperes.}$$

Thus the current through the 30-ohm resistor is 4 amperes. But since the 50-ohm resistor and the 20-ohm resistance are in series with the 30-ohm resistor, the same current must pass through them. Therefore, we can find the voltage across the 20-ohm resistor as follows:

$$E_{20} = I_{20}R_{20}, \quad \text{or}$$

$$\text{Voltage (across 20-ohm resistor)} = \text{current (through 20-ohm resistor)} \times \text{resistance (of 20-ohm resistor)} = 4 \times 20 = 80 \text{ volts.}$$

Also $$E_{50} = I_{50} \times R_{50}, \quad \text{or}$$

$$\text{Voltage (across 50-ohm resistor)} = \text{current (through 50-ohm resistor)} \times \text{resistance (of 50-ohm resistor)} = 4 \times 50 = 200 \text{ volts.}$$

Since this is a series circuit, the voltage across the whole circuit, that is, across $AB$, equals the sum of the voltages across the separate parts, or

$$\text{Voltage across } AB = 120 + 80 + 200 = 400 \text{ volts.}$$

The answers then are

(a) Current through each part = 4 amperes.
(b) Voltage across 20-ohm resistor = 80 volts.
Voltage across 50-ohm resistor = 200 volts.
(c) Voltage across $AB$ = 400 volts.

Note that after we had found the current of the **whole** circuit, we might have added up the resistances and found the resistance of the **whole** circuit, thus, $30 + 20 + 50 = 100$ ohms.

Then voltage (across total circuit) = current (through total circuit) × resistance (of total circuit);

$$4 \times 100 = 400 \text{ volts.}$$

This checks with the total voltage as found by the first method.

**Prob. 33-1.** Lamp $L$, Fig. 37-1, requires 0.2 ampere. Voltage of the generator is 115 volts:

(a) How many volts are used to send current through the line wires?

(b) What voltage is there across the lamp?

(c) What is the resistance of the lamp?

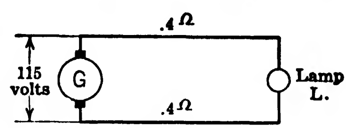



FIG. 37-1. The lamp L is in series with the two .4-ohm wires.

**Prob. 34-1.** The resistor in Fig. 38-1 has a resistance of 100 ohms and the line wires 0.2 ohm each.

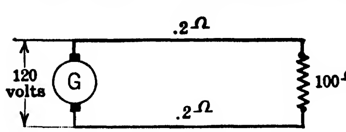



FIG. 38-1. The 100-ohm resistor is in series with two .2-ohm wires.

(a) What is the current in the line?

(b) What is the voltage across the resistor?

(c) How much voltage is used up in sending the current through the line wires?

**Prob. 35-1.** A 10-ohm resistor is joined in series with a 25-ohm resistor.

(a) What is the resistance of the combination?

(b) How much voltage across the combination is required to force 12 amperes through the resistors?

**Prob. 36-1.** Six arc lamps are joined in series; each has a virtual resistance of 14 ohms. If the line wires have a total resistance of 7 ohms, how much voltage is required to send 6 amperes current through the lamps and line?

**Prob. 37-1.** (a) What is the voltage across each lamp in Prob. 36-1?
(b) What voltage is used to send the current through the line wires?

**19. Parallel Circuit. Voltage.** In Fig. 39–1 we have two pipes *R* and *S* at different levels (at different potentials), joined by means of three parallel pipes *A*, *B* and *C*. If we measured the pressure at each of the three valves by means of pressure gauges

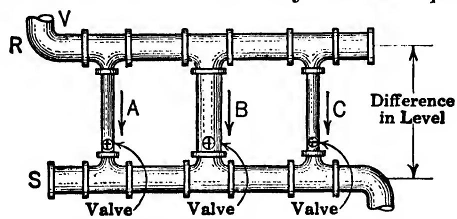



FIG. 39–1. The pressure on the three valves is the same.

attached, we would find the pressure at each valve to be the same. If the valves were opened, the **same pressure** would force current through all three pipes.

Similarly, in Fig. 40–1, three resistors *A*, *B* and *C* are joined in parallel between two electric pressure mains *R* and *S*. The same pressure forces current through all three resistors. That is, we would have 120 volts forcing current through resistor *A*, through resistor *B* and through resistor *C*.

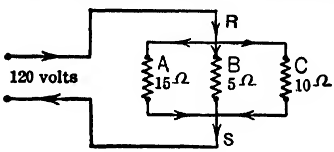



FIG. 40–1. The voltage across resistors A, B and C is the same. The three resistors are in parallel.

The first thing to be noted regarding a parallel circuit is that:

**The voltage across a parallel combination is the same as the voltage across each branch.**

This can be written

$$E_{\text{total}} = E_1 = E_2 = E_3 \tag{7}$$

where $E_1$, $E_2$ and $E_3$ are the voltages across the several branches.

**20. Parallel Circuit. Current.** Turning again to Fig. 39–1, we see that since the pipes *A*, *B* and *C* are not joined in series they are independent of one another as far as current is concerned and do not have to carry the same current. In fact, it is very evident that the largest pipe will carry the largest current and the smallest pipe, the smallest current. Water and electricity **do not** take the path of least resistance **only.** They take **all** paths. Where the path is of small resistance a heavy current flows. But the water or the electricity is sure to make use of **both** the low resistance and the high resistance paths and force as much current as possible

through both. The total current that flows through is merely the sum of the currents in the separate paths.

**Example 7.** In Fig. 40–1, since the resistance of the branch $A$ is high, a small current only will flow through it. By applying Ohm's law to this branch alone, we can find this current. We have seen that the voltage across $A$ is 120 volts. Now the resistance is 15 ohms and Ohm's law applies as follows to find the current:

$$I_A = \frac{E_A}{R_A} \text{ or current (through } A) = \frac{\text{voltage (across } A)}{\text{resistance (of } A)} = \frac{120}{15} = 8 \text{ amperes.}$$

Similarly we can find the current through $B$:

$$I_B = \frac{E_B}{R_B} \text{ or current (through } B) = \frac{\text{voltage (across } B)}{\text{resistance (of } B)} = \frac{120}{5} = 24 \text{ amperes}$$

and also the current through $C$:

$$I_C = \frac{E_C}{R_C} \text{ or current (through } C) = \frac{\text{voltage (across } C)}{\text{resistance (of } C)} = \frac{120}{10} = 12 \text{ amperes.}$$

The current then flowing between $R$ and $S$ by all three paths is merely the sum of these currents or $8 + 24 + 12 = 44$ amperes. This is also the current in the two main wires.

The second fact to notice in a parallel circuit is that:

**The current flowing through the parallel combination is merely the sum of the currents in the separate branches or paths.**

Which can be written:

$$I_{\text{total}} = I_1 + I_2 + I_3 \qquad (8)$$

where $I_1$, $I_2$ and $I_3$ are the currents in the several branches.

**Prob. 38–1.** Through resistor $A$, Fig. 41–1, 4 amperes flow. Through resistor $A'$, 12 amperes flow. What is the total current in the line wires?

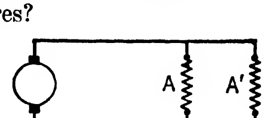



FIG. 41–1. The resistors A and A' are in parallel.

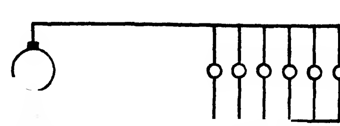


FIG. 42–1. The six lamps are in parallel.

**Prob. 39–1.** Each lamp in Fig. 42–1 takes 0.24 ampere. How much current flows in the main line?

**21. Parallel Circuit. Resistance.** Suppose it is desired to find the resistance of the parallel combination of $A$, $B$ and $C$ in Fig. 43–1, which is the same as Fig. 40–1. We have seen that:

$$\text{The current through } A = \frac{120}{15} = 8 \text{ amperes.}$$

$$\text{The current through } B = \frac{120}{5} = 24 \text{ amperes.}$$

$$\text{The current through } C = \frac{120}{10} = 12 \text{ amperes.}$$

The total current in the combined circuit = 8 + 24 + 12 = 44 amperes.

Now since we know the **current through the combination** (44 amperes) and the **voltage across the combination** (120 volts) we can find the **resistance of the combination.**

$$R_{\text{comb.}} = \frac{E_{\text{comb.}}}{I_{\text{comb.}}}$$

or

$$\textbf{Resistance} \text{ (of combination)} = \frac{\text{voltage (across combination)}}{\text{current (through combination)}}$$

$$= \frac{120}{44} = 2.73 \text{ ohms.}$$

At first sight it may seem strange that the resistance of a combination of three pieces of 5, 10 and 15 ohms should be only 2.73 ohms. But the apparent difficulty disappears when we consider that the more paths we have in parallel for the current to flow from one point to another, the more opportunity there is for electricity to flow, and the lower must be the resistance between those two points. Thus, if there had been only the 5-ohm path between the points $R$ and $S$ then the resistance between these points would have been 5 ohms. But when another path of 10 ohms was run between the same points $R$ and $S$, more current could flow and thus the resistance between the points became less than 5 ohms. And when a

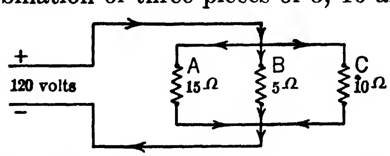



FIG. 43–1. The resistance of the parallel combination of A, B and C equals the voltage across the combination divided by the current through the combination.

third path of 15 ohms resistance was added, the resistance became still smaller. Thus we may see that the resistance of any parallel combination is less than the resistance of the path of smallest resistance. The path having the smallest resistance in this case is the 5-ohm path, and the combined resistance of the three parallel paths amounts to but 2.73 ohms.

Or, referring to Fig. 39–1, it can be seen that the more pipes there are connected in parallel between the main pipes *R* and *S*, the easier it is for the water to get from one main pipe to the other. Hence the smaller the resistance between the main pipes. Note the example below.

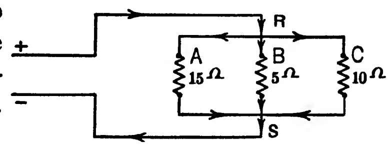



FIG. 44–1. The resistance of this combination can be found by assuming 1 volt to be put across it.

**Example 8.** Suppose we were given merely the three parallel resistances, *A*, *B* and *C*, Fig. 44–1, and there were no mention made of any voltage across them. We could find the resistance of the parallel combination as follows:

First find the current that one volt would force through each branch.

$$\text{Current (per volt) through } A, \frac{1}{15} = 0.0667 \text{ ampere.}$$

$$\text{Current (per volt) through } B, \frac{1}{5} = 0.2 \quad \text{ampere.}$$

$$\text{Current (per volt) through } C, \frac{1}{10} = \underline{0.1\quad\;} \text{ ampere.}$$

Current through combination = sum = 0.3667 ampere.

Now if one volt would force 0.3667 ampere through the combination, the resistance (of combination) =

$$\frac{\text{voltage (across combination)}}{\text{current (through combination)}} = \frac{1}{0.3667} = 2.73 \text{ ohms.}$$

This checks with the value found above.

We have used the expression "**Current per volt**" in the above example; for instance,

$$\text{current per volt through } A = \frac{1}{15}\text{ampere}$$

and

$$\text{current per volt through } B = \frac{1}{5} \text{ ampere.}$$

Note that this value $\frac{1}{15}$ is the reciprocal (or inverse) of the resistance of $A$ (15 ohms), and that $\frac{1}{5}$ is the reciprocal of the resistance of $B$ (5 ohms). This **current per volt** or reciprocal of the resistance is called the **conductance.** The name **mho** has been given to the unit of conductance. Thus, in Example 8, the resistor of 15 ohms has $\frac{1}{15} = 0.0667$ mho conductance, the resistor of 5 ohms has $\frac{1}{5}$ or 0.2 mho conductance, and the resistor of 10 ohms resistance has $\frac{1}{10}$ or 0.1 mho conductance. The **larger** the resistance is, the **smaller** the conductance becomes. The conductance of a parallel combination is merely the **sum** of the **conductances** of the separate branches. Thus, $0.0667 + 0.2 + 0.1 = 0.3667$ mho, the conductance of the parallel combination in the example above.

Conductance, then, is merely another name for the term "amperes per volt," or "current per volt."

In solving for the resistance of a parallel circuit we can, for convenience, express all the above operations in one equation,

$$R_{\text{total}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} \tag{9}$$

where 1 volt is assumed on the circuit and $\frac{1}{R_1}$, $\frac{1}{R_2}$, etc., are the conductances, or currents per volt, of the separate branches. Thus, in the above example, the resistance of the circuit equals

$$\frac{1}{\frac{1}{15} + \frac{1}{5} + \frac{1}{10}} = \frac{1}{0.0667 + 0.2 + 0.1} = \frac{1}{0.3667} = 2.73 \text{ ohms.}$$

The third fact to note about a parallel circuit is that:

**The resistance of a parallel combination is found by, first, finding the current through each branch. Then add these currents to find the total current through the combination. The resistance of the combination then equals the voltage on the combination divided by the total current through the combination. When the voltage across the combination is not known, use one volt.**

Stated in another way, the resistance of a parallel circuit is the reciprocal of the sum of the reciprocals of the resistances in the individual branches. This relation has already been expressed algebraically in equation (9).

**Example 9.** Resistors of 2 ohms, 3 ohms and 4 ohms are connected in parallel. What is the total resistance of the circuit?

Current (per volt) in 2 ohms = $\frac{1}{2}$ = 0.5 ampere or mho.
Current (per volt) in 3 ohms = $\frac{1}{3}$ = 0.333 ampere or mho.
Current (per volt) in 4 ohms = $\frac{1}{4}$ = 0.25 ampere or mho.

Total current per volt = 0.5 + 0.333 + 0.25 = 1.083 amperes or mhos.

$$\text{Resistance} = \frac{1}{1.083} = 0.923 \text{ ohm, or}$$

$$R = \frac{1}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = \frac{1}{0.5 + 0.333 + 0.25} = \frac{1}{1.083} = 0.923 \text{ ohm.}$$

**22. Parallel Circuit. Voltage, Current, Resistance.** The three facts which should be learned with regard to a parallel circuit may be tabulated as follows:

**VOLTAGE across a parallel circuit is the same as the voltage across each branch.**

**CURRENT in a parallel circuit equals the sum of the currents in the separate branches.**

**RESISTANCE of a parallel circuit is LESS THAN the resistance of the branch having the smallest resistance, and can be calculated by Ohm's Law;**

**or by the conductance method,** $R_t = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$.

**Prob. 40-1.** A parallel circuit has resistors in the several branches of 3, 4, 6 and 10 ohms, respectively. What is the conductance of the circuit? What is the resistance?

**Prob. 41-1.** If a pressure of 60 volts is impressed across the circuit in Prob. 40, what current will flow in each branch? What will be the total current through the circuit?

**Prob. 42-1.** What pressure is required to force 12 amperes through a parallel circuit consisting of three branches of 4, 12 and 18 ohms, respectively? What will be the current in each branch?

**Prob. 43-1.** A circuit, Fig. 45-1, consists of three parallel branches of 1 ohm, 2 ohms and 3 ohms, respectively. What is the resistance of the circuit?

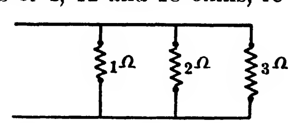



Fig. 45-1. The three resistors are in parallel.

**Prob. 44-1.** The sum of the currents in the 1-ohm and 2-ohm branches of the circuit in Prob. 43 is 8 amperes.

(*a*) What is the voltage across the circuit?

(*b*) What current is flowing through the 3-ohm branch?

**Prob. 45-1.** Three lamps, each having the same resistance, are connected in parallel across a 115-volt circuit. The sum of the currents taken by the three lamps is 1.2 amperes. What is the resistance of each lamp?

**23. Series Parallel Circuits.** As has been indicated in Art. 13 some circuits are neither purely series nor purely parallel, but a combination of the two. Many such circuits exist in practice. The general method of solving them is, first, to find the resistance of each parallel group of resistors, and second, to treat the result as a series circuit. That is, the circuit is reduced to the equivalent series circuit.

**Example 10.** Consider the circuit in Fig. 46–1, where two resistors, of 4 and 12 ohms in parallel, are connected in series with an 8-ohm resistor.

Conductance of parallel group $BC = \frac{1}{4} + \frac{1}{12} = 0.25 + 0.0833 = 0.3333$ mho.

Resistance of group $BC = \frac{1}{0.3333} = 3$ ohms.

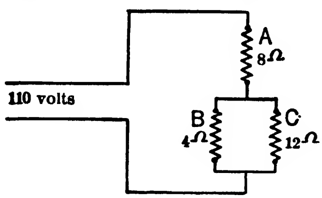



FIG. 46–1. The resistors, B and C in parallel, are in series with a third resistor A.

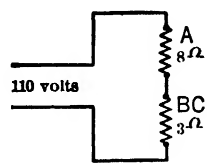



FIG. 47–1. The equivalent circuit to Fig. 46–1.

The circuit now is equivalent to that in Fig. 47–1, a series circuit.

Total $R$ is now $8 + 3 = 11$ ohms.
Total $I = \frac{110}{11} = 10$ amperes.
Voltage across $A = 10 \times 8 = 80$ volts.
Voltage across $BC = 10 \times 3 = 30$ volts.
Current through $A = 10$ amperes.
Current through $B = \frac{30}{4} = 7.5$ amperes.
Current through $C = \frac{30}{12} = 2.5$ amperes.

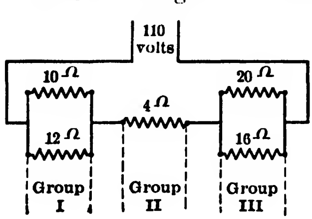



FIG. 48–1. A series-parallel circuit.

**Example 11.** Determine the total resistance and total current through the circuit in Fig. 48–1. Also determine the current through each resistor and the voltage across each resistor.

Resistance of Group I = $R_1$.

$$\frac{1}{R_I} = \frac{1}{10} + \frac{1}{12} = 0.1 + 0.0833 = 0.1833 \text{ mho}$$

$$R_I = \frac{1}{0.1833} = 5.45 \text{ ohms.}$$

Resistance of Group II = $R_{II}$ = 4.0 ohms.
Resistance of Group III = $R_{III}$

$$\frac{1}{R_{III}} = \frac{1}{20} + \frac{1}{16} = 0.05 + 0.0625 = 0.1125 \text{ mho.}$$

$$R_{III} = \frac{1}{0.1125} = 8.88 \text{ ohms.}$$

The equivalent series circuit now becomes that in Fig. 49–1.

Total Resistance = 5.45 + 4 + 8.88
= 18.33 ohms.

$$\text{Total Current} = \frac{110}{18.33} = 6 \text{ amperes.}$$

$$E_I = 6 \times 5.45 = 32.7 \text{ volts.}$$
$$E_{II} = 6 \times 4 = 24 \text{ volts.}$$
$$E_{III} = 6 \times 8.88 = 53.3 \text{ volts.}$$

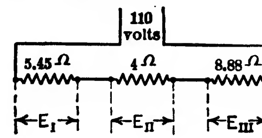



FIG. 49–1. The equivalent circuit to Fig. 48–1.

Now $E_I + E_{II} + E_{III}$ = 110 volts
and 32.7 + 24 + 53.3 = 110 volts — check.

$$I_{10} = \frac{32.7}{10} = 3.27 \text{ amperes.}$$

$$I_{12} = \frac{32.7}{12} = 2.73 \text{ amperes.}$$

Now $I_{10} + I_{12} = I_{total}$ = 6 amperes
and 3.27 + 2.73 = 6 amperes — check.

$$I_4 = I_{total} = 6 \text{ amperes.}$$

$$I_{20} = \frac{53.3}{20} = 2.67 \text{ amperes.}$$

$$I_{16} = \frac{53.3}{16} = 3.33 \text{ amperes.}$$

$$I_{20} + I_{16} = I_{total} = 6 \text{ amperes.}$$

2.67 + 3.33 = 6 amperes — check.

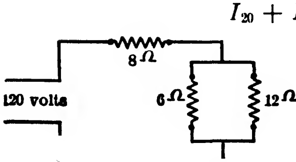



FIG. 50–1. A series-parallel circuit.

**Prob. 46–1.** (*a*) Find the total resistance and total current in the circuit in Fig. 50–1. (*b*) Find the current through each resistor, and the voltage across each resistor.

**Prob. 47–1.** In Fig. 51–1

Resistance of $A$ = 160 ohms.
Resistance of $B$ = 120 ohms.
Resistance of $C$ = 100 ohms.

Find:

(*a*) Current through each resistor.
(*b*) Resistance of parallel combination (*A* and *B*).
(*c*) Combined resistance of system.
(*d*) Voltage across each resistor.

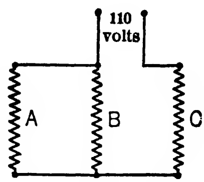



FIG. 51–1. Resistor C is in series with a parallel combination of A and B.

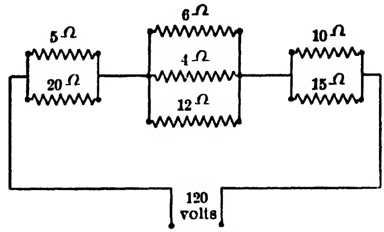



FIG. 52–1. A more complicated series-parallel circuit.

**Prob. 48–1.** In Fig. 52–1 find (*a*) total resistance of the circuit: (*b*) total current; (*c*) current through each resistor; (*d*) voltage across each resistor.

**Prob. 49–1.** In Fig. 53–1 the voltage from *A* to *B* is 50 volts. Current through resistance $x$ is 3.2 amperes.

Resistance $y$ = 5 ohms.
Resistance $z$ = ·4 ohms.

Find:

Current through $y$. Resistance of $x$.
Current through $z$. Voltage from $B$ to $C$.

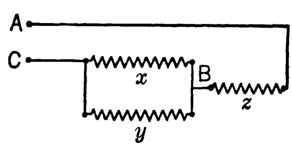



FIG. 53–1. Resistor $z$ is in series with the parallel combination of $x$ and $y$.

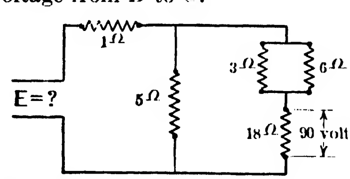



FIG. 54–1. There is 90 volts across the 18-ohm resistor.

**Prob. 50–1.** In Fig. 54–1, the voltage across the 18-ohm resistor is 90 volts. (*a*) What is the total voltage across the combined circuit?
(*b*) What is the current through each resistor?
(*c*) What is the total resistance of the combined circuit?

**24. Electromotive Force and Terminal Voltage.** Every generator must in its windings contain resistance, which is called the internal resistance of the generator. Where the generator is connected to an external circuit this internal resistance forms part of the resistance of the complete electric circuit. It is evident,

then, that the generator must generate enough voltage to force the current through both this internal resistance and the resistance of the external circuit. This generated voltage is called the **Electromotive force** of the generator. The term is usually abbreviated to the letters **emf.**

The voltage necessary to force the current through the resistance of the external circuit is called the **Terminal voltage,** and is the voltage which is measured by a voltmeter connected to the terminals of the generator. It is important that we clearly understand that the terminal voltage of a generator (or battery) is the voltage across the external, or outside, circuit, and is **not** the voltage developed by the machine (unless the outside circuit is open).

Note the following examples:

**Example 12.** The generator in Fig. 55–1 has an internal resistance of 0.2 ohm and is to force 6 amperes through an external circuit consisting of two resistors of 8 ohms and 10 ohms, respectively, connected in series.

(*a*) What is the electromotive force of the generator?

(*b*) The terminal voltage?

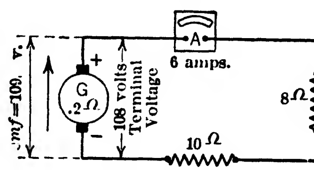



FIG. 55–1. The two resistors are in series with the internal resistance of the generator.

**Solution.**

(*a*) Total resistance of complete circuit =

$$0.2 + 8 + 10 = 18.2 \text{ ohms.}$$

Voltage necessary to send 6 amperes through the circuit, or the **electromotive force** of the generator =

$$6 \times 18.2 \text{ ohms} = 109.2 \text{ volts.}$$

This is the total voltage which must be developed by the generator.

(*b*) Voltage necessary to force 6 amperes through the external circuit, or the **terminal voltage** =

$$6\ (8 + 10) = 6 \times 18 = 108 \text{ volts.}$$

Terminal voltage can also be found by subtracting from the emf, the voltage necessary to send the 6 amperes through the 0.2 ohm internal resistance.

Thus $109.2 - 6 \times 0.2 = 109.2 - 1.2 = 108$ volts (terminal voltage).

**Example 13.** A storage battery, Fig. 56–1, has an electromotive force of 2.2 volts and an internal resistance of 0.02 ohm. It is connected through a switch to an external resistance of 0.42 ohm.

When the switch is open, the voltmeter across the battery terminals measures 2.2 volts, or the emf of the battery (no current flowing in the circuit).

When the switch is closed the resistance of the complete circuit is $0.02 + 0.42 = 0.44$ ohm.

$$\text{Current flowing} = \frac{2.2}{0.44} = 5 \text{ amperes.}$$

To force 5 amperes through 0.02 ohm internal resistance requires

$$0.02 \times 5 = 0.1 \text{ volt.}$$

The voltmeter will now read, $2.2 - 0.1 = 2.1$ volts, terminal voltage. Or, voltage across the external circuit =

$$5 \times 0.42 \text{ ohm} = 2.1 \text{ volts (terminal voltage).}$$

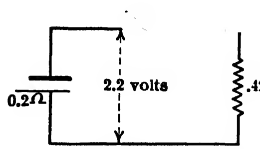



FIG. 56–1. A battery with an emf of 2.2 volts, connected through a switch to a resistor.

Note that when the external circuit is **open** the voltmeter across the terminals of the generator (or battery) measures emf. When the external circuit is **closed** the voltmeter measures the terminal voltage, or voltage across the external circuit; this voltage is always less than the emf.

**Prob. 51–1.** If the emf of the generator in Fig. 55–1, Example 12, remains at 109.2 volts after the external resistance has been changed so that 8 amperes flows in the circuit, what will be the terminal voltage? What will be the external resistance?

**Prob. 52–1.** In Fig. 57–1, the ammeter reads 2.6 amperes; the voltmeter reads 30 volts; internal resistance of the generator is 0.5 ohm. Find:

(*a*) Resistance of *R*.
(*b*) Voltage across the 36-ohm resistance.
(*c*) Voltage across the 42-ohm resistance.
(*d*) Terminal voltage of generator.
(*e*) Emf of generator.

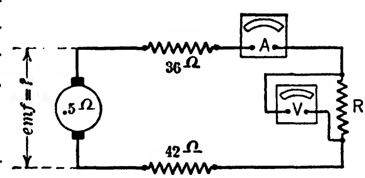



FIG. 57–1. The voltmeter measures the voltage across resistor R, and the ammeter measures the current through R.

## SUMMARY OF CHAPTER I

**ELECTRICITY has been found to flow as a current through a conductor much as water flows through a pipe.**

**AN ELECTRIC CIRCUIT is a path in which ELECTRICITY can flow.**

**THE CURRENT of electricity is measured in AMPERES, which state the QUANTITY of electricity passing through a conductor IN ONE SECOND.**

**THE PRESSURE which causes the current to flow is measured in VOLTS; corresponds to "pounds per square inch."**

**THE RESISTANCE which a conductor offers to the current is measured in OHMS; corresponds to FRICTION in a water pipe.**

RESISTORS are conductors usually made of iron or nickel alloys, or German silver, in the form of wire or ribbon, or cast in the form of grids.

RHEOSTATS are resistors arranged so that the amount of the resistance in the circuit can be adjusted.

When two points in an electric circuit are marked, one (+) and the other (−), it indicates that the current will flow along the circuit from the (+) to the (−), from a higher potential to a lower potential, if there are no generators or batteries between the points so marked.

OHM'S LAW states the relation which exists in an electric circuit regarding current, pressure and resistance. It is written in THREE forms.

1. $\text{Amperes} = \dfrac{\text{Volts}}{\text{Ohms}}$, or $\text{Current} = \dfrac{\text{Pressure}}{\text{Resistance}}$, or $I = \dfrac{E}{R}$.

2. Volts = Amperes × Ohms, or Pressure = Current × Resistance, or $E = IR$.

3. $\text{Ohms} = \dfrac{\text{Volts}}{\text{Amperes}}$, or $\text{Resistance} = \dfrac{\text{Pressure}}{\text{Current}}$, or $R = \dfrac{E}{I}$.

CURRENT is measured by INSERTING a low resistance AMMETER INTO the circuit.

VOLTAGE is measured by TAPPING a high resistance VOLTMETER across TWO POINTS in the circuit.

RESISTANCE is found by dividing the VOLTMETER reading by the AMMETER reading according to Ohm's Law. These voltmeter and ammeter readings must be taken at the SAME TIME.

CAUTION. Be careful not to tap an ammeter to a circuit as you would a voltmeter. Always BREAK the circuit and INSERT the ammeter.

Electrical pieces, or resistors, connected in TANDEM, or END TO END, are said to be in SERIES.

Electrical pieces, or resistors, connected SIDE BY SIDE, so that the current DIVIDES between them, are said to be in PARALLEL.

## Series Combination

CURRENT through a series combination is the SAME as the current through the separate parts.

RESISTANCE of a series combination is the SUM of the resistances of the separate parts.

VOLTAGE across a series combination is the SUM of the voltages across the separate parts.

## Parallel Combination

VOLTAGE across a parallel combination is the SAME as the voltage across each branch.

CURRENT through a parallel combination is the SUM of the currents through the separate branches.

RESISTANCE of a parallel combination is LESS than the resistance

**of the branch of smallest resistance. It is found by using Ohm's Law or by the conductance method.**

**Can be written algebraically as** $R_t = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$.

## Series Parallel Combination

**Some circuits are a combination of both series and parallel circuits. Can be solved by reducing the circuit to the EQUIVALENT series circuit. Then solving this as a series circuit.**

**Ohm's Law applied to any electric circuit always means: The amperes through any PART of a circuit equal the volts across that same PART of the circuit, divided by the ohms of that same PART of the circuit.**

**ELECTROMOTIVE FORCE, abbreviated EMF, is the voltage developed by a battery or by a generator.**

**TERMINAL VOLTAGE is the voltage at the terminals of a battery, or generator, when it is supplying current to a circuit. The terminal voltage is always the voltage across the circuit outside of the generator; not the emf of the generator (unless the outside circuit is open).**

## PROBLEMS ON CHAPTER I

As a visual aid in solving the following problems, the student is advised, wherever possible, to draw a diagram of the circuit, on which he should indicate all given data.

**Prob. 53–1.** What voltage must a generator develop to supply a current of 30 amperes through an electroplating vat? The circuit has a virtual resistance of 0.18 ohm.

**Prob. 54–1.** If the resistance of an electric bell is 65 ohms and it requires 0.2 ampere to ring it, will a battery of 10 volts be sufficient?

**Prob. 55–1.** Which resistance is the greater, one which requires 15 volts to force 6.26 amperes through it, or one which requires 550 volts to force 285 amperes through it?

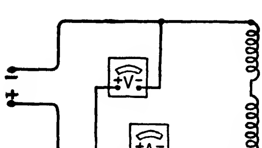



Fig. 58–1. The voltmeter indicates the voltage across the motor field coils and the ammeter, the current through them.

**Prob. 56–1.** The field coils of a motor, Fig. 58–1, have a resistance of 420 ohms. What will the ammeter indicate if the voltmeter indicates a pressure of 220 volts?

**Prob. 57–1.** An ammeter made to measure 5 amperes has a resistance of 0.009 ohm. How much current will flow through the ammeter, if by mistake, it is used as a voltmeter across 115 volts?

**Prob. 58–1.** A voltmeter made to measure 150 volts has a resistance of 15,000 ohms. How much current flows through it when it is placed across a 115-volt circuit?

**Prob. 59–1.** In Fig. 59–1, the voltmeter, $V$, indicates 5 volts and the ammeter, $A$, 50 amperes. What is the resistance of the motor armature? The armature is held stationary

**Prob. 60–1.** What is the resistance of the rheostat, $R$, in the circuit in Fig. 59–1?

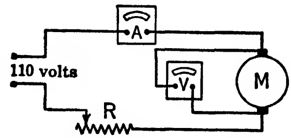



FIG. 59–1. The resistance of the motor armature can be computed from the readings of the voltmeter and ammeter.

**Prob. 61–1.** A 220-volt lamp has a resistance of 484 ohms. A 110-volt lamp has a resistance of 225 ohms. Which takes the larger current when operating on its proper voltage?

**Prob. 62–1.** In Fig. 60–1, the resistance of $R$ is 21.3 ohms. The voltmeter, $V$, reads 43.6 volts. What should the ammeter read?

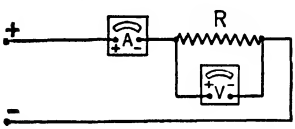



FIG. 60–1. Given the resistance of R and the reading of the voltmeter to compute the reading of the ammeter.

**Prob. 63–1.** An arc lamp has a virtual resistance of 12.5 ohms and requires a current of 6.5 amperes. How much resistance must be added to it, if it is to take just 6.5 amperes when operating on a 110-volt line?

**Prob. 64–1.** A short circuit is made accidentally by placing a wire of 0.010 ohm resistance across 110 volts. What was the momentary current in the short-circuiting wire?

**Prob. 65–1.** A magneto-generator for ringing a telephone bell develops a pressure of 50 volts at its terminals when it is connected to a circuit consisting of a 15-ohm line and a 300-ohm bell. What current flows in the circuit?

**Prob. 66–1.** In Fig. 61–1, all three resistors have the same resistance. Voltmeter, $V$, reads 112 volts. Ammeter, $A$, reads 0.45 ampere.
Find:

(*a*) Resistance of each resistor.
(*b*) Current in the main line.
(*c*) Voltage across each resistor.
(*d*) Current through each resistor.

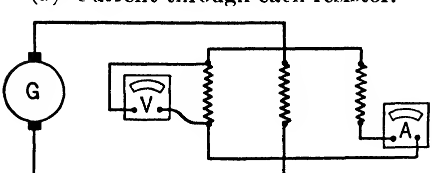



FIG. 61–1. Three identical resistors are in parallel.

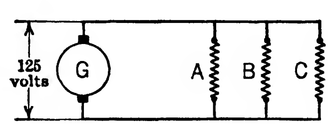



FIG. 62–1. Three parallel resistors.

**Prob. 67–1.** In Fig. 62–1, resistors $A$, $B$ and $C$ are in parallel across a 125-volt circuit. Resistance of $A$ = 60 ohms; resistance of $B$ = 40 ohms; resistance of $C$ = 90 ohms.

Find:
(*a*) Voltage across each.
(*b*) Current through each.
(*c*) Resistance of the combination.
(*d*) Current through the combination.

**Prob. 68-1.** (*a*) What is the resistance of 8 lamps of 226 ohms each, connected in parallel across 110 volts? (*b*) What is the conductance of the combination? (*c*) What is the current through each lamp? (*d*) What is the total current in the circuit?

**Prob. 69-1.** A circuit has three parallel branches of 6, 8 and 12 ohms, respectively. If 5 amperes flow in the 12-ohm branch, what current will flow in each of the others? What is the resistance of the combined circuit?

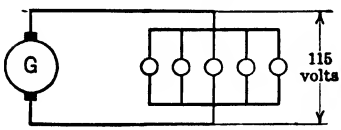


Fig. 63-1. The five lamps are in parallel.

**Prob. 70-1.** The 5 lamps in Fig. 63-1 together take a total current of 3.5 amperes. The lamps are alike and have 115 volts across them. (*a*) What is the current through each lamp? (*b*) The resistance of each lamp? (*c*) The resistance of the combination?

**Prob. 71-1.** A divided circuit has two branches of 1 ohm and 0.25 ohm, respectively. What is the conductance of the combined circuit? What is the resistance?

**Prob. 72-1.** The ammeter in Fig. 64-1 reads 5.5 amperes. If the voltage across each arc lamp is 80 volts, what is the voltage across the resistor, *R*? What is the resistance of *R*? What is the resistance of each lamp? Voltage across the line is 450 volts.

**Prob. 73-1.** If one lamp in Prob. 72, Fig. 64-1, becomes short-circuited, what resistance will *R* have to be made to keep the current at 5.5 amperes?

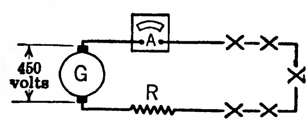


Fig. 64-1. Five arc lamps in series with resistor R.

**Prob. 74-1.** Three resistors, one of 18 ohms, one of 12 ohms, and a third unknown, are connected in parallel. The resistance of the combination is 4.32 ohms. What is the resistance of the third resistor?

**Prob. 75-1.** If the 18-ohm resistor of Prob. 74 is carrying 3.2 amperes, how much current is each of the other two resistors carrying?

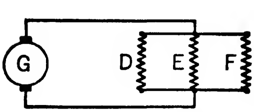


Fig. 65-1. The three resistors carry a total current of 12.5 amperes.

**Prob. 76-1.** What pressure will be required to force 15 amperes through a parallel combination of 8 ohms, and 20 ohms?

**Prob. 77-1.** In Fig. 65-1, resistor *D* has a resistance of 27 ohms, resistor *E* has a resistance of 75 ohms and a current of 1.53 amperes. The total current through the combination is 12.5 amperes.

Find:
(*a*) Voltage across the combination.
(*b*) Current through the resistor *F*.
(*c*) Resistance of *F*.
(*d*) Resistance of the combination.

**Prob. 78–1.** The voltage between the trolley wire and the rails of an electric railway is 600 volts. A car is to be lighted by five 110-volt, 0.25-ampere tungsten lamps connected in series, as in Fig. 66–1. How much resistance must be connected in series with these lamps to make them operate properly on this circuit?

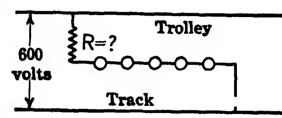


Fig. 66–1. Five lamps connected in series between the trolley wire and the track.

**Prob. 79–1.** What voltage is required across a line on which there are 10 arc lamps in series, each of 13 ohms virtual resistance? The line wires have a resistance of 3 ohms. Lamps require a current of 6.5 amperes.

**Prob. 80–1.** Three resistors, one of 13.5 ohms, one of 3.2 ohms, and a third of 23.75 ohms are connected in series across a 125-volt circuit. If 1.75 amperes flows in the circuit what must be the resistance of the wires used to connect the pieces?

**Prob. 81–1.** The resistance of a parallel circuit of two branches is 6 ohms. The resistance of one of the branches is 16 ohms. What is the resistance of the other?

**Prob. 82–1.** If 5.5 amperes flows in the 16-ohm branch in Prob. 81, what total current flows through the combination?

**Prob. 83–1.** The average resistance of the human body is 10,000 ohms. About 0.1 ampere through the body is usually fatal. From these data what voltage would ordinarily be sufficient to kill a person?

**Prob. 84–1.** A rheostat, Fig. 67–1, is connected in series with a 40-ohm field coil. How much must the resistance of the rheostat be increased to reduce the current from 1.75 amperes to 1.34 amperes?

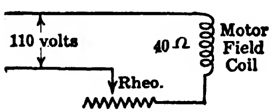


Fig. 67–1. A rheostat in series with a field coil to control the current.

**Prob. 85–1.** Three resistors of 6 ohms, 4 ohms and 18 ohms are connected in series. There is a pressure of 50 volts across the 4-ohm resistor. (*a*) What is the voltage across the entire circuit? (*b*) What is the voltage across each of the other two resistors?

**Prob. 86–1.** What voltage would be required to send 8 amperes through the circuit in Prob. 85? What would be the voltage across each resistor in this case?

**Prob. 87–1.** (*a*) What would be the combined resistance of the resistors in Prob. 85 if they were joined in parallel? (*b*) What voltage would be required to send 8 amperes through this parallel circuit? (*c*) What would be the current through each resistor?

**Prob. 88–1.** A circuit consists of the following 4 parts joined in series. First part: 2 coils, one of 6 ohms and one of 8 ohms connected in parallel. Second part: a wire of 0.65 ohm resistance. Third part: 8 lamps in parallel each having 220 ohms resistance. Fourth part: a wire of 0.8 ohm resistance. (*a*) What voltage is required to send 4 amperes through this circuit? (*b*) What will be the voltage across each part?

**Prob. 89–1.** A storage battery has an emf of 2.2 volts. What is the terminal voltage when it is delivering 20 amperes, if the internal resistance is 0.0025 ohm?

**Prob. 90–1.** What is the resistance of the circuit in Fig. 68–1?

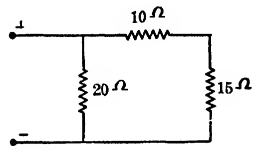



Fig. 68–1. Find the resistance of the circuit.

**Prob. 91–1.** Find the current through each resistor in Fig. 69–1.

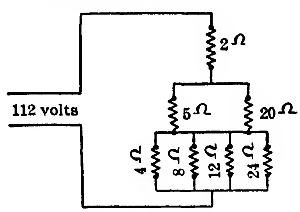



Fig. 69–1. Find the current through each resistor.

**Prob. 92–1.** What is the total resistance of the circuit in Fig. 70–1?

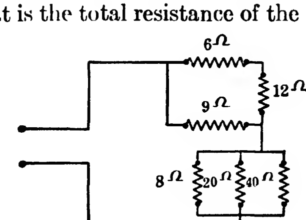



Fig. 70–1. Find the total resistance of the combined circuit.

# CHAPTER II

## DISTRIBUTING SYSTEMS

When water, or gas, or electricity is delivered to our house, the important factor is the pressure at which each of these services is rendered.

We pay for water delivered at a certain pressure to the pipes in our house. If the pressure is too low, we are unable to get the water we need — if too high, our piping may leak or even burst.

If the gas pressure is too low, we cannot get the necessary heat for cooking — if too high, there is danger of leakage in the piping, or the burner blows itself out.

Also, if the electrical pressure, voltage, is low, our lights burn dimly — if too high, they soon burn out.

In any of the above cases, then, if there is much variation in the pressure, we say the service is poor.

Thus the voltage at which electricity is delivered for use is of prime importance. And, in any circuit, or system, for the distribution of electricity, it is necessary to be able to calculate the voltage distribution throughout the system in order to determine what the pressure at the lamps, or other apparatus, will be. We need, also, to know the voltage the generator must develop, in order to have the proper voltage across the lamps, or load.

**1. Simple Parallel Lighting System. Line Drop.** In practice, it is seldom that we find a simple series circuit or a simple parallel circuit. The two arrangements are usually combined into a more or less complicated system. But by considering each part of the circuit by itself and applying Ohm's law to each part separately, it is usually easy to find the current, voltage, and resistance distribution throughout the entire circuit of any electrical system. The same rules we have already learned apply, and apply in the same way. A little practice is all that is needed in order to solve even the more difficult of such arrangements.

For instance, consider the following example:

**Example 1.** In the lighting system, shown in Fig. 1–2, each lamp takes 1 ampere. The line wires *AB* and *CD* each have a resistance of

0.5 ohm. We wish to find the voltage which the generator must develop at its terminals, to force this current through the lamps.

**Solution.** The parallel combination of the 6 lamps is really in series with the two line wires, and there are three definite parts to the circuit, the line wire $AB$, the lamps, and the line wire $CD$.

First — We find the current distribution throughout the entire circuit, and mark it on the diagram as in Fig. 1–2. We start with the

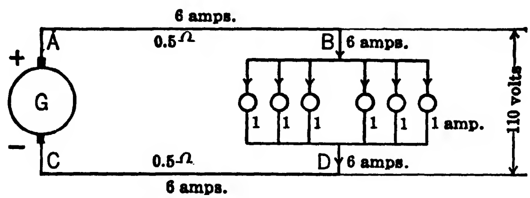



FIG. 1–2. The current in the line wires equals the sum of the currents in the lamps.

current in the lamps. Since they are in parallel, the total current taken by the lamps is $6 \times 1$ or 6 amperes. Now, since the line wires $AB$ and $CD$ are both in series with the lamps, 6 amperes flows through each wire, as well as through the generator. We now have the current distribution.

Second — We can now find the voltage distribution in the same way. The voltage necessary to force 6 amperes through the line wire $AB = 6 \times 0.5 = 3$ volts. Thus it requires 3 volts to force the current out to the lamps. Similarly, it requires 3 volts to force the current from the lamps back to the generator through the line wire $CD$. Thus we have the voltages across the three parts of the circuit which are in series. The voltage across the whole series system is then the sum of the voltages across the separate parts. The voltage distribution is shown in Fig. 2–2.

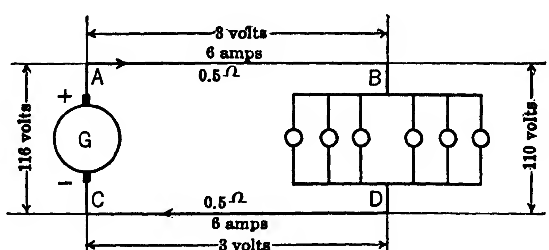



FIG. 2–2. The voltage at the generator terminals equals the voltage across the lamps plus the voltage drop in the two line wires.

Thus voltage across the system = voltage across $AB$ + voltage across the lamps + voltage across $CD$, or:

$$3 + 110 + 3 = 116 \text{ volts.}$$

Therefore, the generator must develop 116 volts at its terminals, in order to have 110 volts across the lamps, since 6 volts are used in forcing the current out to the lamps and back to the generator.

The volts used to send the current through the line wires (3 + 3 or 6 volts) is said to be the " volts used in the line " or the " Line Drop."

The voltage drops and potential differences in Fig. 2-2 are shown graphically in Fig. 3-2. The voltage at the generator is 116 volts. Due to the resistance or the " line drop " in line *AB*, the potential at point *B* has dropped 3 volts below that at *A*; and due to the " line

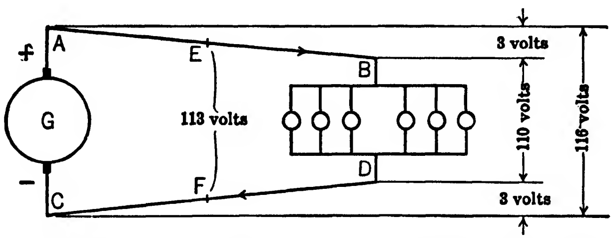



FIG. 3-2. Graphical representation of the voltage drops in Fig. 2-2.

drop " in line *CD*, the point *D* has risen 3 volts above that at *C*. This is so, because there must be a difference of potential between two points in a circuit, in order that current will flow (Art. 8), and current always flows from a higher to a lower potential. This leaves a difference of potential between points *B* and *C* of 110 volts which is the voltage across the lamps.

The voltage drop is uniform along the wires increasing from 0 to 3 volts in each wire. At the points *EF*, halfway from the generator, the drop in each wire is 1.5 volts and a voltmeter, placed across these points, will read 113 volts.

**Note, from the above example, that the general method of attack for this type of problem may be summarized as follows:**

**First.** Find the current distribution throughout the circuit.

**Second.** Find the voltage distribution throughout the circuit.

**Third.** Combine the voltages according to the rules for a series circuit.

Also note that the voltage at the lamps (the load) is less than at the terminals of the generator.

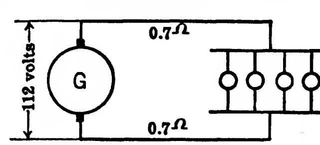



FIG. 4-2. Given the current in each lamp, to find the voltage across the lamps.

**Prob. 1-2.** The generator in Fig. 4-2 has a terminal voltage of 112 volts, and each lamp takes 1 ampere. (*a*) What is the "line drop"? (*b*) What is the voltage across the lamps?

**Prob. 2–2.** Two motors are operating in a shop at the end of a line as indicated in Fig. 5–2. The voltage across the motors is 220 volts and the generator terminal voltage is 230 volts. Motor I takes 20 amperes and motor II takes 15 amperes. (*a*) What is the line drop? (*b*) What is the resistance of each line?

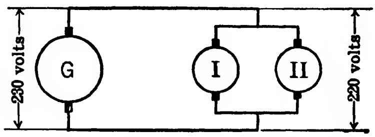



Fig. 5–2. Two motors in parallel at the end of a line.

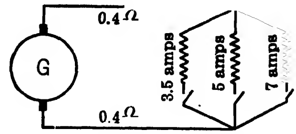



Fig. 6–2. Three electric heaters in parallel.

**Prob. 3–2.** Three electric heaters, Fig. 6–2, taking 3.5, 5 and 7 amperes, respectively, are operated from line wires having a resistance of 0.4 ohm each. What is the line drop: (*a*) When the 3.5-ampere heater alone is used? (*b*) When both the 5-ampere and 7-ampere heaters are used together? (*c*) When all three heaters are in use?

**2. More Complicated Grouping. Line Drop.** In Example 1, and the problems above, it will be noted that the lamps (or loads) are concentrated at one place in the circuit, and that all parts of the line wires carry the same current.

Systems are usually arranged with the loads grouped at different points; as, for instance, groups of lights in two different houses. In such systems the different sections of the line wires carry different currents, the sections nearest the generator carrying more than the sections farther away.

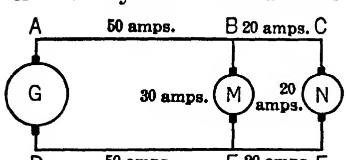



Fig. 7–2. The current delivered to the line by the generator is the sum of the currents taken by each of the motors.

Consider Fig. 7–2. The system is so arranged that motor $M$, taking 30 amperes, is much nearer the generator than motor $N$, which takes 20 amperes. The line wires $BC$ and $FE$ must carry 20 amperes to supply motor $N$. But the line wires $AB$ and $ED$ must carry enough current to supply both 20 amperes to motor $N$, and 30 amperes to motor $M$, or 50 amperes. At point $B$, this current divides, 30 amperes going through motor $M$, and 20 amperes flowing on through wire $BC$, and through motor $N$. Similarly, at point $E$, the two currents join and 50 amperes flow back through the wire $ED$ to the generator.

The circuit in Fig. 7–2 is very similar to the water circuit in Fig. 8–2 — the pipes, $M$ and $N$, carrying respectively 30 and 20

gallons per second. The section of the pipe line $AB$ and $ED$ must carry 50 gallons per second to supply both pipes $M$ and $N$, while the sections $BC$ and $FE$, supplying pipe $N$, have to carry but 20 gallons per second. At point $B$, the current of 50 gallons per second divides, and 30 gallons per second flows down through $M$,

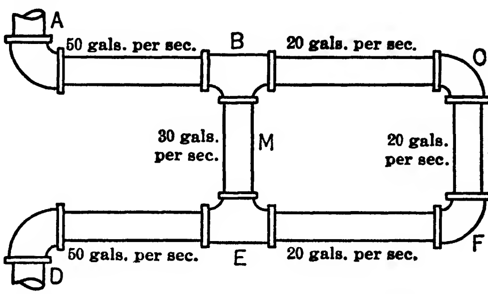



FIG. 8–2. The current flowing into the system at A, and out at D, equals the sum of the currents through pipes M and N.

while 20 gallons per second flows on through pipe $BC$ and through pipe $N$. Similarly, at point $E$, the two currents join, and pipe $ED$ carries 50 gallons per second, which pours out the end of the pipe at $D$

The method for solving this type of system for line drop and voltage on the loads is shown in the example below.

**Example 2.** Consider Fig. 9–2, which is the same as Fig. 7–2 except that the resistance of the line wires and terminal voltage of the generator are given. Resistance of line wires $AB$ and $DE$ is each 0.08 ohm. Resist-

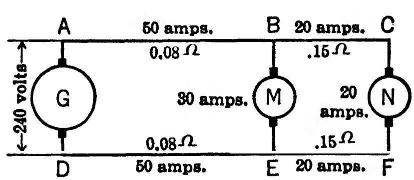



FIG. 9–2. The voltage at the generator minus the line drop between G and M is the voltage across M. The voltage across M minus the line drop between M and N is the voltage across N.

ance of lines $BC$ and $EF$ is each 0.15 ohm. Terminal voltage of the generator is 240 volts. We desire to find the line drop and the voltage across each motor.

**Solution.**

First step. Current Distribution. As in Fig. 7–2, the current in the wires $BC$ and $EF$ is 20 amperes, and the current in line wires $AB$ and $DE$ is 50 amperes.

Second step. Voltage Drop in Line.

$$\begin{aligned}
\text{Voltage drop in line } AB &= 50 \times 0.08 = 4 \text{ volts.}\\
\text{Voltage drop in line } ED &= 50 \times 0.08 = \underline{4} \text{ volts.}\\
\text{Voltage drop in first section of line} &= 8 \text{ volts.}
\end{aligned}$$

That is, it takes 4 volts to force the current of 50 amperes out to point *B*, and also 4 volts to force the current back to the generator from point *E*.

Third step. Voltage on the first motor.

Since the voltage at the generator terminals is 240 volts and 8 volts are used in forcing current through the first section of the line, the voltage across the first motor, *M*, is

$$240 - 8 = 232 \text{ volts.}$$

In the same way:

$$\begin{aligned}
\text{Voltage drop in line } BC &= 20 \times 0.15 = 3 \text{ volts.}\\
\text{Voltage drop in line } EF &= 20 \times 0.15 = \underline{3} \text{ volts.}\\
\text{Voltage drop in second section of line} &= 6 \text{ volts.}
\end{aligned}$$

That is, it takes 6 volts to force 20 amperes from point *B* out to point *C* and back again from point *F* to point *E*.

Since the voltage across the points *BE*, at motor *M*, is 232, and 6 volts are used in forcing current through the second section of the line, the voltage across the motor, *N*, is

$$232 - 6 = 226 \text{ volts.}$$

Since the voltage at the generator terminals is 240 volts and the voltage on the load at the end of the line is 226 volts, the total line drop is

$$240 - 226 = 14 \text{ volts}$$

$$\begin{aligned}
\text{or drop in first section of line} &= 8 \text{ volts}\\
\text{and drop in second section of line} &= \underline{6} \text{ volts.}\\
\text{Then — total line drop} &= 14 \text{ volts.}
\end{aligned}$$

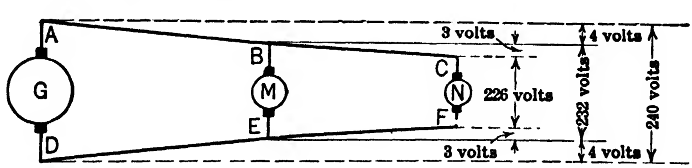



FIG. 10–2. Graphical representation of voltage distribution in Fig. 9–2, Example 2.

The voltage distribution throughout the entire circuit of Example 2 is shown graphically in Fig. 10–2.

Consider Fig. 11–2, which more nearly represents a distribution system for lighting than any yet discussed.

Here, we have the lamps in two groups, at different distances from the generator, and each group made up of several lamps in parallel. Each lamp is here assumed to take a current of one ampere.*

Group I then takes 4 amperes and Group II takes 3 amperes.

The generator must then supply 7 amperes to the two groups, and the sections of the line *AB* and *DE* must also carry 7 amperes,

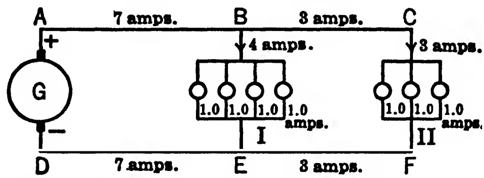



FIG. 11–2. The current through AB and DE is the sum of the currents through Groups I and II.

while the sections *BC* and *EF*, supplying Group II only, carry 3 amperes.

In solving circuits, or systems, similar to those in Figs. 9–2 and 11–2, it is convenient to start with the current in the load farthest from the generator.

The method of solving systems of this type for line drop is shown in Examples 3 and 4 below.

*First Illustration.* When the voltage at the generator is known and the voltage on the loads is to be determined:

**Example 3.** The voltage at the terminals of the generator in Fig. 12–2 is 125 volts. Resistance of line wires *AB* and *ED* is 0.2 ohm each. Resistance of line wires *BC* and *FE* is 0.3 ohm each. Each lamp takes 1 ampere. (*a*) What is the line drop in each section of the line? (*b*) What are the voltages at the loads?

**Solution** (Remember the 3 steps in the solving of distribution systems).

First — Current Distribution (starting with the load farthest from the generator).

Current in Group II $= 4 \times 1 = 4$ amperes.
Hence current in $BC$ = current in $EF$ $= 4$ amperes.
Current in Group I $= 4 \times 1 = 4$ amperes.
Hence current in $AB$ = current in $DE$ $= 4 + 4 = 8$ amperes.

* Modern incandescent lamps are usually installed in parallel. The resistances of all the lamps, even of the same make and the same rating, are not exactly the same. Nor is the voltage across all the lamps when installed the same. Still, for the convenience in calculating the voltage necessary to send current through the line, that is, the " line drop," each lamp is assumed to take the same current, if the lamps have the same rating. The error, introduced by this assumption, is usually too small to be taken into account.

*Second — Voltage Distribution.*

Voltage drop in line $AB = 8 \times 0.2 = 1.6$ volts.
Voltage drop in line $DE = 8 \times 0.2 = 1.6$ volts.
Voltage drop in lines between generator and Group I, or **Line Drop** in first section of line $= 1.6 + 1.6 =$ **3.2 volts.**
Voltage drop in line $BC = 4 \times 0.3 = 1.2$ volts.
Voltage drop in line $EF = 4 \times 0.3 = 1.2$ volts.
Voltage drop in lines between Groups I and II or **Line Drop** in second section of line $= 1.2 + 1.2 =$ **2.4 volts.**

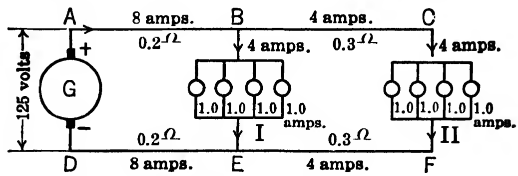



FIG. 12–2. The voltage across Groups I and II is to be determined.

Third — Voltage on Group I $= 125 - 3.2 = 121.8$ volts.

This is also the voltage on the rest of the circuit from $B$ to $C$, Group II, and from $F$ to $E$, therefore, voltage on Group II $= 121.8 - 2.4 = 119.4$ volts.

Note that the voltage at the loads is less than at the generator and that the voltage on Group II is less than on Group I.

*Second Illustration.* When the voltage on the load farthest from the generator is known, and the voltage at the other load and at the generator must be determined:

**Example 4.** The voltage on the lamps in Group II, Fig. 13–2, is 110 volts. Each lamp takes 1 ampere. Resistance of line wires, $BC$ and $EF$, are 0.4 ohm each. Resistance of line wires $AB$ and $DE$ are 0.2 ohm each. What is the **line drop** in each section of the line? What is the voltage across the lamps in Group I, and the terminal voltage of the generator?

**Solution** (Again apply the same three steps in the solution).

First — Current Distribution.

Current in Group II = current in line $BC$ = current in line $EF = 3$ amperes.
Current in Group I $= 4$ amperes. Hence the current in line $AB$ = current in $DE = 4 + 3 = 7$ amperes.

Second — Voltage Distribution.

Voltage drop in line $BC = 3 \times 0.4 = 1.2$ volts.
Voltage drop in line $EF = 3 \times 0.4 = 1.2$ volts.
Voltage drop in second section of line or line drop between Groups II and I $= 1.2 + 1.2 = 2.4$ volts.
Voltage drop in line $AB = 7 \times 0.2 = 1.4$ volts.
Voltage drop in line $DE = 7 \times 0.2 = 1.4$ volts.
Line drop between Group I and generator equals $1.4 + 1.4 = 2.8$ volts.

Third — Voltage on Group I = 110 + 2.4 = 112.4 volts.

Voltage at the terminals of generator = 112.4 + 2.8 = 115.2 volts.

Note again that the voltage on the loads is less than at the generator, and that the voltage at Group II is less than at Group I.

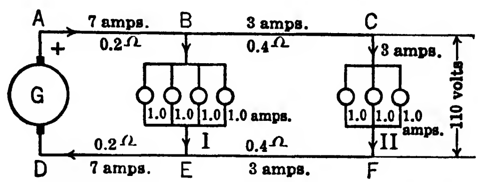



FIG. 13-2. The voltage across Group I equals the voltage across Group II plus the line drop between Groups I and II. Voltage at the generator equals the voltage across Group I plus the line drop between Group I and G.

**Prob. 4-2.** The motor in Fig. 14-2 takes 15 amperes, and the lamps take 0.8 ampere each. (*a*) What is the current in each section of the line? (*b*) Line drop in each section of the line? (*c*) Voltage across motor? (*d*) Voltage across the lamps?

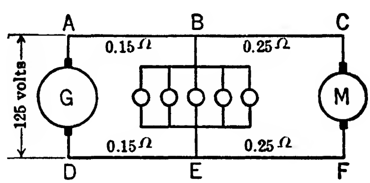



FIG. 14-2. The current through the motor and the lamps is known. To determine the voltage across each.

**Prob. 5-2.** What is the resistance of each lamp in Prob. 4?

**Prob. 6-2.** Each lamp in Fig. 15-2 takes a current of 1.5 amperes, and the voltage across *BE* is 115 volts. (*a*) What is the current in each

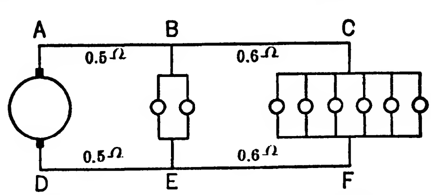



FIG. 15-2. The voltage at BE is known. To determine the voltage at AD, and at CF.

section of the line? (*b*) Line drop in each section of the line? (*c*) Voltage across *CF*? (*d*) Voltage across *AD*?

**Prob. 7-2.** What is the resistance of each lamp in Prob. 6?

## SUMMARY OF CHAPTER II

**Distribution systems, which comprise combinations of series and parallel arrangements, can be separated into definite sections. Then the current, voltage, and resistance distribution can be determined by applying Ohm's law to each part separately.**

**METHOD. (First Step.) Find the current distribution, beginning at the point farthest from the generator.**

**(Second Step.) Compute, by means of Ohm's law, the line drop.**

**(Third Step.) Combine the line drop with known voltages, according to the rules for series and parallel circuits.**

**In computing the current taken by lamp loads, all lamps of the same rating are each assumed to take the same current, even though the voltage across all of them may not be the same.**

## PROBLEMS ON CHAPTER II

As an aid in solving the following problems, draw a diagram of the circuit, and mark the current distribution on the diagram. Then compute the line drop in the different sections of the line, and mark it on the diagram.

**Prob. 8-2.** The voltage across the generator in Fig. 16-2 is 600 volts. Car I takes 65 amperes. Car II takes 70 amperes. Car III takes 50 amperes.

Resistance of trolley wire between generator and Car I = 0.4 ohm.
Resistance of trolley wire between Car I and Car II = 0.6 ohm.
Resistance of trolley wire between Car II and Car III = 0.8 ohm.
Resistance of track between generator and Car I = 0.03 ohm.
Resistance of track between Car I and Car II = 0.05 ohm.
Resistance of track between Car II and Car III = 0.1 ohm.

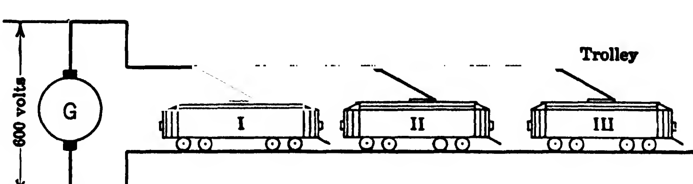



FIG. 16-2. The three cars are in parallel between the trolley and the track.

(*a*) What is the current in the section of track and trolley between Car II and Car III? Between Car I and Car II? Between the generator and Car I?

(*b*) What is the line drop in each section of the trolley?

(*c*) What is the line drop in each section of the track?

(*d*) What is the voltage across each car?

**Prob. 9–2.** Assume that the voltage of the generator in Fig. 16–2 is unknown but that the voltage across Car III is 500 volts. Other data as in Prob. 8–2.

Find the voltage across the generator and the other two cars.

**Prob. 10–2.** Each resistor in Fig. 17–2 has a resistance of 225 ohms and operates across 120 volts.

Find:

(*a*) Total current through the resistors.
(*b*) Volts used in the line, "line drop."
(*c*) Voltage at terminals of the generator.
(*d*) Volts used in the generator.
(*e*) Electromotive force of the generator.

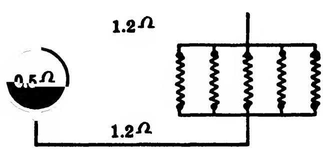



FIG. 17–2. The five parallel-connected resistors are in series with the line.

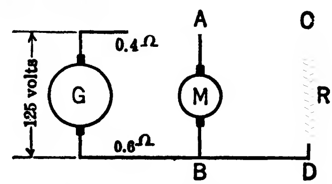



FIG. 18–2. The resistor, R, and motor, M, are connected at different points on the line.

**Prob. 11–2.** In Fig. 18–2, $R$ has a resistance of 0.50 ohm and requires a current of 30 amperes; the motor, $M$, requires 20 amperes.

(*a*) What is the voltage across $R$ and across $M$?
(*b*) What is the combined resistance of $AC$ and $BD$?

**Prob. 12–2.** In Fig. 19–2, the motor takes 15.5 amperes and the lamps each take 1.6 amperes. The voltage across the lamps must be 112 volts.

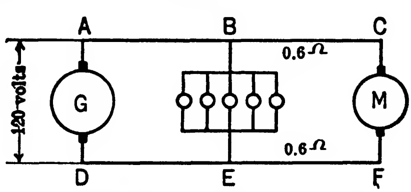



FIG. 19–2. The voltage across the lamps is 112 volts. Voltage across the motor to be determined.

Find:

(*a*) The resistance of the line wires $AB$ and $DE$.
(*b*) The volts drop in the line between lamps and motor.
(*c*) The voltage across the motor.
(*d*) The average resistance of the lamps.

**Prob. 13–2.** Assume the resistance of the line wires $AB$ and $DE$ in Fig. 19–2 is each 0.17 ohm, and the voltage across the lamps is unknown. What will be the voltage across the motor, if the lamps are turned off? The motor is taking 15.5 amperes.

**Prob. 14–2.** What will be the voltage across the lamps (*BE*) in Prob. 13–2 when the motor is not running and the lamps are turned on? Each lamp takes 1.6 amperes.

**Prob. 15–2.** What will be the voltage across *BE* when the motor in Prob. 13–2 is running, but the lamps are turned off?

**Prob. 16–2.** What will be the voltage across the lamps and across the motor in Prob. 13–2, if the lamps and motor interchange places on the line?

**Prob. 17–2.** The voltage at the generator end of a line is 230 volts. When a load of 250 amperes flows, the voltage at the consumer's end of the line is 220 volts. What is the resistance of the line?

**Prob. 18–2.** There is a motor on the consumer's end of the line in Prob. 17–2. When the motor is started alone on the line, the voltage at the motor drops to 214 volts. When the motor gets up speed, the voltage at the motor remains steady at 218 volts.

(*a*) What current does the motor take at starting?
(*b*) What current does the motor take when running?

**Prob. 19–2.** The two motors in Fig. 20–2 each take 8 amperes. Motor *A* is 500 feet from the generator and motor *B* is 1000 feet from the generator. Two coils of wire are available for line wires; one, having a resistance of 0.5 ohm and the other, 1.5 ohm. There are 1000 feet of wire in each coil.

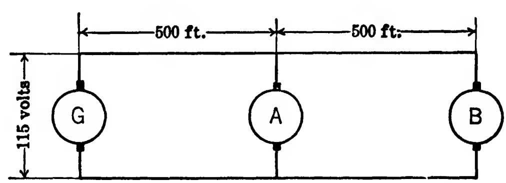



FIG. 20–2. A and B each represent a motor taking 8 amperes.

(*a*) What will be the voltage on the motors, if the 1.5-ohm coil is used between the generator and motor *A*, and the other coil is used between the two motors?
(*b*) What will be the voltage on the motors, if the arrangement of the two coils is reversed?
(*c*) Which is the better arrangement?

**Prob. 20–2.** In the system in Fig. 21–2, each lamp takes 1.3 amperes and the voltage across the generator is 120 volts. Find:

(*a*) The amount and direction of the current in the sections of the lines, *AB*, *BC*, *EF* and *DE*.
(*b*) The voltage drop in each of the above sections of the line.
(*c*) The voltage across each group of lamps.

**Prob. 21–2.** If the voltage across Group II in Fig. 21–2 is 115 volts, and each lamp takes 1.3 amperes, find the voltage across the generator and Group I.

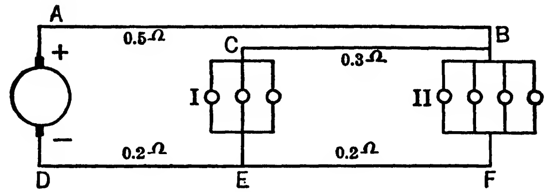



FIG. 21–2. Given the current through each lamp and the voltage at the generator, to find the current distribution and voltage across the lamps.

**Prob. 22–2.** Each lamp in Fig. 22–2 takes 1.5 amperes. The voltage across the switch is 110 volts. The resistance of each section of wire between the lamps, and from the switch to the lamps, is 0.2 ohm. What is:

(*a*) The amount and direction of the current flowing in each section of the wire?

(*b*) Voltage drop in each section of the wire?

(*c*) Voltage across each lamp?

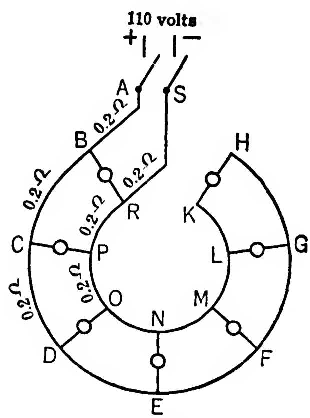



FIG. 22–2. A parallel arrangement of lamps supplied from the same ends of the line.

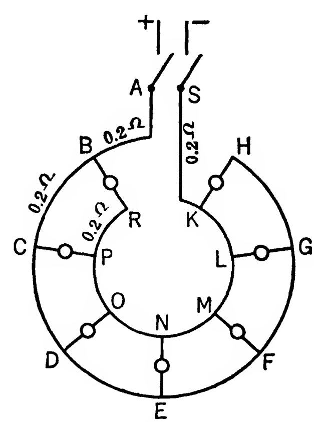



FIG. 23–2. The same parallel arrangement of lamps as in Fig. 22–2, but supplied from opposite ends of the line.

**Prob 23–2.** If the wiring of Prob. 22–2 were arranged as in Fig. 23–2, what would be:

(*a*) The amount and direction of current in each section of the wire?

(*b*) Voltage drop in each section of the wire?

(*c*) Voltage across each lamp?

(*d*) Which is the better method of making connection, this, or that of Prob. 22–2?

**Prob. 24–2.** If the wiring of Prob. 22–2 were arranged as in Fig. 24–2 what would be:

(*a*) Amount and direction of current in each section of the wire?
(*b*) Voltage drop in each section?
(*c*) Voltage across each lamp?
(*d*) Which of the three methods of making connections is the best, this, or that in Prob. 22–2 or in Prob. 23–2?

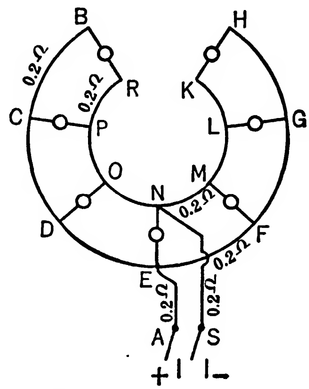



FIG. 24–2. The same arrangement of lamps as in Fig. 22–2, but supplied from the middle points of the line.

(*e*) If the points *BH* and *RK* in Fig. 24–2 were each connected with a wire of 0.2 ohm resistance, what would be the current in these two sections and how would the voltage on the lamps be affected?

**Prob. 25–2.** In Fig. 25–2, voltage across the generator is 124 volts and voltage across Group I is 121 volts. Resistance of each lamp in Group I is 220 ohms. Find:

(*a*) Line drop between Groups I and II.
(*b*) Voltage across Group II.
(*c*) Current through each lamp in I and each lamp in II.
(*d*) Resistance of each lamp in II.

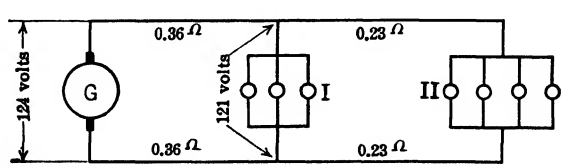



FIG. 25–2. The resistance of the lamps in Group I is known and the voltage at Group II is to be determined.

**Prob. 26–2.** What would the values in Prob. 25–2 become, if the resistance between *G* and Group I were 0.36 ohm for each line wire, and from Group I to Group II, 0.23 ohm for each wire?

**Prob. 27-2.** A street lighting circuit contains 25 arc lamps in series. Each lamp has 13.1 ohms virtual resistance and takes 6.5 amperes. If the resistance of the generator armature is 3 ohms, and of the line is 6 ohms, what electromotive force must the generator develop? What is the terminal voltage?

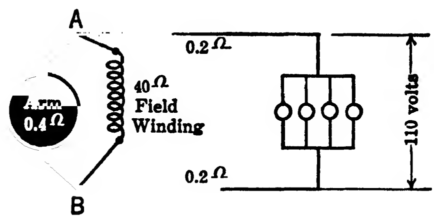



FIG. 26-2. A "shunt" generator connected to a line and supplying a group of lamps.

**Prob. 28-2.** The "shunt" generator in Fig. 26-2 supplies 25 amperes to a lamp load at 110 volts. The resistance of the generator armature is 0.4 ohm and the resistance of the field winding is 40 ohms. What is the voltage at the terminals ($AB$) of the generator? What is the current in the field winding? What current does the generator armature supply? What electromotive force does the generator develop?

# CHAPTER III

## ELECTRIC POWER AND ENERGY

Most electrical apparatus is rated as to the voltage of the circuit on which it can run and also as to the amount of power required to operate it.

Thus one incandescent lamp may be rated as a 115-volt, 50-watt lamp, while another may be rated as a 115-volt, 25-watt lamp. This means that both lamps are intended to be operated on a 115-volt circuit, but that twice as much power is required to operate the first lamp as is required to operate the second.

This chapter takes up the consideration of electric power and energy.

**1. Unit of Power. Watt.** The flow of an electric current has been likened to the flow of water through a pipe. A current of water is measured by the number of gallons, or pounds, flowing per minute; a current of electricity, by the number of amperes, or coulombs per second. The power required to keep a current of water flowing is the product obtained by multiplying the current in **pounds per minute** by the head, or pressure, in **feet.** This gives the power in **foot-pounds per minute.** To reduce to horsepower, it is necessary merely to divide by 33,000; i.e.,

$$\frac{(\text{pounds per min.}) \times (\text{feet})}{33{,}000} = \text{horsepower.}$$

In exactly the same way, the **power** required to keep a current of electricity flowing is the product of the current in **amperes** by the pressure in **volts.** This gives the power in **watts.**

$$\textbf{Watts} = \textbf{Amperes} \times \textbf{Volts.}$$

In the form of an algebraic equation this becomes,

$$P = IE \qquad (1)$$

where $P$ = power in **watts**

$I$ = current in **amperes**

$E$ = pressure in **volts.**

The **watt** is a standard unit of power and denotes the power used when **one volt** causes **one ampere** of current to flow. The watts involved when any given current flows under any pressure can always be found by multiplying the current in amperes by the pressure in volts. Thus, if an incandescent lamp takes 0.5 ampere when burning on a 110-volt line, the power consumed equals

$$0.5 \times 110 = 55 \text{ watts.}$$

That is,

$$P = IE$$

$$\text{power} = \text{current} \times \text{pressure}$$

or

$$\text{watts} = \text{amperes} \times \text{volts.}$$

**Example 1.** What power is consumed by a motor which runs on a 220-volt circuit if it takes 4 amperes?

$$\begin{aligned} \text{Watts} &= \text{amperes} \times \text{volts} \\ &= 4 \times 220. \\ \text{Power} &= 880 \text{ watts.} \end{aligned}$$

**Prob. 1-3.** A 20-candlepower tungsten lamp takes 0.227 ampere when on a 110-volt line. What power is consumed?

**Prob. 2-3.** An arc lamp takes 5 amperes at 110 volts. What power is consumed?

**Prob. 3-3.** What would be the power rating of a 32-candlepower incandescent lamp which used 1.5 watts per candlepower?

**Prob. 4-3.** What current does a motor take which uses 440 watts on a 110-volt line?

**Prob. 5-3.** What current does a 20-candlepower tungsten lamp take on a 112-volt circuit, if it requires 1.25 watts per candlepower?

**Prob. 6-3.** A motor is rated to take 5 amperes and to consume 1200 watts. For what voltage is it built?

**Prob. 7-3.** A 50-candlepower lamp rated at 1.5 watts per candlepower is to take 0.341 ampere. On what voltage should it be run?

**2. Variations of the Power Equation.** Just as Ohm's law has the three forms, (1) $I = \frac{E}{R}$, (2) $E = IR$, (3) $R = \frac{E}{I}$, so this power equation, $P = IE$, may have three forms, found as follows:

$$P = IE \tag{1}$$

but

$$E = IR \qquad \text{Ohm's law.}$$

Therefore,

$$P = I(IR) \text{ or } I^2R, \text{ substituting in (1).}$$

$$P = I^2R \tag{2}$$

Again $$I = \frac{E}{R} \qquad \text{Ohm's law.}$$

Therefore, $$P = \left(\frac{E}{R}\right)E = \frac{E^2}{R}, \text{ substituting in (1).}$$

$$P = \frac{E^2}{R}. \tag{3}$$

It is well to learn this equation in its three forms:

$$P = IE = I^2R = \frac{E^2}{R}.$$

When the voltage and current are known, equation (1) is the most convenient to use. When the current and resistance are known, equation (2) is the most convenient, and when the voltage and resistance are known, equation (3) is the most convenient to use.

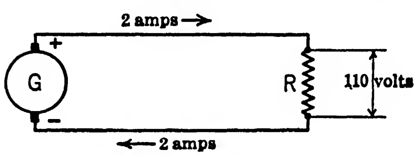



FIG. 1-3. The power taken by resistor, R, can be computed from any one of the three forms of the power equation.

**Example 2.** Generator, $G$, Fig. 1-3, is supplying 2 amperes to the resistor, $R$, at 110 volts. What power is consumed by the resistor?

(*a*) Applying equation (1)

$$P = IE = 2 \times 110 = 220 \text{ watts.}$$

(*b*) Or, by finding the resistance of $R\left(\frac{110}{2} = 55 \text{ ohms}\right)$, and applying equation (2)

$$P = I^2R = 2^2 \times 55 = 220 \text{ watts.}$$

(*c*) Applying equation (3)

$$P = \frac{E^2}{R} = \frac{110^2}{55} = 220 \text{ watts.}$$

Note that the use of any one of the three methods gives the same result. It is the same as though we used Ohm's law first to find the amperes and volts, and then multiplied. In this case, it is best to use equation (1) since the volts and amperes are given.

**Prob. 8-3.** What power does a 20-ohm resistor consume when 2.5 amperes are forced through it?

**Prob. 9-3.** The field winding of a motor has 43 ohms resistance and takes a current of 3.2 amperes. What power does it consume?

**Prob. 10-3.** What power is being used by an incandescent lamp which takes 0.652 ampere at 115 volts?

**Prob. 11–3.** What power does a car heater use if its resistance is 120 ohms and the pressure across it is 550 volts?

**Prob. 12–3.** What power does an electric flatiron use on a 115-volt circuit, if the resistance is 30 ohms?

**Prob. 13–3.** An automobile windshield defroster takes 4.8 amperes from a 6-volt circuit. How much power does this device consume?

**3. To Determine the Power in an Electric Circuit.** If we wish to know the power that is being consumed in **a certain part** of an electric circuit, we merely have to insert an ammeter to measure the current in **that part** of the circuit, and apply a voltmeter to measure the voltage across **that part** of the circuit, and multiply the ammeter reading by the voltmeter reading. This gives us our power directly in watts. This is usually stated as an equation as follows:

$$\text{Watts} = \text{Volts} \times \text{Amperes}.$$

The same precautions must be observed in the use of this equation as in the use of Ohm's law. That is, the Voltage and the Current must be measured for the **same part** of the circuit at the same time, and their product is the power consumed in **that part** of the circuit alone. The following example illustrates the use of the equation.

**Example 3.** A generator $G$, Fig. 2–3, is furnishing a current of 4 amperes to the line at a pressure of 120 volts. There is in the circuit a resistor $R$, which requires 5 volts to force the current through it, and a motor $M$, which requires 115 volts. How much power does the resistor $R$ consume, and how much does the motor $M$ consume?

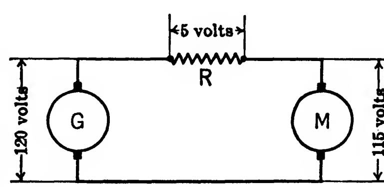


FIG. 2–3. The power taken by resistor, R, plus that taken by the motor, M, is the total power used in the circuit.

**Solution.**

(*a*) The resistor $R$ consumes:

$$P_R = E_R I_R = 5 \times 4 = 20 \text{ watts.}$$

(*b*) The motor $M$ consumes:

$$P_M = E_M I_M = 115 \times 4 = 460 \text{ watts.}$$

(*c*) Power consumed by both $R$ and $M$ = 20 + 460 = 480 watts.

(*d*) Total Power delivered to the circuit:

$P_{\text{total}} = E$ (total volts across circuit) $\times$ $I$ (total current in circuit)
$= 120 \times 4 = 480$ watts. This checks with value in (*c*).

**Prob. 14–3.** Find the power consumed in resistor $A$, in resistor $B$, and in resistor $C$, in Fig. 3–3, if 5 amperes flows in the circuit. What is the total power delivered to the circuit?

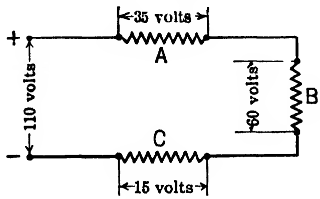



FIG. 3–3. Each resistor consumes part of the power delivered to the circuit.

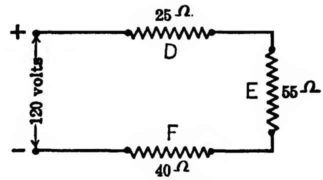



FIG. 4–3. The total power delivered to the circuit is the sum of the watts consumed in each resistor.

**Prob. 15–3.** What is the power used in each of the resistors in Fig. 4–3? What is the total power used in the circuit?

**Prob. 16–3.** A shunt motor, Fig. 5–3, takes 22 amperes at 110 volts. ($a$) What power is taken by the armature? ($b$) What power is consumed in the field coil? ($c$) What is the total power delivered to the motor?

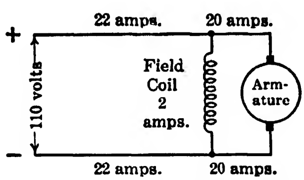



FIG. 5–3. Power taken by a motor.

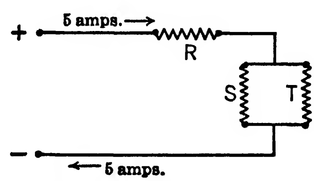



FIG. 6–3. Power consumed by resistors in a series-parallel circuit.

**Prob. 17–3.** In Fig. 6–3

Resistor $R$ has 10 ohms resistance.
Resistor $S$ has 16 ohms resistance.
Resistor $T$ has 48 ohms resistance.

5 amperes are flowing through resistor $R$. What power is consumed in each resistor? What is the voltage across the circuit?

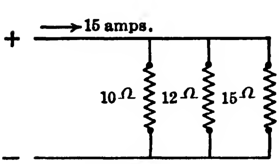



FIG. 7–3. Power consumed by resistors in parallel.

**Prob. 18–3.** ($a$) What power is consumed in each of the resistors in Fig. 7–3? ($b$) What is the total power delivered to the circuit? ($c$) What is the voltage across circuit?

**4. Kilowatt and Horsepower.** Since the watt is a unit of power too small to express conveniently the output of modern electrical

machinery, a unit called the **Kilowatt**, equal to 1000 watts, is generally used. Thus:

$$2500 \text{ watts would be } \frac{1}{1000} \text{ of } 2500 \text{ or } 2.5 \text{ kilowatts}$$

and

$$450 \text{ watts would be } \frac{1}{1000} \text{ of } 450 \text{ or } 0.45 \text{ kilowatt.}$$

**Example 4.** What power does a motor consume which takes 20 amperes at 220 volts?

$$\begin{aligned} \text{Watts} &= 20 \times 220 \\ &= 4400 \text{ watts.} \\ \text{Kilowatts} &= \frac{4400}{1000} = 4.4 \text{ kilowatts.} \end{aligned}$$

Since a kilowatt is a unit of power it can be reduced to horsepower.

1 kilowatt = 1.34 horsepower or (approx.) $1\frac{1}{3}$ horsepower. (4)

1 horsepower = 0.746 kilowatt or (approx.) $\frac{3}{4}$ kilowatt. (5)

746 watts = 1 horsepower. (6)

**Example 5.** What horsepower does the motor of Example 4 above consume?

$$\begin{aligned} 1 \text{ kilowatt} &= 1.34 \text{ horsepower.} \\ 4.4 \text{ kilowatts} &= 4.4 \times 1.34 \text{ horsepower} \\ &= 5.89 \text{ horsepower.} \end{aligned}$$

**Example 6.** The power of a 10-horsepower motor can be expressed in kilowatts.

$$\begin{aligned} 1 \text{ horsepower} &= 0.746 \text{ kilowatt.} \\ 10 \text{ horsepower} &= 10 \times 0.746 \text{ kilowatt} \\ &= 7.46 \text{ kilowatts.} \end{aligned}$$

**Prob. 19–3.** What electric power is required to drive a pump which must raise water 100 feet and supply 1500 gallons per hour? (1 gallon of water weighs 8.3 pounds.)

**Prob. 20–3.** What horsepower is required to drive a generator which must deliver 4 amperes at 120 volts?

**Prob. 21–3.** What horsepower do 10 lamps consume if each takes 0.25 ampere at 112 volts?

**Prob. 22–3.** A steam engine is rated as a 250-horsepower engine. What would be its kilowatt rating?

**5. Use of the Wattmeter.** Instead of using two separate instruments, an ammeter and a voltmeter, to measure the power con-

sumed in a certain part of a circuit, we may use a single instrument called a **wattmeter.** This instrument is a combination of an ammeter and a voltmeter and thus has an ammeter part of **low resistance** to measure the current, and a voltmeter part of **high resistance** to take the voltage. The indicator shows the product of the volts times the amperes, that is, the **watts.**

A wattmeter, therefore, usually has **four** terminals, two for the ammeter leads and two for the voltmeter leads. The utmost care must always be exercised to use the proper terminals, as the arrangement of terminals is not the same in all makes of wattmeters. An error in this respect will ruin a very expensive instrument. Also care must be taken not to connect a wattmeter in a circuit where the current flowing exceeds the rating of the current part of the instrument, nor where the voltage exceeds the rating of the voltmeter part. The maximum current and voltage rating of a wattmeter is always marked on the instrument.

Figure 8–3 shows the correct method of connecting a wattmeter in the circuit to measure the power taken by the motor $M$. The ammeter terminals $AA$ are put in **series** with the motor $M$, and the voltmeter terminals $VV$ are connected in **parallel across** the motor. For a detailed discussion of the wattmeter see Chapter XVII.

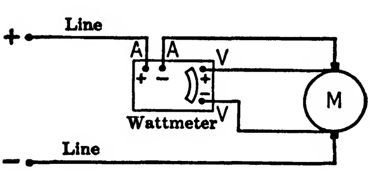



FIG. 8–3. A wattmeter connected to indicate the power taken by the motor, M.

**6. Line Loss.** Since it requires power to keep a current flowing through a resistance, there must be some power used in keeping the current flowing through the line wires of any system. Of course, all the power used in this way is wasted and it is therefore called the **Line Loss.** Line loss is measured in watts just as any other electrical power and is the product of the volts times the amperes, or the product of the resistance of the line wires times the square of the amperes in the line wires. That is,

$$P_L = I_L \times E_L$$

in which

$P_L$ = Power lost in line

$I_L$ = Current in line

$E_L$ = Voltage drop in line

or

watts (lost in line) = volts (drop in line wires) × amperes (through line wires)

or

$$P_L = I_L^2 R_L$$

watts (lost in line) = amperes (in the line) squared × ohms (in the line).

**Example 7.** The generator $G$, Fig. 9–3, has a terminal voltage of 115 volts and supplies 20 amperes to the motor at 110 volts. What is the line loss?

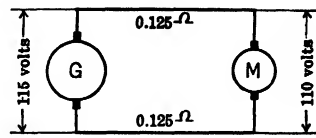



FIG. 9–3. The watts lost in the line wires may be computed by three different methods.

**Solution.**

**First method.** Volts drop in the line wires = 115 − 110 = 5 volts. Watts lost in the line wires = 5 × 20 = **100 watts.**

**Second method.** The generator $G$ supplies 115 × 20 = 2300 watts. The motor $M$ takes 110 × 20 = 2200 watts. Watts lost in the line = 2300 − 2200 = **100 watts.**

**Third method.** The generator supplies 20 amperes to the motor through line wires having a total resistance of 0.125 + 0.125 or 0.25 ohm. Power lost in the line wires = $\overline{20}^2 \times 0.25$ = **100 watts.**

**Prob. 23–3.** How many watts are consumed in the lead wires in Fig. 10–3, if their resistance is 0.5 ohm each and the motor takes 1.4 kilowatts at 220 volts?

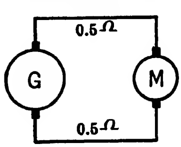



FIG. 10–3. The power lost in the line can be computed from the line resistance and line current.

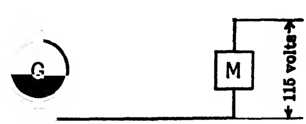



FIG. 11–3. The generator, G, delivers power to the line, where some of it is lost in heat and the remainder delivered to the motor.

**Prob. 24–3.** The generator, Fig. 11–3, delivers 6 kilowatts to the line. The motor uses 5.7 kilowatts. Find:

(*a*) Watts lost in line.
(*b*) Voltage and current of motor.
(*c*) Terminal voltage of generator.

**7. The Difference Between Power and Work (or Energy). Laboratory and Commercial Units.** When we pay our electric-light bill we say we pay so much for "Power." This is not strictly

true, for we pay for **work done** in supplying electricity to our home, *or for the **energy** we used. Our bill is for **energy** used in **kilowatt-hours**, not for **power** in **kilowatts**.*

The difference between **work** and **power** is so often confused it is important that we get the distinction clearly.

**Work** is defined as force multiplied by the distance through which the force acts.

**Power** (rate of doing work) is force multiplied by the distance through which it acts, divided by the time the force is applied.

$$\text{Work} = \text{force in lbs} \times \text{distance in ft} = \text{foot-pounds} \tag{7}$$

$$\text{Power} = \frac{\text{force in lbs} \times \text{distance in ft}}{\text{seconds}} = \text{foot-pounds per second.} \tag{8}$$

Note in the above equation that **work divided by time** = power; and conversely that **power multiplied by time** = work, that is, $\dfrac{\text{pounds} \times \text{feet}}{\text{seconds}} \times \text{seconds} = \text{foot-pounds} = \text{work.}$

Note the examples below.

**Example 8.** (*a*) A 10-lb weight lifted 5 ft = $10 \times 5$ or 50 foot-pounds of work done. (*b*) But if the **rate** of doing work is brought into the problem, and we say the weight was lifted 5 feet in 1 second, then the power used will be $\dfrac{10 \times 5}{1}$ = 50 **ft-lbs per second.** (*c*) If the weight is lifted 5 ft in 2 seconds then the power used is $\dfrac{10 \times 5}{2}$ = 25 **ft-lbs per second.** (*d*) If power is expended in (*b*) at the rate of $\dfrac{10 \times 5}{1}$ or 50 ft-lbs per second, and this rate of using energy (power) is kept up for 30 seconds, then the **energy used** or **work done** will be $\dfrac{10 \times 5}{1\text{ sec}} \times 30\text{ secs} = 50 \times 30$ or 1500 ft-lbs. (Of course, during this time the 10-lb weight would be lifted 150 ft.)

Similarly, in the electric circuit we have seen that when 1 ampere flows through a conductor under a pressure of 1 volt the **power** expended is 1 watt.

$$P = EI.$$

But, $\quad I = \text{coulombs per second} = \dfrac{Q}{t}$

where $\quad Q = \text{coulombs and } t = \text{time in seconds.}$

Therefore:

$$P = EI = E\frac{Q}{t} = \text{watts, or volt-coulombs per second.} \quad \text{(Power)} \quad (9)$$

Now, power multiplied by time is work, therefore:

$$\text{Work} = Pt = EIt = E\frac{Q}{t} \times t = EQ. \quad \text{(Work)} \quad (10)$$

Or **work** = **volt-coulombs** (amount of electricity transferred under a pressure $E$. No rate of transfer expressed).

Also work = $EIt$ = watt-seconds (no rate of transfer implied).

A **watt-second** is called a **joule.**

**Example 9.** If 10 volts were used in forcing 5 amperes through a conductor, the power consumed would be 10 × 5 or **50 watts;** and if this **power,** or rate of using energy, were kept up for 30 seconds, the **energy used** or **work done** would be 50 × 30 or 1500 **watt-seconds** or **joules.**

Note again that **work done per second** is **power,** but **work done per second** multiplied by **time in seconds** (during which the power is supplied) is **work.**

From the above, we see that **foot-pounds,** the mechanical unit of **work,** corresponds to the **watt-seconds** or **joules.** And that **foot-pounds per second** corresponds to the **watt** — both units of **power.**

The scientist, Joule, determined in 1843 that,

1 **watt-second** = 0.74 foot-pound.

The units discussed above are too small for general use and may be called **laboratory units.**

In commercial units of work, time in hours during which power is used, is employed rather than time in seconds.

$$\begin{aligned} \text{1 Horsepower} &= \frac{\text{33,000 ft-lbs}}{\text{minute}} \\ &= \text{33,000 ft-lbs per minute.} \qquad \text{(Power)} \quad (11) \\ \text{1 Horsepower-hour} &= \frac{\text{33,000 ft-lbs}}{\text{minute}} \times \text{60 minutes} \\ &= \text{1,980,000 ft-lbs.} \qquad \text{(Work)} \quad (12) \\ \text{1 Kilowatt} &= \text{1000 watts.} \qquad \text{(Power)} \quad (13) \\ \text{1 Watt-hour} &= 60 \times 60 \text{ watt-seconds} = 3600 \text{ watt-seconds.} \qquad \text{(Work)} \quad (14) \end{aligned}$$

$$1 \text{ Kilowatthour} = 1000 \times 60 \times 60 \text{ watt-seconds}$$
$$= 3{,}600{,}000 \text{ watt-seconds.} \qquad \text{(Work)} \qquad (15)$$

The horsepower-hour and the kilowatthour are the commercial units of work.

**8. Application of the Commercial Units of Work.** We have seen from Art. 7 that, in order to compute the amount of work done by a given engine, it is necessary to know the time it has been running and the power it has been supplying, that is, its rate of doing work. If the power is measured in **horsepower** and the time in **hours,** the work done is measured in **horsepower-hours** and is the product of the **horsepower** by the **hours.**

Similarly, if the power is measured in **kilowatts** and the time in **hours,** the work done is measured in **kilowatthours** and is merely the product of the **kilowatts** by the **hours.**

When a man buys mechanical power to run his shop he has to pay not only according to the horsepower he uses but also according to the number of hours he uses the power. For instance, he may use 40 horsepower for 1 hour and pay $1.20 for it, that is, at the rate of 3 cents for each horsepower-hour. If he used 40 horsepower for 2 hours he would have to pay twice as much because he has used twice as much energy or work. Another way of stating the same fact is to say that he used twice as many horsepower-hours. For in the first instance he used 40 × 1, or 40 horsepower-hours, and in the second 40 × 2, or 80 horsepower-hours. In other words, he did twice as much work in the second case as he did in the first, or received twice as much energy. The unit of work or energy then is the **horsepower-hour,** and is the work done in 1 hour by a 1-horsepower machine.

$$1 \text{ horsepower-hour} = .746 \text{ kilowatthour.} \qquad (16)$$

$$1 \text{ kilowatthour} = 1.34 \text{ horsepower-hour.} \qquad (17)$$

**Example 10.** How much work is done by a machine delivering 15 horsepower when it is run for 8 hours?

1 horsepower in 1 hour does 1 horsepower-hour,
15 horsepower in 1 hour do 15 horsepower-hours,
15 horsepower in 8 hours do 8 × 15, or 120 horsepower-hours,

or

$$\text{work} = \text{horsepower} \times \text{hours},$$
$$15 \times 8 = 120 \text{ horsepower-hours.}$$

**Example 11.** How much work is done in one day of 8 hours by a 150-kilowatt generator running at full load?

$$150 \times 8 = 1200 \text{ kilowatthours,}$$
$$1200 \times 1.34 = 1610 \text{ horsepower-hours.}$$

**Example 12.** At 15 cents per kilowatthour, what is the cost of burning 100 lamps for 8 hours if each lamp consumes 50 watts?

Power consumed in 100 lamps =

$$100 \times 50 = 5000 \text{ watts} = 5 \text{ kilowatts.}$$
$$\text{Energy} = 8 \times 5 = 40 \text{ kilowatthours.}$$
$$40 \times 15 \text{ cents} = \$6.00.$$

**Prob. 25–3.** What does it cost to run a 20-horsepower engine for 125 hours at 1.75 cents per horsepower-hour?

**Prob. 26–3.** A man's "power" bill for 44 hours was $27.80. If he paid at the rate of 2 cents per horsepower-hour, what average power was delivered to him?

**Prob. 27–3.** How much will it cost at 5 cents per kilowatthour to run a 110-volt motor for 16 hours? Motor takes 55 amperes.

**Prob. 28–3.** If work in Prob. 27–3 were paid for at the rate of 5 cents per horsepower-hour, how much would it cost?

**Prob. 29–3.** How much work is done when a 60-watt lamp is burned for 5 hours?

**Prob. 30–3.** What work is done in maintaining for 12 hours a current of 100 amperes in a wire of 2.3 ohms resistance?

**Prob. 31–3.** A bill for electric energy was $18.20 for 130 hours. If the price was 8 cents per kilowatthour, what was the average power used?

**Prob. 32–3.** When 120 incandescent lamps had been burned on a 115-volt circuit for 5 hours, a bill of $2.76 was presented, computed on the rate of 8 cents per kilowatthour. What was the average current taken by each lamp?

**Prob. 33–3.** At 5 cents per kilowatthour how much does it cost per year for transmission losses in a line which carries a current of 200 amperes and has a resistance of 0.42 ohm? The line is used 5 hours each day.

**Prob. 34–3.** By how many foot-pounds are 1,000,000 watt-seconds greater or less than 3 horsepower-hours?

**9. Electrical Energy and Heat Energy.** An electric current may be used in one part of a circuit to produce mechanical motion as in a motor; in another part of the circuit to produce electrolytic action as in an electroplating vat; in another part to produce light as in an electric lamp. In any part of the circuit where it is doing

no one of these things, all the energy consumed goes into producing heat. It even produces heat in the portions of the circuit where it is also producing some other form of energy. A motor never gives out in mechanical energy all that it receives in the form of electrical energy. Some of the electrical energy is turned into heat energy. This heat is produced in overcoming the electrical **resistance,** just as heat is produced in a machine in overcoming mechanical **friction.**

It is safe to say that in any part of an electric circuit where there is no transformation to other forms of energy, the whole of the electrical energy consumed is turned into heat. Thus in line wires all the electrical energy consumed in forcing a current through them goes into heat.

The resistance of an electric circuit is similar to the friction of a machine. Just as the power used to overcome the resistance of the wire appears in the heat generated in the wire, so the power used to overcome the friction of a bearing appears in the heat generated in the bearing. The mechanical engineer strives to reduce the amount of power wasted in heat by reducing the friction of the bearings. The electrical engineer may reduce the power wasted in heat by reducing the resistance of the wire used to transmit a given current.

In designing electrical machinery, a great deal of care is given to provide means for keeping the heat losses as low as possible. This is done by constructing machines with ventilating ducts in the iron frames and cores, by attaching fan blades to rotating parts and by providing sufficient surface for radiation, so that the heat generated may not cause an excessive rise in temperature.

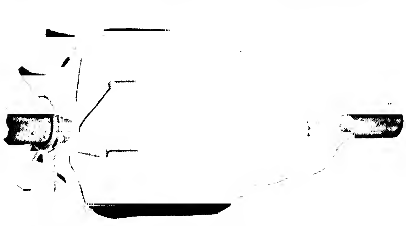


FIG. 12-3. An armature equipped with fan blades to force air through the frame of the machine. This helps to carry away, or dissipate, the heat generated by the current. *General Electric Co.*

Figure 12-3 shows the armature of a generator equipped with a fan. This provides a draught of air through openings in the core and over the surface of the armature, to carry away the heat generated in the copper windings.

The following examples show how this power lost in heat may be computed.

**Example 13.** The generator $G$ in Fig. 13-3 maintains a pressure of 240 volts across the line. The line wires have a total resistance of 0.5 ohm. The motor $M$ requires 40 amperes at 220 volts. How much electrical power is lost in heat in the line?

**Solution.** Since the line wire is delivering no other form of power, all the electrical power consumed in it must be used in overcoming its resistance and must go into heating it. The power used in the line wire is found as follows:

$$I = 40 \text{ amperes}$$
$$R = 0.5 \text{ ohm}$$
$$P = I^2R = 40 \times 40 \times 0.5 = 800 \text{ watts.}$$

Thus 800 watt-seconds of electrical energy are turned into heat each second. If this heat is not allowed to radiate, the wire will soon become hot enough to destroy the insulation and finally fuse the copper itself. Thus is seen the necessity for arranging wires in the field and armature coils of motors in such a way as to allow for the dissipating of the heat generated in overcoming the resistance of the coils.

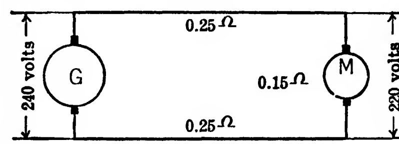



FIG. 13-3. Some electrical energy is changed into heat energy, both in the wires connecting the motor with the generator, and also in the windings of the motor.

This heat loss of 800 watts in Example 13 is called the **I²R loss.** Thus the **line loss** explained in Art. 6 is a heat loss.

**Example 14.** What is the heat loss in the armature of the motor in Fig. 13-3, Example 13? The resistance of the armature is 0.15 ohm.

**Solution.** Motor armature takes 40 amperes.

$$P = I^2R = 40^2 \times 0.15 = 240 \text{ watts.}$$

Note in Example 14 that the motor armature takes 40 amperes at 220 volts, and thus uses $220 \times 40$ or 8800 watts. Of this amount 240 watts are transformed into heat energy and $8800 - 240$ or 8560 watts are converted into mechanical energy to run the motor. The 240 watts is the **I²R loss,** sometimes called the **Copper Loss,** since electrical conductors are generally copper.

Note particularly that $220 \times 40$ or 8800 watts is **not all** heat loss, because 220 volts are not used in forcing 40 amperes through the 0.15 ohm resistance of the armature. When a motor is running it develops a **Counter EMF** or **Back Voltage** and the 220 volts across the motor must overcome this voltage as well as force the 40 amperes through the 0.15 ohm in the armature.

The voltage used to force the current through the resistance of the armature is $40 \times 0.15$ or 6 volts.

Now the heat loss can also be figured thus:

$$P = EI = 6 \times 40 = 240 \text{ watts}$$

or

$$P = \frac{E^2}{R} = \frac{6^2}{0.15} = \frac{36}{0.15} = 240 \text{ watts.}$$

These results check with the calculation in Example 14.

**Note,** that all three of the power equations apply to the power consumed by the resistance, if we use only the voltage and current which apply to the resistance alone.

**Prob. 35-3.** How much power (copper loss) is consumed in heating the armature of a generator which delivers 15 kilowatts at 220 volts? Armature resistance is 0.2 ohm.

**Prob. 36-3.** How many kilowatthours of electricity are used up in a week in heating the line wires of a transmission system carrying an average current of 65 amperes, 24 hours per day every day of the week? Total resistance of line wires is 0.6 ohm.

**Prob. 37-3.** A 125-volt motor requires 25 amperes. The resistance of the armature is 0.18 ohm.

(*a*) How many horsepower does the motor take?

(*b*) How many horsepower are used up in heating the armature wires ($I^2R$ loss)?

**10. Electrical Equivalent of Heat.** Since electrical energy can be transformed into heat energy as well as into mechanical energy, electrical units of energy can be reduced to heat units of energy as well as to mechanical. The common unit of heat energy is the **British thermal unit,** generally abbreviated to **Btu.** The small or laboratory unit of heat is the **Calorie.**

One British thermal unit is the heat necessary to raise one pound of water one degree Fahrenheit. Thus to raise 5 pounds of water 8 degrees requires 5 × 8 or 40 Btu.

One calorie is the quantity of heat necessary to raise one gram of water 1° C. One Btu equals 252 calories.*

**Example 15.** How much heat is required to heat 300 pounds of water from 60° to 200° F?

**Solution.**

Water must be raised 200° − 60 = 140°.

To raise 1 pound water 1° requires 1 Btu.

To raise 300 pounds water 1° requires 300 × 1 = 300 Btu.

To raise 300 pounds water 140° requires 300 × 140 = 42,000 Btu.

* To change Centigrade Temperature to Fahrenheit — $(C° \times \frac{9}{5}) + 32 = F°$.
To change Fahrenheit Temperature to Centigrade — $(F° - 32) \times \frac{5}{9} = C°$.

By immersing a coil of resistance wire in water and measuring the number of watt-seconds required to raise the temperature of a pound of water one degree, we can determine the number of watt-seconds of electrical energy which are equivalent to one British thermal unit. It has been found that one British thermal unit equals 1055 watt-seconds. Accordingly it would take 1055 seconds for an ampere flowing through a 1-ohm coil to raise 1 pound of water one degree. Similarly, it would take but 10.55 seconds for one ampere flowing through a 100-ohm coil to raise 1 pound of water one degree.

Since it takes 1055 watt-seconds to produce one British thermal unit, one watt-second would produce $\frac{1}{1055}$ or 0.0009479 Btu. **The factor 1055 is called the electrical equivalent of heat. The factor 0.0009479 is called the heat equivalent of electricity.**

Similarly one watt-second = 0.24 calorie, or one calorie = 4.2 watt-seconds.

These relations are usually expressed in the form of equations

$$H = \frac{I^2Rt}{1055}\text{ Btu} \tag{18}$$

or

$$H = 0.24\ I^2Rt\text{ cal.} \tag{19}$$

**Example 16.** How many heat units are generated when 50 amperes flow for 10 minutes through a 2-ohm wire?

$$\begin{aligned}\text{Power} &= I^2R\\ &= 50 \times 50 \times 2\\ &= 5000\text{ watts.}\end{aligned}$$

Electrical Work or energy:

$$\begin{aligned}\text{Measured in Electrical units} &= 5000 \times 10 \times 60\\ &= 3{,}000{,}000\text{ watt-seconds.}\\ \text{Measured in Heat units} &= \frac{3{,}000{,}000}{1055}\\ &= 2844\text{ Btu.}\end{aligned}$$

**Example 17.** How long will it take to heat 12 pounds of water from 50° to 120° F using an electric current of 20 amperes and a 6-ohm coil?

**Solution.**

$$\begin{aligned}\text{Temperature rise} &= 120° - 50° = 70°\text{ rise.}\\ \text{Heat required in heat units} &= 12 \times 70 \qquad = 840\text{ Btu.}\\ \text{Heat required in electrical units} &= 840 \times 1055 = 886{,}200\text{ watt-sec.}\end{aligned}$$

Electrical power consumed in heat:

$$\begin{aligned}P &= I^2R\\ &= 20 \times 20 \times 6 = 2400\text{ watts.}\end{aligned}$$

To produce 886,200 watt-seconds at the rate of 2400 watts would require $\frac{886{,}200}{2400}$ = 368 seconds or 6.13 minutes.

**Example 18.** A bank of 50-watt incandescent lamps takes 10 amperes at 115 volts. If 90 per cent of the energy received by the lamps is given off in heat, how many Btu are thus given off in 1 hour?

**Solution.**

$$115 \times 10 = 1150 \text{ watts (delivered to the lamps);}$$
$$1150 \times 0.90 = 1035 \text{ watts (given off in heat);}$$
$$1 \text{ hour} = 60 \times 60 = 3600 \text{ seconds;}$$

$$\text{Heat (in electrical units)} = 1035 \times 3600 = 3{,}726{,}000 \text{ watt-seconds.}$$

$$\text{Heat (in heat units)} = \frac{3{,}726{,}000}{1055} = 3531 \text{ Btu;}$$

$$\text{or} \quad = 0.24 \times 3{,}726{,}000 = 894{,}240 \text{ calories.}$$

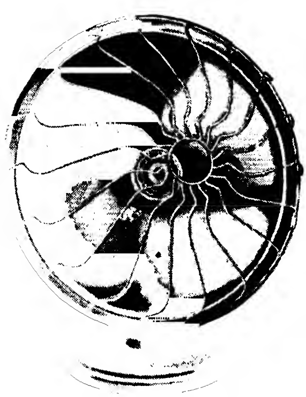


FIG. 14-3. A "radiant" heater. The energy consumed in the heater element is changed into heat. The bowl reflects the heat and sends it in any desired direction. *General Electric Co.*

**Prob. 38-3.** How much heat is generated per hour in an electric flatiron using 4.5 amperes at 115 volts? Calculate this both in Btu and in calories.

**Prob. 39-3.** If the price of the electrical energy used in Prob. 38-3 is 8 cents per kilowatthour, what is the price per Btu?

**Prob. 40-3.** How many Btu per hour are given off by a car heater which takes 5.5 amperes at 600 volts?

**Prob. 41-3.** A 115-volt, 200-watt lamp, has a resistance of 66 ohms. If only 12 per cent of the electrical energy consumed in the lamp goes into light, how many Btu of heat does the lamp give off in burning 24 hours?

**Prob. 42-3.** The radiant heater in Fig. 14-3 takes 5.5 amperes at 110 volts. How many Btu in heat does it give off per hour?

**Prob. 43-3.** How long would it take a current of 5 amperes under a pressure of 120 volts to supply enough heat to just melt 12 pounds of ice? It requires 144 Btu to melt 1 pound of ice.

**11. Power Units and Work Units.** We have considered the Power units and the Work units in common use in electrical, mechanical and heat measurements. It is well to take some time to get the relations among them firmly fixed in the mind.

First, it must be clearly understood which are the power units and which are the work or energy units.

The Power units are

$$(a)\ \text{Electrical}\ \begin{cases}\text{Watt}\\ \text{Kilowatt} = 1000\ \text{watts}\end{cases}$$

$$(b)\ \text{Mechanical}\ \begin{cases}\text{Foot-pounds per minute}\\ \text{Horsepower} = 33{,}000\ \text{foot-pounds per minute}\end{cases}$$

$$(c)\ \text{Heat}\ \begin{cases}\text{Btu per second}\\ \text{Calories per second}\end{cases}$$

Note that in the case of electrical, mechanical and heat power there are two units each, one the small unit, the other the large unit. Since all are units of power, a quantity of power expressed in any one unit may be reduced to any of the other units of power. Thus, 5000 watts may be expressed as a certain number of kilowatts, a certain number of foot-pounds per minute, a certain number of horsepower or a certain number of Btu per second as desired. It is merely necessary to have the heat and the mechanical equivalents of the electrical unit.

The common units of Work or Energy are:

$$\text{Electrical}\ \begin{cases}\text{Watt-second}\\ \text{Kilowatthour}\end{cases}$$

$$\text{Mechanical}\ \begin{cases}\text{Foot-pound}\\ \text{Horsepower-hour}\end{cases}$$

$$\text{Heat}\ \begin{cases}\text{British Thermal Unit}\\ \text{Calorie}\end{cases}$$

Note that there are two electrical, two mechanical and two heat units of work, a large and a small one, just as in the case of Power units.

A quantity of energy expressed in any one of these units can be reduced to any of the other units of energy. Again it is merely necessary to have the heat and mechanical equivalents of the electrical unit.

It is not necessary to memorize the electrical, mechanical and heat units of both power and energy equivalents. If one work equivalent and one power equivalent are learned, the rest can be derived as desired. The three most useful equivalents are:

$$\begin{aligned}1\ \text{kilowatt} &= 1.34\ \text{horsepower}\\ 1\ \text{Btu} &= 1055\ \text{watt-seconds}\\ 1\ \text{watt-second} &= 0.24\ \text{cal.}\end{aligned}$$

**Caution.** There are no equivalents between power and energy. They are different things and therefore cannot be reduced one to the other. Thus, British thermal units which are **work** units cannot be reduced to an equivalent number of watts or horsepower, because watts and horsepower are units of **power** and not of **work.** The greatest care is necessary not to confuse the **work** with the **power** units. Keep these clearly defined and no difficulty will be met in the change from one system to another.

The following table gives the equivalents of the units in the three systems for both power and energy.

| | Power | Work or Energy |
|---|---|---|
| Electrical | Watt = 44.2 ft-lbs per min<br>Kw = 1000 watts<br>= 1.34 hp | 1 watt-sec = 0.000948 Btu<br>= 0.74 ft-lb<br>1 Kwhr = 3,600,000 watt-sec<br>= 1.34 hp-hr<br>= 3412 Btu |
| Mechanical | 1 ft-lb per min = 0.0226 watt<br>1 hp = 33,000 ft-lb per min<br>= 0.746 kw | 1 ft-lb = 1.36 watt-sec<br>= 0.00129 Btu<br>= 0.325 calorie<br>1 hp-hr = 1,980,000 ft-lbs<br>= 0.746 kwhr<br>= 2560 Btu<br>= 645,000 calories |
| Heat | 1 Btu per sec = 1055 watts<br>= 46,600 ft-lb per min<br>= 1.41 hp<br>= 1.055 kw<br>1 calorie per sec = 4.17 watts | 1 Btu = 1055 watt-sec<br>= 778 ft-lb<br>= 0.000293 kwhr<br>= 0.000393 hp-hr<br>1 calorie = 4.17 watt-sec |

**12. Efficiency of Electrical Appliances.** Since all pieces of electrical apparatus have resistance, there must always be some heat generated when a current is sent through them. Unless the apparatus is to be used for heating purposes, this energy is wasted, just as the energy is wasted which goes toward overcoming mechanical friction. Thus no electrical machine gives out all the power it receives. The percentage which it does give out is called its **efficiency.** Accordingly a motor that gives out 9 kilowatts for every 10 kilowatts it receives is said to have an efficiency

of 90 per cent. If it gives out only 8 kilowatts for every 10 kilowatts it receives, it has an efficiency of only 80 per cent.

The power a machine receives is called the **input.** The power it gives out is called the **output.** The efficiency may be said to be the ratio of the output to the input, or

$$\text{efficiency} = \frac{\text{output}}{\text{input}}. \tag{20}$$

Since the output is always smaller than the input, the fraction $\frac{\text{output}}{\text{input}}$ is always less than unity, and is stated as a percentage, as 75 per cent, 90 per cent, etc. The efficiency of any device is always less than 100 per cent. Of course, the output and the input must always be stated in the same units. We cannot compare the input of a motor in kilowatts with the output in horsepower. They must both be reduced either to kilowatts or to horsepower.

**Example 19.** A 3-horsepower motor requires 2.4 kilowatts to drive it. What is its efficiency?

$$\begin{aligned} \text{Output} &= 3.0 \text{ horsepower} \\ \text{Input} &= 2.4 \times 1.34 = 3.2 \text{ horsepower} \\ \text{Efficiency} &= \frac{\text{Output}}{\text{Input}} = \frac{3.0}{3.2} = 0.94 \text{ or } 94 \text{ per cent.} \end{aligned}$$

Power machines are always rated in terms of their full-load **output.** When we speak of a 90-horsepower motor, we mean it will **deliver** 90 horsepower at full load.*

A 10-horsepower motor with an efficiency of 80 per cent is a motor that will **deliver** or **develop** 10 horsepower at full load, which is 80 per cent or 0.80 of the **input.** The losses are therefore 20 per cent or 0.20 of the **input** and we do not know the input. We **cannot** add 0.20 of 10 or 2 horsepower to the 10 horsepower **output** and call the input 10 + 2 or 12 horsepower, for 2 hp is

* The power rating of any electrical machine is determined by the amount of heat generated in it, i.e., upon the temperature at which it operates. The greater the load on it, the higher the temperature will be. If the temperature is too high, the insulation on the windings may be charred, or the soldered connections may even melt. The American Institute of Electrical Engineers specifies the safe temperature at which various types of machines may be allowed to operate; and this temperature determines how large a load it can carry continuously.

For instance, the 90-hp motor above, or any motor or generator, will carry more load than the rating given on the name plate, but the temperature will rise above the allowable limit and the life of the machine will be shortened.

20 per cent of the **output.** The proper solution is shown in Example 19 below.

**Example 20.** What is the input of a 10-horsepower motor which has an efficiency of 80 per cent at full load?

**Solution.**

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

$$0.80 = \frac{10}{\text{input}}, \quad \text{therefore,} \quad \text{input} = \frac{10}{0.80} = 12.5 \text{ horsepower.}$$

For a discussion of the losses and efficiency of motors and generators, see Chapter XII.

**Prob. 44–3.** What efficiency has a 15-horsepower motor which requires 60 amperes at 230 volts?

**Prob. 45–3.** The efficiency of a certain generator is 80 per cent. What current can it deliver at a pressure of 120 volts when it receives 12 horsepower from the driving engine?

**Prob. 46–3.** (*a*) What is the horsepower input to a 50-horsepower motor, which has an efficiency of 89 per cent at full load? (*b*) What is the kilowatt input to the motor?

**Prob. 47–3.** What is the efficiency of a motor which does 600,000 foot-pounds of work in 5 minutes, if the input is 3.3 kilowatts?

**Prob. 48–3.** A 25 horsepower 220-volt motor has an efficiency of 90 per cent. How much current does it take from the line?

**Prob. 49–3.** When the motor in Prob. 48–3 is operating at half load, the efficiency is 76 per cent. What current does it take?

**13. Efficiency of Transmission.** When power is transmitted from one place to another, some energy is always lost in the process. If it be by mechanical means, as a belt and pulleys, there is a friction loss; if by a steam line, there is the heat lost by radiation; or if by electrical transmission, there is a heat loss, or $I^2R$ loss in the line. This is the loss we have already discussed. When distances are considerable, it is generally more convenient and cheaper to transmit energy by electrical means; and this fact has had more influence in the wide use of electricity in industry than perhaps any other one thing. But even here, we have seen we lose energy. Some of the energy put into a transmission line never reaches its destination, but is dissipated in heat on the way.

The ratio of the power delivered to the loads by a transmission line, to the power put into the line, is called the efficiency of transmission.

Thus:

$$\frac{\text{Kilowatts delivered to the load}}{\text{Kilowatts delivered to the line}} \times 100$$
$$= \text{percentage efficiency of transmission.}$$

This may also be expressed as energy; as

$$\frac{\text{Kilowatthours delivered to the load}}{\text{Kilowatthours delivered to the line}} \times 100$$
$$= \text{percentage efficiency of transmission.}$$

**Example 21.** The group of lamps, Fig. 15–3, uses 14 amperes. Resistance of each line wire is 0.2 ohm. Voltage at the generator is 125 volts. What is the efficiency of the transmission? That is, what percentage of the total power, delivered to the line, is delivered to the lamps?

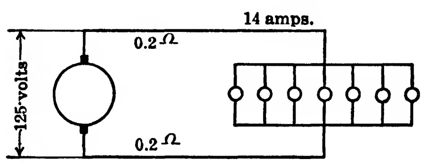



FIG. 15–3. The ratio of the power delivered to the lamps to the power supplied by the generator to the line is the efficiency of the line.

**Solution.**

Total power = $(EI)$ = $125 \times 14 = 1750$ watts put into line.
Watts lost in the line = $(I^2R) = 14^2 \times 0.4 = 78.4$ watts.
Watts left for lamps = $1750 - 78.4 = 1671.6$ watts.

$$\text{Efficiency of transmission} = \frac{1671.6}{1750} = 0.954 = 95.4 \text{ per cent.}$$

Or,

Volts used in line = $(IR) = 14 \times 0.4 = 5.6$ volts.
Volts across lamps = $125 - 5.6 = 119.4$ volts.
Watts delivered to lamps = $119.4 \times 14 = 1671.6$ watts.

$$\text{Efficiency of transmission} = \frac{1671.6}{1750} = 0.954 = 95.4 \text{ per cent.}$$

Note that in the example above, the ratio of the voltage at the load to the voltage at the generator also gives the efficiency $\left(\frac{119.4}{125} = 0.954\right)$; but this is simply because the generator current and load current are the same, and the ratio $\left(\frac{119.4}{125}\right)$ is really the ratio of the power output to power input.

**Note particularly that when more than one load is fed from a line, the ratio of voltages does NOT give the ratio of the output to input.** The total **power** taken by the various loads, or total **power output** of the line, must be determined.

**Prob. 50–3.** What is the efficiency of transmission for a system, which receives 15 kilowatts from the generator, and delivers 13.2 kilowatts to a motor at the other end?

**Prob. 51–3.** Each wire in a line, feeding a group of 6 lamps, has a resistance of 1.2 ohms. The hot resistance of each lamp is 275 ohms, and each lamp takes 0.4 ampere. What is the efficiency of transmission?

**Prob. 52–3.** In Fig. 16–3, load $A$ takes 10 amperes and load $B$ takes 15 amperes. The voltage at the generator is 125 volts. What is the efficiency of transmission?

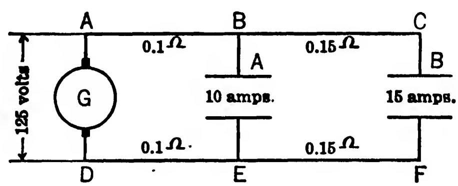



FIG. 16–3. Given the voltage at the generator, the current taken by each load and the resistance of the line wires, to compute the efficiency of transmission.

**Prob. 53–3.** Each lamp in Fig. 17–3 takes 1.8 amperes, and the motor takes 25.5 amperes at 110 volts. What is the efficiency of transmission?

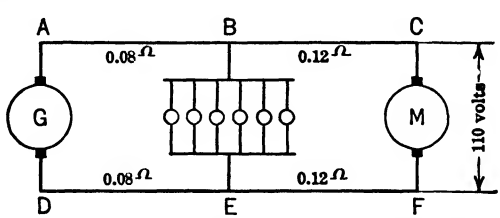



FIG. 17–3. To find the efficiency of transmission.

**Prob. 54–3.** If each lamp in Fig. 17–3 takes 2 amperes, and the motor has a load of 3.5 hp, and an efficiency of 73 per cent, and all other data remain as in Prob. 53–3, what is the efficiency of transmission?

**14. Electric Heating.** The heat developed when an electric current flows through the resistance in the circuits of electrical apparatus, or through a transmission line, is a loss and lowers the efficiency of the equipment. However, use is made of this heating

effect in many electrical devices, the most common probably being the fuse, one form of which is shown in Fig. 18–3. A lead conductor in the fiber tube connects the two terminals. If the current in the circuit becomes too great, the heat developed melts the wire, or the fuse "blows," and opens the circuit. It is an electrical safety valve. Fuses are manufactured to "blow" at any desired current from a fraction of an ampere to several hundred amperes.

Fig. 18–3. A fuse; an electric safety valve. In case of too great a current, a lead wire in the fiber case is melted. This opens the circuit.

Electric toasters, flatirons and stoves also make use of this heating energy, as do many industrial furnaces.

Figure 19*a*–3 is a view of a self-regulating flatiron. The heating element in the base of the iron is shown in 19*b*–3 and the Spencer automatic switch for regulating the temperature, in Fig. 19*c*–3. This switch consists of a thermostatic (bimetallic) metal disc with

Fig. 19a–3. A modern flatiron, the temperature of which is automatically regulated by the Spencer automatic switch. *Westinghouse Electric & Mfg. Co.*

contacts mounted on a base. At the maximum temperature desired (up to 700° Fahrenheit), the expansion and the subsequent curling of this disc cause it to snap, suddenly reversing its curvature and instantly breaking the contact.

Since all the energy consumed in a resistance is converted into heat, the efficiency of an electric heater is practically 100 per cent. There are no waste gases created which carry off a certain percentage of the heat. It is true, however, that the heat-conducting material, surrounding the heating element itself, and the material

upon which it is mounted, do, to some extent, reduce its effectiveness. However, from the standpoint of cleanliness, convenience,

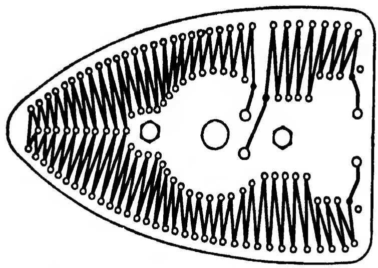


Fig. 19b-3. The heating element in the base of an electric flatiron. *Westinghouse Electric & Mfg. Co.*

ease of control and efficiency, it probably surpasses any other form of heating.

The question, of course, arises as to why it is not used universally for the heating of buildings. The answer is that it is too expensive. By far the greater part of our electrical power to-day is supplied by steam plants with low thermal efficiency, in which probably not more than 20 to 25 per cent of the heat contained in the coal is converted into electrical energy. Even when power is obtained from hydro-electric plants, the cost is high, because of the great first cost of dams and reservoirs, etc.

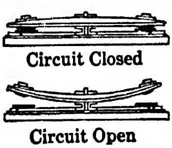



Fig. 19c-3. The Spencer automatic snap switch, which regulates the temperature of the flatiron.

It is true that where power can be obtained cheaply enough, the electrical heating of houses is feasible. In fact, in some towns in the United States, the dwelling houses are all heated electrically.

Two types of electric house heaters are shown in Figs. 20*a*-3 and 20*b*-3.

## SUMMARY OF CHAPTER III

**USE OF WATTMETER. Measures power; is a combination of ammeter and voltmeter. Ammeter side is connected in series with circuit and voltmeter side across circuit. Great care must be used not to get ammeter side across the circuit.**

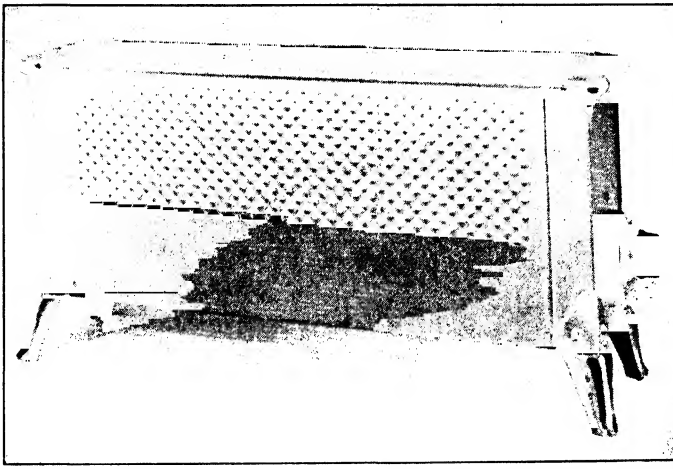


FIG. 20a–3. A 3-kw electric house heater arranged for three-heat operation. *Westinghouse Electric & Mfg. Co.*

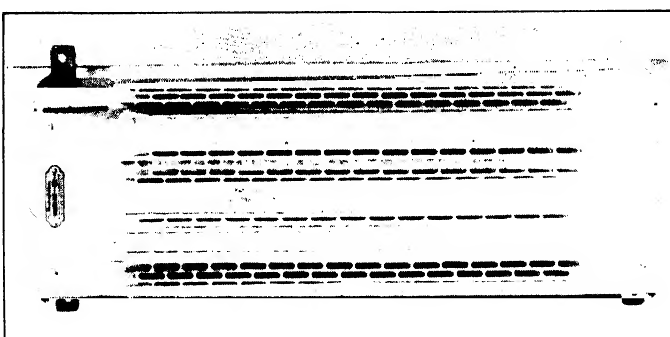


FIG. 20b–3. A heater adapted for wall mounting. *Westinghouse Electric & Mfg. Co.*

## COMPUTATION OF ELECTRICAL POWER

**Watts (power) = amperes × volts.**

**This should be understood to mean that the Watts consumed in any part of a circuit equals the product of the Amperes flowing through that same part of the circuit times the Volts across that same part. Just as Ohm's Law may be written in three ways, so the power equation may be written in three ways, thus:**

$$\mathbf{P = IE = I^2R = \frac{E^2}{R}}.$$

**ELECTRICAL EQUIVALENT OF HEAT ENERGY**

$$
\begin{aligned}
1\ \text{Btu} &= 1055\ \text{watt-seconds.}\\
1\ \text{Calorie} &= 4.2\ \text{watt-seconds.}\\
H &= 0.24\ I^2Rt\ \text{calories.}\\
&= \frac{I^2Rt}{1055}\ \text{Btu.}
\end{aligned}
$$

**MECHANICAL EQUIVALENT OF ELECTRICAL POWER**

**1 Kilowatt = 1.34 horsepower = $1\frac{1}{3}$ horsepower (approx.)**
**1 Horsepower = 746 watts = $\frac{3}{4}$ kilowatt (approx.)**

**EFFICIENCY OF ELECTRICAL MACHINES. The commercial efficiency of an electric machine, as of any machine, may be stated as that percentage which the useful energy given out by the machine is of the energy put into the machine. The size, or rating, of a machine is given in terms of the OUTPUT, while the efficiency and the losses are stated as percentages of the INPUT.**

$$\frac{\text{output}}{\text{input}} \times 100 = \text{percentage efficiency.}$$

**Both values must be given in the same power unit. The above is also true of electrical transmission.**

**ELECTRICAL HEATING APPLIANCES. All energy used is converted into heat and the efficiency is practically 100 per cent. The use of electricity for house heating is economically feasible where the cost of energy is cheap enough.**

## PROBLEMS ON CHAPTER III

**Prob. 55–3.** The motor in Fig. 10–3 takes 25 amperes at 115 volts.

(*a*) What is the terminal voltage of the generator *G*?

(*b*) What is the efficiency of transmission?

**Prob. 56–3.** An engine supplies 180 horsepower to a generator, delivering 500 amperes at 240 volts. What is the commercial efficiency of the generator?

**Prob. 57–3.** At 6.5 cents per kilowatthour, what is the monthly bill for using a flatiron which takes 5.5 amperes at 110 volts? The iron is in use an average of 16 hours per month.

**Prob. 58–3.** Electrical energy is supplied at 5 cents per kilowatthour for driving a 10-horsepower motor. The efficiency of the motor at full load is 82 per cent and it is operated 8 hours a day, 26 days a month. Find the monthly cost of operating the motor.

**Prob. 59–3.** A generator, supplying 250 amperes at 500 volts, is driven by a steam engine. If the efficiency of the generator is 92 per cent, how many horsepower must the engine supply?

**Prob. 60–3.** At 6 cents per kilowatthour, how much will it cost per week of 48 hours to run a motor having an average load of 60 horsepower and an average efficiency of 90 per cent?

**Prob. 61-3.** Each lamp in Fig. 21-3 takes 1.2 amperes. Find:

(*a*) Voltage across *CD* and across *EF*.
(*b*) Volts used in *AC* and *BD*, and in *CE* and *DF*.
(*c*) Watts delivered to each group of lamps.
(*d*) Watts lost in the line.
(*e*) Efficiency of transmission.

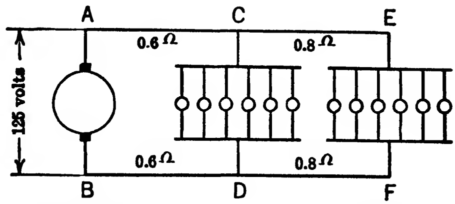



FIG. 21-3. To find the power lost in the line wires and the efficiency of transmission.

**Prob. 62-3.** At Group II, Fig. 22-3, there are five 60-watt 110-volt lamps. At Group I there are four 100-watt, 115-volt lamps. The internal resistance of the generator *G* is 0.4 ohm. Find:

(*a*) Resistance of $AC + BD$.
(*b*) Resistance of $CE + DF$.
(*c*) Emf of the generator.
(*d*) Efficiency of transmission.
(*e*) Average resistance per lamp in Group I.
(*f*) Average resistance per lamp in Group II.

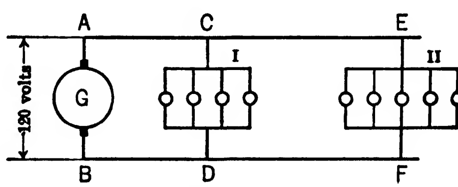



FIG. 22-3. To find the resistance of the line wires and compute the efficiency of transmission.

**Prob. 63-3.** A generator, Fig. 23-3, delivers a current of 90 amperes at a pressure of 120 volts. What power does it supply in kilowatts? What horsepower does it supply?

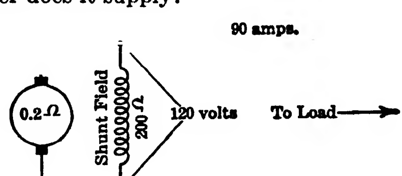



FIG. 23-3. A "shunt" generator delivering 90 amperes. The armature must also supply current to the field winding.

**Prob. 64–3.** The generator in Prob. 63–3, Fig. 23–3, has a shunt-wound field with a resistance of 200 ohms. How many watts are absorbed in the field?

**Prob. 65–3.** The armature of the generator in Prob. 63–3 has a resistance of 0.20 ohm. What is the armature current? What power is lost in the armature?

**Prob. 66–3.** What must be the horsepower of an engine to run a generator, feeding 700 60-watt lamps at 120 volts? Six volts are used in the line; the efficiency of the generator is 92.5 per cent, and the loss in the belt is 1.5 per cent.

**Prob. 67–3.** Assuming an efficiency of 65 per cent for the whole arrangement, what current will be used by a 240-volt motor-driven hoist, when raising 3000 pounds 180 feet per minute?

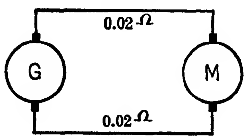


FIG. 24–3. The power supplied by the generator is equal to the power delivered to the motor plus the power lost in the line.

**Prob. 68–3.** The generator, Fig. 24–3, delivers 160 kilowatts to the line. The motor $M$ uses 140 kilowatts. Find:

(*a*) Watts lost in the line. (*b*) Current taken by the motor. (*c*) Voltage on the motor, and terminal voltage of the generator.

**Prob. 69–3.** Assume the internal resistance of the generator in Prob. 68–3 to be 0.045 ohm. The owner of the motor pays the owner of the generator 2.5 cents per kilowatthour for energy delivered to the motor. If it costs the owner of the generator 0.9 cent per kilowatthour to generate electric energy, how much does he earn or lose in a month of 240 hours? Assume that 1.5 kilowatts are used in exciting the shunt field of the generator.

**Prob. 70–3.** The resistance of shunt coil $S$ in the arc lamp in Fig. 25–3 is 380 ohms. Voltage across the terminals of the lamp is 60 volts. The arc, $A$, uses 6.5 amperes at 42 volts. Find:

(*a*) The resistance of $R$.
(*b*) Power consumed by the lamp.
(*c*) Power consumed by each of the resistances $S$ and $R$.
(*d*) The efficiency of the lamp as a machine.

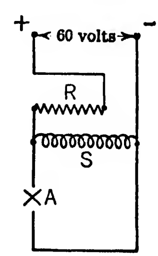


FIG. 25–3. The circuits in an arc lamp. The resistance, R, and the coil, S, consume part of the power supplied to the lamp.

**Prob. 71–3.** What power is consumed in each resistor in Prob. 91–1?

**Prob. 72–3.** A current of 0.5 ampere, flowing through a lamp, generates 0.85 Btu of heat in 16 seconds. (*a*) What is the resistance of the lamp? (*b*) What power is expended in the lamp? Express in watts and in horsepower.

**Prob. 73–3.** A field coil of a generator contains 20 lbs of copper and has a resistance of 200 ohms. Allowing for no radiation of heat, how

fast does the temperature of the coil rise when 2.1 amperes is flowing through the coil? (Specific heat of copper = 0.095, i.e., it takes 0.095 Btu to raise 1 lb of copper 1° F.)

**Prob. 74-3.** A wire having a resistance of 40 ohms is coiled in a vessel containing 300 grams of oil of which the specific heat is 0.75. The vessel is copper and together with the copper wire weighs 200 grams (specific heat = 0.095). What will be the temperature rise after a current of 4.2 amperes has been flowing through the coil for 5 minutes? Neglect radiation.

**Prob. 75-3.** A coil of copper wire immersed in a steel tank containing 100 lbs of water, has a resistance of 20 ohms. The coil weighs 10 lbs and the tank 25 lbs. How long will it take to raise the temperature of the water from 70° to 120° F if 6 amperes is put through the coil? (Specific heat of steel = 0.117.) Neglect radiation.

# CHAPTER IV

## ELECTRICAL PROPERTIES OF WIRE

Transmission lines, distribution systems, power-plant wiring, motors, generators and most electrical equipment are constructed largely of electrical conductors of copper wire or copper bar. We know that all electrical conductors have resistance.

Just as the resistance a pipe line offers to the flow of water depends upon the cross section (area), and the length and smoothness of the pipe; so does the resistance of an electrical conductor depend upon its cross-sectional area, length and the material of which it is made.

In order to use as little voltage as is possible in line wires, in motors and generators, etc., and to keep the heat losses and temperatures of machines as low as possible, it is desirable to construct them with as little resistance as is practicable. Resistors and rheostats are designed and built to have definite amounts of resistance in them.

It is therefore important to be able to determine in advance the resistance of conductors of different sizes and lengths and of different materials.

**1. Area of Conductors. Circular Measure.** The most common form of an electrical conductor is a round copper wire. We will, therefore, consider first the calculation of the resistance of round copper wires of different lengths and diameters. Since the cross section of the wire is round, electrical engineers do not use square units but **circular units** to measure the area. The idea of circular units is a little difficult to grasp, but once mastered, simplifies the calculations to a marked degree.

When we determine the area of a surface, such as the cross-sectional area of a 2 × 4 piece of lumber, we say it has an area of 8 square inches. Its area is equal to that of eight small squares, 1 inch on a side; that is, we get the area in terms of **unit squares**. Note Figs. 1–4 and 2–4.

Suppose it is desired to determine the area of a circle 3 inches in diameter. We might state it in terms of the number of square

inches it contains. Our unit of area again would be the **unit square.** The area of the 3 inch circle would be determined as follows:

$$\text{Area of a circle} = \frac{\pi d^2}{4}, \text{ where } d = \text{the diameter};$$

then

$$\frac{3.1416}{4} \times 3^2 = 0.7854 \times 9 = 7.069 \text{ square inches or } \textbf{unit squares.}$$

But suppose we state the area in circular inches. Our unit of area is then the area of a **circle** one inch in diameter. Our problem then is to determine how many **circles** one inch in diameter are contained in a circle three inches in diameter.

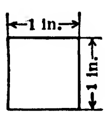


FIG. 1-4. A unit square, or one square inch.

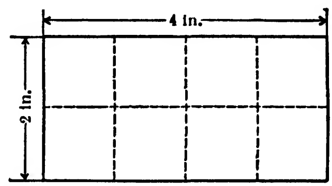


FIG. 2-4. Eight square inches equal to the area of eight unit squares.

The number of square inches in a circular inch equals

$$1 \times 1 \times 0.7854 \text{ or } 0.7854 \text{ square inch.}$$

Since the number of square inches in a circle three inches in diameter equals 7.069 square inches.

The three-inch circle would therefore contain $\frac{7.069}{0.7854}$ or 9 circles one inch in diameter. The number of **circular inches** in a **three-inch circle,** therefore, is the **square of the number of inches in the diameter.** Note Figs. 3-4 and 4-4.

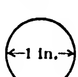


FIG. 3-4. A circle one inch in diameter, or one circular inch or unit circle.

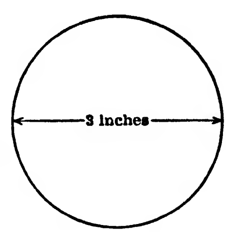


FIG. 4-4. A circle 3 inches in diameter, equal in area to 9 circles one inch in diameter, or to 9 circular inches or unit circles.

In fact it can be shown that the area in circular inches of a circle of any size equals the square of the diameter in inches. The mathematical proof is as follows:

Let $D$ equal diameter of circle.

$$\begin{aligned} \text{Area of circle} &= 0.7854\,D^2 \text{ square inches} \\ \text{Area of circular inch} &= 1 \times 1 \times 0.7854 \\ &= 0.7854 \text{ square inch} \end{aligned}$$

$$\text{Number of circular units in circle} = \frac{0.7854\ D^2}{0.7854} = D^2 \text{ circular inches.}$$

But the circular inch is too large for convenience in measuring the cross-section area of wires, so it is customary to use as a unit, the area of a circle one-thousandth of an inch in diameter. This unit is called the **circular mil,** abbreviated **cir mil.** The term "mil" means one-thousandth of an inch just as the term "mill" in our coinage means one-thousandth of a dollar. We have merely to state the number of thousandths of inches in the diameter in order to express the diameter in mils. Thus a wire having a diameter of 0.025 inch is said to have a diameter of 25 mils; a circle having a diameter of 0.25 inch is said to have a diameter of 250 mils; a circle 2.5 inches is said to have a diameter of 2500 mils.

Since the area of a circle expressed in circular measure is exactly the square of the diameter, the area of a circle expressed in circular mils is the square of the number of mils in the diameter. Thus the circular-mil area of a circle with a 0.025 inch diameter is 25 × 25 or 625 circular mils; of a circle with a 0.25 inch diameter, 250 × 250 or 62,500 circular mils; of a 2.5 inch diameter, 2500 × 2500 or 6,250,000 circular mils.

**Prob. 1–4.** A wire 0.076 inch in diameter has what circular-mil area?

**Prob. 2–4.** What is the circular-mil area of a wire 0.03 inch in diameter?

**Prob. 3–4.** What is the diameter of a wire having a cross-section area of 173,056 circular mils?

**Prob. 4–4.** What is the diameter of a wire containing 3600 circular mils?

**Prob. 5–4.** A wire having an area of 22,500 circular mils has a diameter of how many inches?

**Prob. 6–4.** A wire is $\frac{3}{8}$ inch in diameter. What is the circular-mil area?

Since most electrical conductors are in the form of round wire, the area of **all** conductors, no matter what their shape, is given in circular mils. When the conductor is rectangular or manufactured as a flat ribbon, it is common practice to calculate the area in circular mils. For this reason, it is well to note the relation between circular measure and square measure. Figure 5–4 will help to visualize the difference between a circular mil and a square mil.

Note the relations below:

$$1 \text{ circular mil} = 0.7854 \text{ square mils.} \quad (1)$$

$$1 \text{ square mil} = \frac{1}{0.7854} = 1.273 \text{ circular mils.} \quad (2)$$

$$1 \text{ square inch} = 1{,}000{,}000 \text{ square mils.} \quad (3)$$

$$1 \text{ square inch} = 1{,}273{,}000 \text{ circular mils.} \quad (4)$$

$$1 \text{ circular inch} = 785{,}400 \text{ square mils.} \quad (5)$$

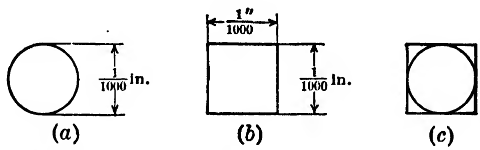



FIG. 5–4. Illustrates the difference between one circular mil and one square mil. (a) A circular mil. (b) A square mil. (c) The circular mil has less area than the square mil.

**Prob. 7–4.** What is the cir-mil area of a square copper conductor $\frac{1}{16}$ inch on a side?

**Prob. 8–4.** A copper bar $\frac{1}{8}$ inch by $\frac{3}{4}$ inch in cross section has how many circular mils area?

**2. Unit Wire. Mil-Foot.** We may, therefore, take for our wire of unit cross section, a wire with a cross-section area of one circular mil. Any wire may then be thought of as composed of a bundle of parallel wires of unit area bound together. The number of circular mils in any given wire is thus merely the number of circular-mil wires which go to make up its cross section.

We know that the resistance of any parallel bundle of similar wires is equal to the resistance of any one of the wires, divided by the number of wires in the bundle. Thus, if we know the resistance of a circular-mil wire of a certain length and material, we can find the resistance of a wire of the same material and length, whatever its diameter. We have merely to divide the resistance of the wire of one circular-mil area by the number of circular mils in the area of the given wire.

**Example 1.** The resistance of a certain wire of one circular mil cross section is 450 ohms. What is the resistance of a wire of the same material and length but having a diameter of 0.016 inch?

$$0.016 \text{ inch} = 16 \text{ mils}$$
$$\text{Area of wire} = 16 \times 16$$
$$= 256 \text{ circular mils}$$

$$\text{Resistance} = \frac{450}{256}$$
$$= 1.76 \text{ ohms.}$$

But the resistance of a wire depends also upon its material and length as well as upon its cross section. It is a familiar fact that the resistance is directly proportional to the length, a wire twice as long as another of the same material and cross section having twice the resistance.

This agrees with our knowledge of the electron theory. In twice the length of the conductor, the electrons would have twice as many collisions with the molecules, and, hence, would meet with twice the opposition to their flow.

We have, therefore, only to know the resistance of one foot of wire having a circular mil cross section, to be able to find the resistance of any length of wire having any cross section and made of the same material. This wire having a cross-section area of one circular mil and a length of one foot is the unit of wire and is said to be a **mil-foot wire.** The resistance of a mil-foot of commercial copper wire at 20° C is 10.37 ohms. For ordinary work, correct to three figures, this value may be taken as 10.4 ohms. Knowing this, we may find the resistance of any length of any size copper wire. It is merely necessary to multiply the resistance of a mil-foot by the length and divide by the circular-mil area.

**Example 2.** What is the resistance of 2000 feet of copper wire 0.064 inch in diameter?

Resistance of 1 foot of copper wire 1 mil in diameter = 10.4 ohms.

Resistance of 2000 feet of copper wire 1 mil in diameter = $2000 \times 10.4 = 20{,}800$ ohms.

Resistance of 2000 feet of copper wire 64 mils in diameter $= \dfrac{20{,}800}{64 \times 64}$ $= 5.08$ ohms.

This may be expressed in the form of an equation.

$$R = \frac{Kl}{d^2}; \tag{6}$$

where $R$ = resistance of wire in ohms;
$K$ = resistance of 1 mil-foot in ohms;
$l$ = length in feet;
$d$ = diameter in mils;

or $d^2$ = section area in circular mils.

The resistance per mil-foot ($K$) is called the **resistivity** or **specific resistance** of a material, and varies with different materials. See Appendix, Table I.*

Using this equation, the above example is solved as follows:

$$R = \frac{Kl}{d^2}$$
$$= \frac{10.4 \times 2000}{64 \times 64}$$
$$= 5.08 \text{ ohms.}$$

**Prob. 9–4.** A copper wire has a resistance of 1.5 ohms and a diameter of 0.05 inch. How long is the wire?

**Prob. 10–4.** What is the resistance of 1200 feet of copper wire, $\frac{5}{32}$ inch in diameter?

**Prob. 11–4.** What diameter must a copper wire have in order that one mile of it may have a resistance of 1.25 ohms?

**Prob. 12–4.** What is the resistance of 3 miles of 42 mil copper wire?

**Prob. 13–4.** 600 feet of copper wire has a resistance of 0.5625 ohm. What is its circular-mil area?

**Prob. 14–4.** What is the diameter of the wire in Prob. 13–4?

**Prob. 15–4.** What is the resistance of 3.5 miles of copper wire $\frac{5}{16}$ inch in diameter?

**Prob. 16–4.** How many miles of copper wire $\frac{5}{8}$ inch in diameter will it take to make 3 ohms?

**Prob. 17–4.** The distance between a motor and a generator is 750 feet. The copper line wires are 0.183 inch in diameter. What is the resistance of the line?

**Prob. 18–4.** Ordinary fixture wire has a diameter of 0.064 inch. What is the resistance per 1000 feet?

**Prob. 19–4.** Annunciator wire generally has a diameter of 0.04 inch. How many feet does it take to make 1.5 ohms resistance?

**Prob. 20–4.** How many feet of the wire in Prob. 18–4 does it take to make 1.25 ohms?

**Prob. 21–4.** What is the resistance of 5 miles of copper wire $\frac{9}{16}$ inch in diameter? Use hard drawn copper.

**Prob. 22–4.** How much current can be transmitted over a copper wire 0.0875 inch in diameter and $\frac{3}{4}$ mile long, with but 12 volts "line drop"?

* Resistivity is often given in **microhms** $\left(\frac{1}{1{,}000{,}000} \text{ ohms}\right)$ per centimeter cube of the material.

**Prob. 23–4.** Each arc lamp, Fig. 6–4, takes 6.5 amperes at 80 volts. The distance between lamps is 300 feet. The lamps nearest the generator are 300 feet from it. What size wire is used for the line wires?

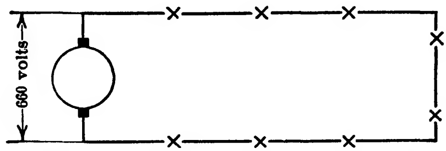



FIG. 6–4. The voltage drop over the lamps plus the line drop equals the terminal voltage of the generator.

**Prob. 24–4.** What line drop is there in a three mile copper trolley wire (hard drawn) carrying 300 amperes, if the wire is 0.85 inch in diameter?

**Prob. 25–4.** Each lamp in Fig. 7–4 takes 1.5 amperes at 115 volts. The line wire is $\frac{1}{13}$ inch in diameter. What is the terminal voltage of the generator?

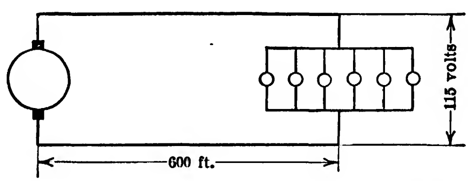



FIG. 7–4. The drop in the line, with any current, can be determined if the diameter and length of wire are known.

**Prob. 26–4.** What will be the drop per mile in a line consisting of hard drawn copper wire $\frac{1}{8}$ inch in diameter, carrying 32 amperes?*

**Prob. 27–4.** 15 kilowatts are transmitted at 240 volts over a copper line 0.824 inch in diameter. What will be the line drop in volts and loss in watts per mile? Use hard drawn wire.

**Prob. 28–4.** A motor takes 26.5 amperes and is 2000 feet from the generator. The line drop must not exceed 5.5 volts. What size hard drawn copper wire must be used?

**3. Specific Resistance of Metals other than Copper.** Although copper, on account of its low resistivity, is the metal most widely used for electrical conductors, aluminum and even galvanized iron are sometimes used. The resistivity of aluminum is 17.0 ohms per mil-foot at 20° C, about 1.6 that of copper. But its low specific gravity more than counterbalances this, so that for equal lengths

* Voltage drop per mile in a transmission line is understood to mean drop in both wires per mile of line, in distinction from drop per mile of line wire, which means the drop in one wire.

and weights aluminum wire has less resistance than copper, and for this reason is coming into more general use.

The resistivity of iron and steel is about seven times that of copper. These materials, therefore, can be used only where a conductor of a large cross section can be installed, as in the case of a third rail, or where very little current is to be transmitted, as in the case of the telegraph. Some alloys of copper, nickel, zinc, manganese, chromium, etc., have a resistance of more than 600 ohms per mil-foot. For the resistivity of various pure metals and alloys, see Appendix, Table I.

**Prob. 29–4.** What is the resistance of a mile of aluminum wire 0.234 inch in diameter?

**Prob. 30–4.** What cross section must a steel rail have if the resistance is to be equal to that of a copper wire 0.64 inch in diameter? Use 90 ohms as specific resistance of the steel rail.

**Prob. 31–4.** How many feet of iron wire 0.125 inch in diameter are required in a coil which is to have a resistance of 11.2 ohms? Use 75 ohms as specific resistance of iron wire.

**4. Effect of Temperature on Resistance. Temperature Coefficient.** It can be shown by experiment that the resistance of all pure metals varies with the temperature. With the increase of current in a conductor more electrons are in motion, with, of course, more collisions with the molecules. Hence, a greater agitation of the molecules results, which in itself increases the temperature of the conductor. The electrons will now have more collisions with the molecules in greater motion, and thus the resistance is increased.

Since electrical machinery operates at temperatures considerably above that of the surrounding air, it is necessary to know the relation between resistance and temperature.

In Art. 2, the resistance of a mil-foot of copper was given as 10.4 ohms at 20° C. For aluminum it is 17 ohms at 20°. All computations based on these values will give the resistance at 20° C.

For each degree (Centigrade) the temperature of copper wire rises above 20° C, the resistance increases 0.393 of 1 per cent. For each degree the temperature drops below 20° C, the resistance decreases 0.393 of 1 per cent. Stating this fact another way — **for every degree change** in temperature **from 20° C** the resistance of the wire **changes 0.00393 ohms for every ohm resistance** it has **at 20° C.**

This change in resistance **per ohm per degree** is called the

**Temperature Coefficient of Resistance.** For all pure metals, this coefficient has nearly the same value. See Appendix, Table I.

**Example 3.** The resistance of a coil of copper wire at 20° C is 48 ohms. What will be the resistance of the coil at 50° C?

Increase in resistance per degree rise = $48 \times 0.00393 = 0.1887$ ohm.
Temperature rise = $50 - 20 = 30°$.
Increase in resistance for 30° rise = $30 \times 0.1887 = 5.66$ ohms.
Resistance at 50° C = $48 + 5.66 = 53.66$ ohms.

**Example 4.** The resistance of a field coil at 20° C is 200 ohms. What will be the resistance at 5° C?

Decrease in resistance per degree fall = $200 \times 0.00393 = 0.786$ ohms.
Temperature fall = $20° - 5° = 15°$.
Decrease in resistance for 15° fall = $15 \times 0.786 = 11.79$ ohms.
Resistance at 5° C = $200 - 11.79 = 188.21$ ohms.

The process used in the examples above may be expressed as an equation.

$$R_{t_1} = R_{t_{20}} + R_{t_{20}} \times 0.00393 \times (t_1 - t_{20}) \tag{7}$$

where $R_{t_1}$ = resistance at any temperature $t_1$
$R_{t_{20}}$ = resistance at temperature of 20° C ($t_{20}$).

Equation (7) can be written

$$R_{t_1} = R_{t_{20}} [1 + 0.00393(t_1 - t_{20})]^* \tag{8}$$

It is suggested that the student solve the examples above by the use of equation (8).

If the resistance at some temperature other than that at 20° C is known, in the use of the above equation, the resistance at 20° must first be found before solving for resistance at the desired temperature.

**Example 5.** The resistance of a coil of copper wire is 150 ohms at 8° C. What will it be at 60° C?

* 20° C or 68° F is here taken as normal room temperature, or **cold** temperature. Other temperatures may be taken as normal provided the coefficient for those temperatures is known. Many resistance tables are based on 0° C or 25° C. Equation (8) may be expressed for any temperature, as follows:

$$R_{t_1} = R_{t_2} [1 + A_{t_2}(t_1 - t_2)]$$

where $R_{t_1}$ = resistance at temperature $t_1$
$R_{t_2}$ = resistance at temperature $t_2$
$A_{t_2}$ = temperature coefficient at temperature $t_2$.

First Step. ($R$ at 20°)

$$150 = R_{20}\,[1 + 0.00393\ (8 - 20)]$$
$$150 = R_{20}\,[1 + 0.00393 \times (-12)] = R_{20}\ (1 - 0.0472)$$
$$150 = R_{20} \times 0.9528. \quad R_{20} = \frac{150}{0.9528} = 157.4 \text{ ohms.}$$

Second Step

$$R_{60} = 157.4\,[1 + 0.00393\ (60 - 20)]$$
$$= 157.4\ (1 + 0.1572 = 15.74 \times 1.1572 = 182.14328 \text{ ohms.}$$

**Method of Similar Triangles.** When the resistance of **copper** is measured through the range of ordinary temperatures, for example, from 100° C to 0° C, and the results plotted as a curve between temperature and resistance, the results lie on a straight line, which if continued, reaches the point of zero resistance at −234.5° C, as shown in Fig. 8–4.

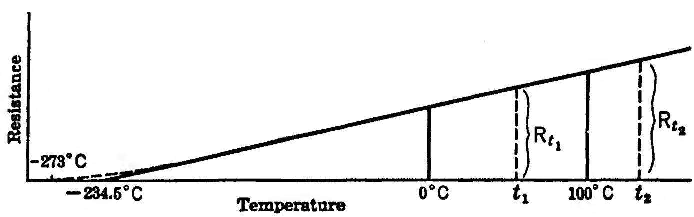



Fig. 8–4. A curve showing the relation between resistance and temperature of copper. Through the ordinary range of temperature this curve is straight, and change in resistance can be computed by the method of similar triangles.

This curve probably bends at very low temperatures and reaches the point of zero resistance at −273° C, or absolute zero. The **slope** of the curve for **other metals than copper** would be **slightly different** from that in Fig. 8–4, and if extended, would reach the zero point of resistance at a slightly different temperature. For aluminum this zero point is −237.7° C. From Fig. 8–4 and the relation of similar triangles we can write:

$$\frac{R_{t_1}}{R_{t_2}} = \frac{234.5 + t_1}{234.5 + t_2} \tag{9}$$

and Example 5 can be solved without the use of a temperature coefficient, as follows:

$$\frac{R_{60}}{150} = \frac{234.5 + 60}{234.5 + 8} \quad \text{or}$$

$$R_{60} = \frac{(234.5 + 60) \times 150}{234.5 + 8} = \frac{294.5 \times 150}{242.5} = 182.14328 \text{ ohms.}$$

**Prob. 32–4.** The resistance of a copper wire is 5.60 ohms at 20°. What is the resistance at 95° C?

**Prob. 33–4.** The resistance of an armature winding is 0.26 ohm at 20°. When the armature becomes heated to 80° C what will the resistance be?

**Prob. 34–4.** If the resistance of a coil of copper wire is 140 ohms at 20° C, what will the resistance become at a temperature of −9° C?

**Prob. 35–4.** What will the resistance of a coil of copper wire become at −10° C if the resistance is 100 ohms at 0° C?

**Prob. 36–4.** A coil of aluminum wire has a resistance 14 ohms at 20° C. What will the resistance be at 90° C.

**Prob. 37–4.** The resistance of a copper field coil is 120 ohms at 16° C. What will the resistance be at 150° C?

**Prob. 38–4.** What will the resistance of a copper line wire be at 5° C, if the resistance at 60° C is 6.24 ohms?

**5. Temperature Change Measured by Change in Resistance.** The size or rating of a generator or motor is determined by the ability of the insulation of the coils and windings to withstand the temperature created by the current, without burning, or deteriorating. Thus the temperature of any machine limits its output. Machines are, therefore, generally rated and sold under a guarantee that the temperature will not rise above a certain value when run continuously under full load. These temperature limits vary with the type of insulation used. It is, therefore, important to know the temperature of the windings.

By measuring the resistance of the coils at room temperature (20° C or 68° F), and then again after the machine has been run, it is possible to calculate the temperature from the resistance readings by use of equation (7) which can be transposed and expressed as,

$$\frac{R_{t_1} - R_{t_{20}}}{R_{t_{20}} \times 0.00393} = t_1 - t_{20} = \text{temperature rise}; \qquad (10)$$

or

$$\frac{R_{t_1} - R_{t_{20}}}{R_{t_{20}} \times 0.00393} + t_{20} = t_1 = \text{temperature of coil.} \qquad (11)$$

**Example 6.** The resistance of the field winding of a motor was 42 ohms at 20 C. After a run of 6 hours the resistance was measured and found to be 48.4 ohms. What is the temperature rise in the coil? What is the temperature of the coil?

$$\frac{48.4 - 42}{42 \times 0.00393} = 38.78° = \text{temperature rise.}$$

$$38.78 + 20 = 58.78° = \text{temperature of the coil.}$$

Equation (9) can also be transposed and written:

$$R_{t_1}\frac{(234.5 + t_2)}{R_{t_2}} - 234.5 = t_1 = \text{temperature of the coil,} \qquad (13)$$

where $t_2$ = initial temperature;

and Example 6 can again be solved as follows,

$$\frac{48.4\ (234.5 + 20)}{42} - 234.5 = 293.27 - 234.5 = 58.78° = \text{temp. of coil.}$$

Temperature rise $= t_1 - t_{20} = 58.78° - 20° = 38.78°$.

**Prob. 39–4.** A coil of copper wire has a resistance of 14.5 ohms at 20° C. How high will the temperature be when the resistance becomes 16.4 ohms?

**Prob. 40–4.** The cold resistance (20° C) of an armature was 0.345 ohm. The hot resistance was 0.493 ohm. What was the temperature rise?

**Prob. 41–4.** It is generally specified that the temperature of the field coils of a dynamo must not rise more than 55° C on continuous full load. The resistance of a set of field coils was 82.5 ohms at 20° C. After a run of 8 hours at rated load the resistance became 93.5 ohms. Did the machine meet the usual specifications?

**Prob. 42–4.** A length of aluminum wire has a resistance of 2.45 ohms at 20° C. What is the temperature of the wire if the resistance increases to 3.15 ohms?

**Prob. 43–4.** The resistance of certain coils in a machine was found to be 8.42 ohms at 40° C. It was specified that on continuous full load, the temperature of the coils should not exceed 95° C (the usual limit). The resistance of the coils measured after a long run, was found to be 10.76 ohms. Did the machine meet this specification?

**6. Temperature Coefficient of Alloys, etc.** It has been stated that the temperature coefficient of resistance for all pure metals is nearly the same, that is, somewhere about 0.4 per cent. Alloys, though of a much higher resistance per mil-foot have much lower coefficients, some having practically zero and even negative coefficients at certain temperatures.

"Manganin," for instance, an alloy consisting of copper, nickel and iron-manganese, has a resistance per mil-foot of from 250 to 450 ohms, according to the proportions of the different metals used, and a temperature coefficient so low as to be practically negligible.

Certain substances, notably carbon, porcelain, glass, electrolytes, etc., decrease in resistance very rapidly when heated. The cold resistance of a carbon lamp filament is about twice as great as the "hot" resistance. The filaments of tungsten lamps

are pure metal and accordingly have a positive coefficient which is about 0.0039.

**7. Copper-Wire Tables.** Tables have been prepared by the Bureau of Standards and adopted by the A. I. E. E. which give the resistance of 1000 feet of standard annealed copper wire of different standard sizes and several temperatures. The sizes are designated by gauge numbers, diameter in mils, and section area in circular mils, etc. There are several standard wire gauges. B. & S. (Brown and Sharpe) is in general use in America, and is commonly called A. W. G. (American Wire Gauge). The B. W. G. (Birmingham Wire Gauge) is in general use in Great Britain. Table No. II in the Appendix, gives the complete data for the American or Brown and Sharpe gauge numbers. By means of these tables it is easy to find the resistance of any length of wire of a given standard section area, etc.

**Example 7.** What copper wire (B. & S. gauge) should be used to transmit electric power 2 miles (out and back); resistance not to exceed 2.7 ohms; temperature to be assumed, 20° C?

**Solution.**

$$\text{2 miles} = 2 \times 5280 = 10{,}560 \text{ feet.}$$

$$\text{2.7 ohms for 2 miles} = \frac{2.7}{10.56} \text{ ohm per thousand feet}$$

$$= 0.256 \text{ ohm per thousand feet.}$$

From wire table:

$$\text{No. } 5 = 0.3133 \text{ ohm per thousand feet;}$$
$$\text{No. } 4 = 0.2485 \text{ ohm per thousand feet.}$$

No. 4 must be used in order not to exceed the limit of 0.256 ohm per thousand feet.

**Note:** If the following simple facts concerning the copper wire table are memorized, the gauge number and resistance and size of any wire can be roughly estimated without reference to the table.

No. 10 wire is practically $\frac{1}{10}$ inch (100 mils) in diameter, 10,000 circular mils in area and has practically 1 ohm per 1000 feet. As the wires grow smaller, every third gauge number **halves** the end area and **doubles** the resistance. For instance, No. 13 has about 5000 circular mils area and 2 ohms resistance; No. 16 has 2500 circular mils area, and 4 ohms per 1000 feet, etc. As the wires increase in size, every third gauge number doubles the circular mils area and halves the resistance; No. 7, for instance, has practically 20,000 circular mils, and 0.5 ohm per 1000 feet, etc.

Another simple method for remembering the approximate resistances and weights of the different gauge sizes of copper wires is given in circular No. 31 of the Bureau of Standards.

Note that the resistance of No. 0 is 0.1 ohm, of No. 1 is 0.125 ohm and of No. 2 is 0.16. The next number in each column is the fourth gauge number and the resistance is doubled in each case.

Similarly for gauge No. 10, 11 and 12, the resistances are just 10 times those for No. 0, 1 and 2, respectively, etc. It is merely necessary to remember that for gauge No. 0, 1 and 2 the resistances are 0.1, 0.125 and 0.16 and that for every fourth number, the resistance doubles.

The weight may be approximated from the fact that 1000 feet of No. 0 weighs approximately 320 pounds and that the weight varies inversely with the resistance.

| Gauge No. | Ohms per 1000 feet | | |
|---|---|---|---|
| 0 | 0.1 | | |
| 1 | | .125 | |
| 2 | | | .16 |
| 3 | .2 | | |
| 4 | | .25 | |
| 5 | | | .32 |
| 6 | .4 | | |
| 7 | | .50 | |
| 8 | | | .64 |
| 9 | .8 | | |
| 10 | 1 | | |
| 11 | | 1.25 | |
| 12 | | | 1.6 |
| 20 | 10 | | |
| 21 | | 12.5 | |
| 22 | | | 16 |

**Example 8.** If the line in Example 7 is to work at a temperature of 45° C, what number wire will be required?

**Solution.** The resistance is not given in the tables at 45° C, but at 20° C. We must then find out what resistance a wire will have at 20° C, which has 2.7 ohms at 45° C.

This can be found as in the first step in Example 5, or by use of equation (9). Using equation (9)

$$\frac{R_{20}}{R_{45}} = \frac{234.5 + t_{20}}{234.5 + t_{45}}$$

$$\frac{R_{20}}{2.7} = \frac{234.5 + 20}{234.5 + 45}$$

or $$R_{20} = \frac{254.5}{279.5} \times 2.7 = 2.46 \text{ ohms for 2 miles at } 20° \text{ C}$$

$$\text{Resistance per 1000 feet} = \frac{2.46}{10.56} = 0.233 \text{ ohm.}$$

The problem thus becomes: What size of wire has a resistance of 0.233 ohm per 1000 feet?

From Table II:

No. 4 has a resistance of 0.2485 ohm per 1000 feet;
No. 3 has a resistance of 0.1470 ohm per 1000 feet.

No. 3, therefore, must be used in order not to exceed 0.233 ohm per 1000 feet.

**Prob. 44–4.** What size wire (B. & S.) will have a resistance of practically 1 ohm per 1000 feet?

**Prob. 45–4.** Thirty 60-watt 115-volt lamps are to be used in a building. 240 feet of feed wires are required (each way) from the generator to the distributing point. What size wire should be used in order that there shall not be more than 2.25 volts drop in these feeders?

**Prob. 46–4.** How far can 40 amperes be transmitted through No. 6 wire, B. & S. gauge, with 6 volts line drop?

**Prob. 47–4.** What size wire, B. & S., has about 1.6 ohms per mile?

**Prob. 48–4.** How many miles of No. 0 wire will it take to make 3 ohms?

**Prob. 49–4.** A transmission line 600 feet long is to carry 100 amperes with not more than 20 volts line drop. The line works at 50° C. What size of wire (B. & S.) should be used?

**Prob. 50–4.** A coil for an electromagnet has 1200 turns of No. 22, B. & S., copper wire. The average length of a turn is 17 inches. What is the resistance of the coil?

**Prob. 51–4.** It is desired to construct a coil having about 120 ohms resistance. The coil must have 500 turns of 18 inches average length. What size wire, B. & S., should be used?

**8. Stranded Wire.** On account of their greater flexibility, stranded cables are often used instead of solid wire. Such a cable is much easier to pull into conduit, and less likely to break when bent at a sharp angle. When a size of wire larger than No. 0000 is required, it is nearly always made in strands rather than solid, but even the smaller sizes are also common in the stranded form.

For instance, instead of using a solid No. 4 wire, having a diameter of 204 mils and an area of 41,700 circular mils, it is much easier to use a cable made up of 7 wires, each 0.077 inch in diameter. Each strand (wire) would then have an area of 77 × 77, or 5930 circular mils. But since the cable is made up of 7 of these strands, the area of the cable would be 7 × 5930 equals 41,600 circular mils, which is practically the area of a No. 4 solid wire. The diameter of a stranded wire will always be slightly greater than that of a solid wire of the equivalent cross section.

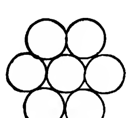


Fig. 9–4. A 7-strand concentric lay cable.

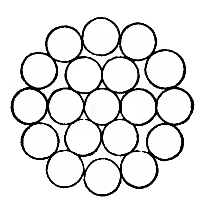


Fig. 10–4. A 19-strand concentric lay cable.

In the grouping of wires to make up a concentric cable, 6 round strands will just encircle one of equal size, making a 7-strand cable. In the next layer 6 + 6 or 12 strands will just encircle 6. Six additional strands are needed in each succeeding layer as shown in Figs. 9–4 and 10–4.

Concentric lay cables are thus built up:

$$1 + 6 = 7 \text{ strands}$$
$$1 + 6 + 12 = 19 \text{ strands}$$
$$1 + 6 + 12 + 18 = 37 \text{ strands}$$
$$1 + 6 + 12 + 18 + 24 = 61 \text{ strands}$$
$$1 + 6 + 12 + 18 + 24 + 30 = 91 \text{ strands, etc.}$$

Table III in the Appendix gives the resistance at 20° C per 1000 ft for the larger sizes of stranded cable.

**Prob. 52–4.** To what size, B. & S., is a stranded cable equivalent, which is made up of 19 strands each 0.059 inch in diameter?

**Prob. 53–4.** It is desired to make a cable of 37 strands, which shall be equivalent to a No. 00 B. & S. solid conductor. What size strands should be used?

**Prob. 54–4.** How many strands 0.061 inch in diameter will it take to make a cable equivalent to a No. 6 wire?

**Prob. 55–4.** It is desired to make a very flexible cable, equivalent to No. 4 B. & S. wire. If strands of No. 22 B. & S. wire are used, how many will be required?

**Prob. 56–4.** If ninety-one No. 18 B. & S. wires are used to make up a cable, to what size wire is the cable equivalent?

**9. Aluminum.** We have noted that although the resistance of aluminum wire is 17 ohms per mil-foot (practically 1.63 times that of copper) its weight is only about 0.3 that of copper. For this reason some transmission lines are strung with aluminum wire. While this necessitates a larger wire for the same resistance per 1000 feet, the weight of such a wire will be less than that of a copper conductor. However, such a line possesses the disadvantage over a copper line in that aluminum melts at a lower temperature than copper. A short circuit which would burn up but a few inches of a copper wire is likely to burn out long sections of an aluminum line. Aluminum line wires also offer greater surface to the wind and will accumulate a greater weight of sleet per foot than will copper conductors.

**Example 9.** What size would an aluminum wire be which has the same resistance as a No. 8 B. & S. copper wire?

**Solution.**

No. 8 copper wire has a resistance of 0.6282 ohm per 1000 feet. The resistance of an aluminum wire is found from the equation

$$R = \frac{17 \times l}{d^2}.$$

Thus, $$0.6282 = \frac{17 \times 1000}{d^2}$$

or, $$d^2 = \frac{17 \times 1000}{0.6282} = 27{,}000 \text{ cir mils.}$$

This would require an aluminum wire of gauge No. 6 of 26,250 circular mils. No. 7 has a cross-section area of only 20,800 circular mils; and No. 5 has a cross section of 33,100 circular mils; a No. 6 aluminum wire would be called the equivalent of No. 8 copper wire.

**Example 10.** How would the weight per 1000 feet of the equivalent aluminum wire in Example 9 compare with the weight of copper wire?

**Solution.**

$$\begin{aligned}
\text{Weight of 1000 feet of No. 8 copper wire} &= 50 \text{ pounds.} \\
\text{Weight of 1000 feet of No. 6 copper wire} &= 79.5 \text{ pounds.} \\
\text{Weight of 1000 feet of No. 6 aluminum wire} &= 0.3 \times 79.5 \\
&= 23.85 \text{ pounds.}
\end{aligned}$$

Thus the weight of the equivalent aluminum wire is only $\frac{23.85}{50}$ or 0.477 that of the copper wire.

**Prob. 57–4.** What size aluminum wire will have the same resistance per mile as a No. 6 B. & S. copper wire?

**Prob. 58–4.** What size copper wire is equivalent to a No. 0 aluminum wire in resistance per 1000 feet?

**Prob. 59–4.** An aluminum cable is to be made up of 19 strands which will be equivalent to a No. 0 solid copper wire. What size must the strands be?

**Prob. 60–4.** If ninety-one No. 19 aluminum wires are used in making up a cable, to what size solid copper is such a cable equivalent?

**10. Copper Clad Steel Wire.** On account of its cheapness and great tensile strength, copper clad steel wire is used to some extent for trolley wires and transmission lines. This type of wire consists of a steel core to which has been welded a covering of copper. The resistance per mil-foot of such wire depends upon the relative size of the copper and steel cross-section areas. One company has put on the market two grades; one called 30 per cent conductivity and the other 40 per cent conductivity. This rating merely means that a copper wire will have 30 per cent and 40 per cent, respectively, the resistance of the copper clad steel wire of the same size.

In the 30 per cent conductivity wire, the area of the steel core is 79.5 per cent of the entire cross section, while the copper has 20.5 per cent area. In the 40 per cent conductivity wire, the steel makes up 68.2 per cent and copper 31.8 per cent of the total area.

**Prob. 61-4.** In a No. 00 " copper clad " trolley wire of 30% conductivity, how many circular mils would there be of steel and copper respectively?

**Prob. 62-4.** What would be the resistance of 1000 feet of the steel core of the trolley wire in Prob. 61-4? Mil-foot resistance of steel = 86.7 ohms.

**Prob. 63-4.** What would be the resistance of 1000 feet of the copper cover of the trolley wire of Prob. 61-4?

**Prob. 64-4.** From the resistance of the steel core and copper covering of 1000 feet of the trolley wire in Prob. 61-4, compute the resistance of the trolley wire. Would 1000 feet of a copper wire of the same size have 30% as much resistance?

**11. Insulated Wire.** For wiring in buildings **Rubber Covered** wire is used almost exclusively, and is specified by the National Board of Fire Underwriters for most work. It consists of a vulcanized rubber compound, moulded over the wire, which has been previously tinned. Over this is a braided covering of cotton thread. The whole makes a rugged protection against mechanical injury and is impervious to moisture.

**Varnished Cambric** insulation is allowed in certain cases. The wire is wrapped spirally with layers of cotton tape, treated with an insulating varnish, and impregnated during wrapping with an insulating compound. This is covered with a cotton braid. Such insulation, however, is **not** impervious to moisture.

**Weatherproof wire** is covered with three braids of fibrous cotton covering, impregnated with a waterproof compound. It is used exclusively for outdoor service.

**Magnet wire** is used for winding the field coils, and often the armatures of generators and motors. It is also used in the construction of transformers, etc. The wire is wrapped with one to three layers of fine cotton or silk thread.

**Enameled wire** has a thin layer of enamel baked on the conductor, and may in addition be wrapped with a cotton or silk covering. This may be used in place of magnet wire.

**Asbestos covered wire** is used in installations where the wire is subjected to comparatively high temperatures, as in rheostat leads, etc. Mechanically, this covering is not strong.

**12. Safe Carrying Capacity for Copper Wires.** In installing wire in buildings, it is necessary to take into account another factor besides the voltage drop when determining the size to be used. It is a fact of common experience that an electric current heats

*any conductor through which it passes. If heat is generated in the* wire faster than it can be radiated from the surface of the wire, the temperature will continue to rise as long as this condition exists. It is necessary, therefore, to select a wire which will radiate the heat generated by the current at such a rate that the temperature never rises high enough to cause the insulation to deteriorate.

Accordingly, the National Board of Fire Underwriters has issued a table of the safe current-carrying capacity of copper wire of the sizes used in house wiring. Wherever local regulations do not specify otherwise, the currents carried by any interior wiring should not exceed the values given in this table. Table IV, Appendix.

Wire smaller than No. 14 must not be used in interior work. This rule is necessary in order that the wires may have sufficient mechanical strength. No wire smaller than No. 18 may be used in fixture wiring, or for flexible cords.

**Example 11.** It is desired to install a conductor to carry 40 amperes. What size copper wire should be used?

From the table, No. 6, rubber insulated wire will carry 50 amperes and is the size to be used.

If weatherproof wire can be used, No. 8 will do.

**Prob. 65-4.** What size rubber-covered wire should be used when it is necessary to carry 15 amperes?

**13. Determination of Right Sizes for Interior Wires.** In deciding upon the wire sizes which should be used in the different parts of any interior distributing system, it is necessary to take into consideration two factors:

First: The size in each section must be such that the current in no wire exceeds the amount given in the Underwriters' Table of safe carrying capacities for wires.

Therefore, it is necessary to determine accurately the current which each wire must carry and select its size from the above table.

Second: The voltage drop throughout the system must then be computed in order to make certain that it does not exceed a certain value. For if lamps are to be operated anywhere on the system a variation of more than 5 per cent in the voltage at the lamps causes an unpleasant variation in the illumination. If the entire load consists of motors, heating appliances, etc., a drop of 10 per

cent is usually allowable. Any greater drop than this, however, would have a bad effect upon the speed of the motors.*

**Example 12.** The panel board in Fig. 11–4 is situated 150 feet from the main switch. From the board run three branch lines. Each branch is supplied with 12 outlets for 50-watt, 110-volt lamps. What size must the mains be?

**Solution.** Each branch line carries $12 \times 50$ or 600 watts. This means $\frac{600}{110}$ or 5.45 amperes in each branch wire.**

**Branch wires. Size according to Table IV.**

Although according to the table, No. 16 wire would carry this current safely, it will be noted that no size smaller than No. 14 can be installed.

**Mains. Size according to Table IV.**

Each main must carry the current in all three branches or $3 \times 5.45 = 16.35$ amperes.

According to Table IV, No. 12 wire must be used as the next size in common use, No. 14 can carry but 15 amperes.

**Check on above sizes for voltage drop.**

| | |
|---|---|
| Distance from panel to load center*** of branches | = 50 feet |
| Length of wire in each branch = $2 \times 50$ | = 100 feet |
| Resistance of 1000 feet No. 14 (Table I) | = 2.525 ohms |
| Resistance of 100 feet of No. 14 = $\frac{1}{10}$ of 2.525 | = 0.253 ohm |
| Voltage drop in each branch = $5.45 \times 0.253$ | = 1.38 volts |
| Length of mains = $2 \times 150$ feet | = 300 feet |
| Resistance of 1000 feet of No. 12 (Table I) | = 1.588 ohms |
| Resistance of 300 feet of No. 12 = 0.3 of 1.588 | = 0.476 ohm |
| Voltage drop in mains = $16.35 \times 0.476$ | = 7.78 volts |
| Total drop from main switch to lamps = $7.78 + 1.38$ | = 9.16 volts |

Percentage line drop $= \frac{9.16}{110} = 8.3$ per cent.

This is nearly twice as great a drop as is allowed in good practice because the brightness of the lamps would vary through wide ranges depending on how many were in use at one time. When only a few lamps were on, the voltage at these lamps would be about the same as that at the main switch, $110 + 9.16$ or about 119 volts. This would cause the lamps to glow far above their rated candle power and would either burn them out at once or shorten their life to a small per cent of the normal rating. It would, therefore, be necessary to install larger than No. 12 mains. Let us try No. 10.

* The drop may be figured as a percentage of either the voltage at the load, or at the service switch.

** In calculating the voltage drop in any system it is customary to figure the current on the basis of rated watts at normal voltage.

*** The load center is that point on the branch line at which for convenience in calculation, all the lamps may be considered to be concentrated.

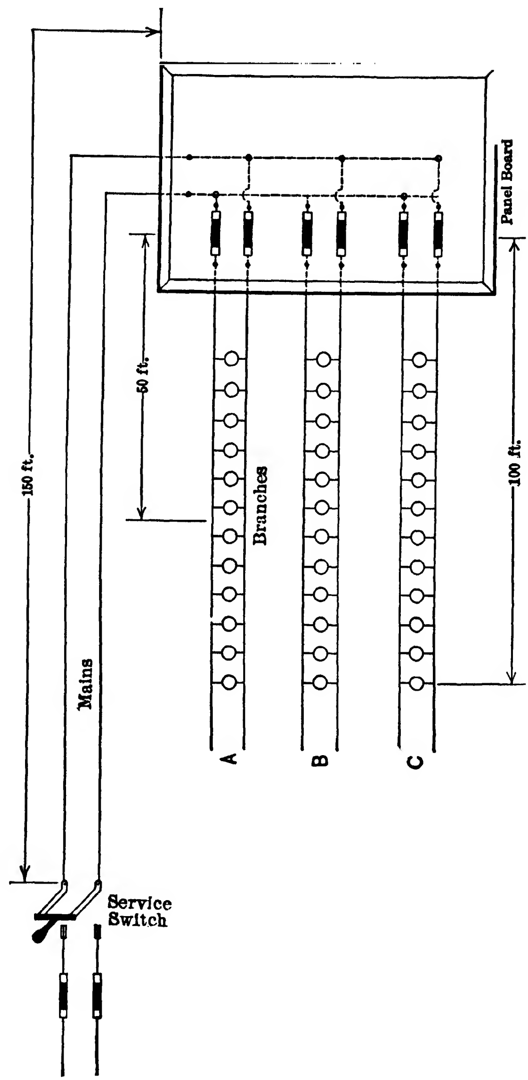



FIG. 11–4. An inside distribution system with panel board.

Resistance of 300 feet of No. 10 = 0.3 of 0.9989 = 0.300 ohm
Drop in main = $0.300 \times 16.35$ = 4.92 volts
Total drop = $4.92 + 1.38$ = 6.30 volts

Percentage drop $\frac{6.30}{110} = 5.73$ per cent.

This is still somewhat too large a drop. It is, therefore, necessary to use No. 8 mains.

**Prob. 66–4.** What size mains would be used in the above example if the panel board were situated 50 feet from the main switch and same loads were on the branch lines?

**Prob. 67–4.** An installation requires seventy 50-watt, 110-volt lamps. The panel board is situated 80 feet from the main switch. What size main should be run? Note that the Underwriter rules do not ordinarily allow more than twelve 50-watt lamps on a single branch line.

**Prob. 68–4.** If the panel board in Prob. 67–4 could be placed 40 feet from the main switch what size mains might be used?

**Prob. 69–4.** If the load on the installation of Prob. 67–4 consisted of motors instead of lamps but using the same total load, what size wire could be used for the mains?

**14. Electric Heating Resistor Elements.** There are two distinct types of resistor elements: first, the non-metallic, made of graphite, carbon or silicon-carbide; second, metal resistors made of resistance wire or ribbon. The second type only will be considered here.

Resistor elements may consist of a round wire wound in grooves in a moulded support, made of an insulating refractory material, or these elements may consist of a round wire in the form of a closely wound helical coil of small diameter fitted into grooves in the insulating refractory material such as the heating elements in radiant heaters, hot plates and electric ranges. A heating element may also consist of flat ribbon mounted on mica strips or other insulating material, as in many types of electric toasters. Flat ribbon is also used in the larger heating devices as space heaters for buildings, and in electric furnaces. Again, both the round straight wire and the flat ribbon are in some cases mounted on insulating supports in open air.

Probably, the most common form of element is the closely wound helical coil of small diameter.

Although, as stated in Chapter III, electrical energy is converted into heat energy at 100 per cent efficiency, the application of this heat involves a loss. The supports for the resistor and the sur-

rounding material absorb some of this heat, the amount depending upon whether the element is mounted as a straight wire, an open coil, or is partially or totally enclosed. A wire which operates at a given current at a bright red temperature in open air will operate at a dull red in strong air currents and will be black if immersed in water; and, if it operates totally enclosed in some insulating refractory material, it will melt.

The design of heating elements for efficient operation depends upon three things: (1) the watts dissipated (quantity of heat needed), (2) the temperature desired, and (3) the rate heat is carried away from the element. The first is a fixed quantity. The second and third depend upon the conditions mentioned in the previous paragraph and it is impossible to calculate these with accuracy. These can only be determined by cut and try methods, or by experience.

Heating elements are worked at high temperatures, and in most metals, if the heating element is not totally enclosed, oxidation takes place at these temperatures and the element deteriorates. Alloys of nickel-chromium, consisting of about 80 per cent nickel and 20 per cent chromium, have been found to be among the most durable up to about 2000° F. The melting point is around 2500° F. Such alloys are now the accepted material for heating elements for electric stoves, furnaces and high grade heating devices. The amount of heat that can be safely dissipated from nickel-chromium elements is approximately from 15 to 35 watts per square inch of surface of the conductor, where these elements are mounted near, or partially enclosed in refractory material. For elements mounted in open air, the amount is considerably greater, from 100 to 200 watts and more per square inch. For directions for the design of such heating elements, see catalogs issued by the makers of nickel-chromium alloys.

## SUMMARY OF CHAPTER IV

**CIRCULAR MEASURE. The cross-sectional area of conductors is expressed in terms of the number of unit circles it contains, i.e., circles one thousandth of an inch, or one mil, in diameter. Such a circle is called a CIRCULAR MIL.**

**THE AREA of a CIRCLE in CIRCULAR MILS equals the square of the diameter in mils. $A = d^2$.**

**MIL-FOOT. A wire one foot long and one circular mil cross-sectional area.**

**SPECIFIC RESISTANCE.** The resistance of a mil-foot of any material. (Sometimes also taken as the resistance of a centimeter *cube, from face to face opposite.)*

The specific resistance of commercial copper at 20° C equals 10.4 ohms. Symbol = K.

**RESISTANCE OF WIRE.** The resistance of a wire equals the specific resistance (resistance per mil-foot) multiplied by the length in feet and divided by the section area in circular mils.

$$R = \frac{Kl}{d^2}$$

For copper at 20° C;

$$R = \frac{10.4l}{d^2}$$

**TEMPERATURE COEFFICIENT OF RESISTANCE.** For every degree centigrade rise in temperature, the resistance of a pure copper wire increases 0.393 of 1% of its resistance at 20° C. This constant 0.00393 is called the Temperature Coefficient of Resistance. It has a much lower value in alloys and a negative value in carbon, porcelain, etc. Other pure metals have about the same coefficient as copper.

**THE RISE IN TEMPERATURE** of windings, etc., can be computed by measuring the cold and hot resistance and applying the temperature coefficient.

**AMERICAN or BROWN AND SHARPE WIRE GAUGE.** The usual sizes of copper wire have been standardized in America according to the gauge used by Brown and Sharpe.

**ALUMINUM WIRE** is sometimes used on account of its small weight and moderate specific resistance. It has 0.3 the weight of copper and 1.63 the resistance.

**STEEL CORE WIRES** are often used on account of their great strength and cheapness.

**SAFE CARRYING CAPACITY OF COPPER WIRES.** In interior wiring, no wire must carry greater current than that specified for that wire in Table IV, prepared by the National Fire Underwriters.

**TO DETERMINE THE PROPER WIRE SIZE** in an interior installation. (1) Determine current to be carried by each section and select a wire size from the table of safe carrying capacities which is recommended for the current as determined. (2) Check the voltage drop in the line. This must not exceed 5% for lamp loads or 10% for motor loads.

**HEATING ELEMENTS.** Nickel-chromium alloys (about 80% nickel and 20% chromium) are almost universally used for electrical heating elements. They do not oxidize up to 2100°. About 15 to 35 watts may be safely dissipated per square inch of conductor surface in partially or totally enclosed elements; while from 100 to 200 watts or more may be dissipated from elements in open air.

## PROBLEMS IN CHAPTER IV

**Prob. 70–4.** How many volts are required to send 6 amperes through 400 feet of copper wire 0.064 inch in diameter?

**Prob. 71–4.** What voltage is required to send 20 amperes through 800 feet of copper wire 0.162 inch in diameter?

**Prob. 72–4.** A $\frac{1}{4}$ inch copper wire carries 25 amperes. What is the voltage drop per mile?

**Prob. 73–4.** A copper wire carries 120 amperes for 1600 feet. If its diameter is 0.464 inch, what is the voltage drop?

**Prob. 74–4.** How many amperes can be forced through one thousand feet of copper wire $\frac{1}{8}$ inch in diameter with a voltage drop of 6 volts? (Hard drawn wire.)

**Prob. 75–4.** What size copper wire must be used if 3 volts are to be used in forcing 15 amperes through one mile of wire?

**Prob. 76–4.** If 14 volts are used in forcing 25 amperes through a line wire $\frac{3}{16}$ inch in diameter, what is the length of the wire?

**Prob. 77–4.** How far will a pair of copper line wires transmit 40 amperes with a line drop of 6 volts, if the wire is 0.262 inch in diameter?

**Prob. 78–4.** What resistance will a one-mile copper wire have which is a quarter inch in diameter? Use hard drawn wire.

**Prob. 79–4.** If the wire in Prob. 78–4 is of aluminum what resistance will it have?

**Prob. 80–4.** If the wire in Prob. 78–4 is of iron, what resistance will it have? Use 75 as the mil-foot resistance of iron.

**Prob. 81–4.** What size iron wire will have the same resistance per mile as a No. 0 B. & S. copper wire?

**Prob. 82–4.** What size iron wire will have the same resistance per mile as a No. 0 B. & S. aluminum wire?

**Prob. 83–4.** To what size copper wire is a stranded aluminum cable equivalent which is made up of 19 strands each 0.059 inch in diameter?

**Prob. 84–4.** How many strands of aluminum wire 0.064 inch in diameter will it take to make a cable equivalent to a No. 1 copper wire?

**Prob. 85–4.** It is desired to make a flexible aluminum cable equivalent to No. 4 copper wire. If strands of No. 17 B. & S. wire are used how many will be required? Cables are usually made of 7, 19, 37 or 61 strands. Use one of these numbers.

**Prob. 86–4.** Each lamp, Fig. 12–4, takes 2 amperes at 112 volts. The motor is 110 volts, 2 horsepower, 75% efficiency. What size wire must be used between the motor and lamps?

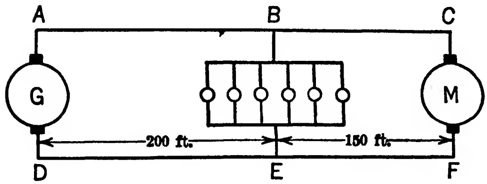



FIG. 12–4. A generator feeding a group of lamps and a motor.

**Prob. 87–4.** If No. 4 wire is used between the lamps and generator, what is the voltage of the generator in Prob. 86–4?

**Prob. 88–4.** What power is lost in the line in Probs. 86–4 and 87–4?

**Prob. 89–4.** A copper wire is 500 feet long and 0.38 inch in diameter. How many volts will it take to send 1.5 amperes through it?

**Prob. 90–4.** What will be the drop per mile in a line wire consisting of No. 10 B. & S. copper carrying 32 amperes?

**Prob. 91–4.** What will be the line drop in voltage and loss in watts per mile in transmitting 10 kilowatts at 550 volts if a No. 00 copper wire is used?

**Prob. 92–4.** A group of incandescent lamps takes 12 amperes. The line voltage drop is not to exceed 1.6 volts. What must be the size of the copper wire to be used if the lamps are 500 feet from the generator?

**Prob. 93–4.** A rough rule for the safe carrying capacity of copper is, "1000 amperes per square inch of cross section." According to this rule, what should be the diameter of a round wire capable of carrying 350 amperes?

**Prob. 94–4.** According to rule in Prob. 93–4, what should be safe carrying capacity of No. 0000 B. & S.? Compare value with that in Table IV.

**Prob. 95–4.** At 65° C, what is the resistance per 1000 feet of No. 6 copper wire?

**Prob. 96–4.** A trolley wire consists of No. 0 hard-drawn copper. What will be the drop per mile on a day when the temperature of the wire is −10° C, if the wire carries 45 amperes?

**Prob. 97–4.** A generator is supplying 10 amperes to 50 arc lamps in series. Drop across each lamp = 45 volts. Twelve miles of No. 10 wire are used for line. Find terminal voltage of generator if temperature of wire is 25° C.

**Prob. 98–4.** If the generator of Prob. 97–4 has 65 ohms resistance and line wire is at temperature of −5° C, what emf and terminal voltage must be developed in Prob. 97–4?

**Prob. 99–4.** Compute efficiency of transmission in Prob. 97–4 and Prob. 98–4.

**Prob. 100–4.** A coil of No. 20 wire is found to have a resistance of 42 ohms. How many feet are there in the coil?

**Prob. 101–4.** In Fig. 13–4, the wires used are No. 6 B. & S. copper.

$$AB = 1700 \text{ feet};$$
$$BC = HG = 800 \text{ feet};$$
$$CD = GF = 800 \text{ feet};$$
$$EF = 100 \text{ feet}.$$

Each lamp takes 2 amperes. The motor takes 10 amperes.
Find:

(1) Line loss in each section.
(2) Voltage across each group.
(3) Efficiency of transmission.

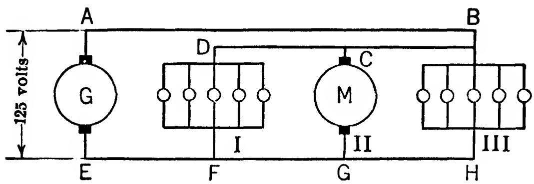



FIG. 13–4. A wiring scheme which helps equalize the voltage across the various loads.

**Prob. 102–4.** A line consisting of copper wire has a resistance of 2 ohms at 0° F. What is its resistance at 85° F?

**Prob. 103–4.** An electric soldering iron has a resistance composed of commercial iron wire of 80 ohms when cold (20° C). When hot, the resistance rises to 150 ohms. What temperature is reached, assuming a constant temperature coefficient?

**Prob. 104–4.** The resistance of a car heater when cold (20° C) is 120 ohms. If the temperature rises to 150° C when hot, how much less current does it take when hot than when cold? Material of heater is iron wire. Voltage is 550.

**Prob. 105–4.** It is desired to keep the current constant through a coil of wire, which is on a constant voltage circuit. The resistance of coil at 95° C is 40 ohms. How many feet of Nichrome V wire, No. 18 B. & S., must be added to copper wire when the temperature falls to 25° C in order that current may not change?

**Prob. 106–4.** A motor 2500 feet from the generator requires 50 amperes at 550 volts. What wire should be used according to rule in Prob. 93–4? What is terminal voltage of the generator and the power lost in line? Efficiency of transmission?

**Prob. 107–4.** If wire of twice the cross section is used in Prob. 106–4, what will the answers be?

**Prob. 108–4.** A 110-volt 25-horsepower motor of 90 per cent efficiency is situated 400 feet from the generator. No. 0000 B. & S. copper

wire is used for line. What must terminal voltage of generator be? Efficiency of transmission?

**Prob. 109–4.** What size aluminum wire could have been used in Prob. 108–4 and have the same line drop?

**Prob. 110–4.** What size of copper wire is required between a 115-volt generator and a 110-volt, 10-horsepower motor of 85 per cent efficiency? The motor and generator are 800 feet apart.

**Prob. 111–4.** A building situated 300 feet from a 115-volt generator is to be supplied with sufficient current from the generator to light 500 lamps, each taking .45 ampere. The efficiency of transmission must be 97 per cent. What size copper wire must be used?

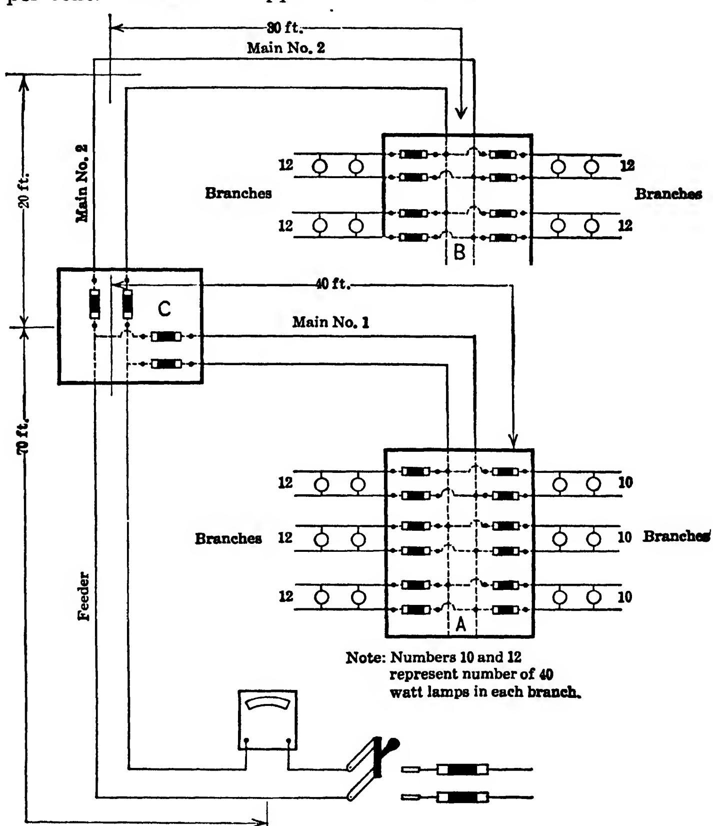



FIG. 14–4. An inside distribution system with three panel boxes.

**Prob. 112–4.** Calculate the voltage at the lamps fed from Panel B in Fig. 14–4 using No. 8 wire for Main No. 2 and proper size for feeder. All lamps are on, and are rated at 110 volts. The average length of branches is 50 ft. Voltage at the service switch is 112 volts.

**Prob. 113–4.** (*a*) What will be the voltage across the lamps fed from Panel A in Prob. 112–4 with all circuits loaded to capacity? (*b*) What will be the voltage across a single lamp fed from Panel A, if that lamp is the only one in the system turned on? Use No. 8 wire for Main No. 1.

**Prob. 114–4.** Using Fig. 14–4 as a basis, but changing the dimensions to the following, determine the size of the feeders and mains to meet requirements of Fire Underwriters, and check for voltage drop to lamps fed from Panel B. All lamps rated 115 volts.

| | |
|---|---|
| Voltage at lamps of Panel B .................... | 114 volts |
| Length of feeder, to Main No. 1 tap ............ | 60 feet |
| Length of Main No. 1.......................... | 65 feet |
| Length of Main No. 2.......................... | 95 feet |

Panel B has six 660-watt branches.
Panel A has five 660-watt branches.
Average length of branch, 50 ft.

**Prob. 115–4.** Check the voltage drop to be fed from Panel A for the system of Fig. 14–4 when installed with the wire sizes determined in Prob. 114–4.

**Prob. 116–4.** If the number of branches on Panel B of Prob. 114 were doubled, determine the wire sizes to be used and check voltage drop to lamps.

**Prob. 117–4.** A two-wire line of 600,000 circular-mil cable delivers 450 amperes at 440 volts to a distance of 800 feet. If the voltage at input end of line is maintained constant, by what percentage will the voltage at the output end rise as the load is reduced from 450 amperes to zero?

**Prob. 118–4.** What must be the size (Underwriters' Table) of two (equal) smaller wires in parallel, to deliver the same load as the single large wire of Prob. 117? Rubber-insulated wire in both cases. Assuming the cost of conductor to be in direct proportion to weight of metal, what per cent is saved by this substitution?

**Prob. 119–4.** Calculate for the parallel feeders of Prob. 118–4 the per cent rise of voltage at output end of feeders from full load to zero load, voltage at input end being maintained constant. Compare this result with the corresponding figure for Prob. 117–4, and draw conclusions.

**Prob. 120–4.** Try to explain why the heavier conductors in Table IV cannot be permitted to carry as large a current, in relation to their cross-section area, as the smaller conductors.

**Prob. 121–4.** A No. 0 two-wire feeder was originally installed to carry a load of 120 amperes, but the load grew to 220 amperes and the station operator attempted to meet the situation by paralleling another two-wire feeder of No. 1 size. Both feeders have rubber insulation. Do you think that together they are sufficient?

**Prob. 122–4.** If the feeder of No. 1 wires of Prob. 121–4 were carrying the largest current permitted for it by N. E. C. (Table IV), what would be the total voltage drop along the wires? The feeders are 500 feet long. How many amperes must the No. 0 parallel feeder then be carrying, since the drop must be identically the same along both feeders? What ratio exists between current in each feeder and total current? How do these results bear on Prob. 121–4?

**Prob. 123–4.** A cable is made up of 19 strands of 40% conductivity copper clad wires. Total cross-section area is 246,920 circular mils. What is the drop per 1000 feet when carrying 12 amperes?

## CHAPTER V

### METHODS OF MEASURING RESISTANCE

Resistance measurements are widely used in the industries to determine the condition of an appliance or to discover any faults which may have been made in the construction of it. This chapter will discuss only those methods in general use.

**1. Ammeter-Voltmeter Method.** Ordinarily, the simplest means of determining the resistance of an appliance is to send a current through it and measure the current with an ammeter, and the voltage with a voltmeter. The resistance is then found by Ohm's law.

$$R = \frac{E}{I}$$

Simple as this method is, certain precautions must be taken in the use of the instruments. Both the ammeter and voltmeter in themselves consume energy when connected to the circuit and, therefore, introduce errors in the measurement of current and voltage. But by properly connecting them, we can usually reduce these errors to insignificant quantities.

Such measurements fall into **two** general cases: **First,** where the **current** is **small** and the **voltage drop high;** and **Second,** where the **current** is **large** and the **voltage drop low.**

The conditions in the first case are illustrated in Examples 1 and 2 below.

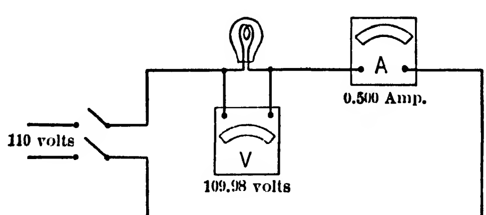



FIG. 1–5. A poor arrangement of instruments for measuring the resistance of the lamp.

**Example 1.** It is desired to measure the resistance of an ordinary incandescent lamp. Let us first try the connections of Fig. 1–5. $A$ is a Weston Milliameter, resistance 0.045 ohm. $V$ is a Weston Voltmeter,

resistance 15,000 ohms. Assume both instruments indicate correctly. Suppose there is a pressure of 110 volts on the circuit and the ammeter reads 0.050 ampere.

Voltage drop over the ammeter $= 0.045 \times 0.5 = 0.0225$ volts.
Voltmeter will read $110 - 0.0225 = 109.98$ volts.

This reading is the actual voltage on the lamps and is correct, but the ammeter reads high, since it reads the current through both the lamp and the voltmeter.

$$\text{Current through the voltmeter} = \frac{109.98}{15{,}000} = 0.00733 \text{ ampere.}$$

Then the current through the lamp $= 0.50 - 0.00733 = 0.4927$ amperes.

$$\text{Actual resistance of the lamp} = \frac{109.98}{0.4927} = 223.22 \text{ ohms.}$$

Resistance of the lamp computed from the instrument readings

$$\frac{109.98}{0.50} \quad 219.96 \text{ ohms.}$$

$$\text{Per cent error} = \frac{223.22 - 219.96}{223.22} \times 100 = 1.46 \text{ per cent.}$$

This error is entirely too large for any such simple measurement.

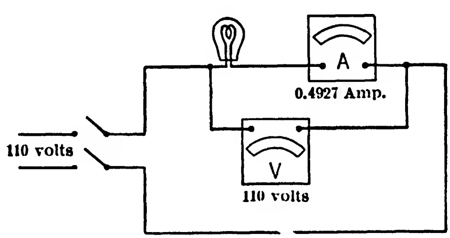



FIG. 2–5. A better arrangement of instruments for measuring the lamp resistance.

**Example 2.** Now suppose we connect the same instruments as in Fig. 2–5, with the terminals of the voltmeter connected across both the lamp and the ammeter. The ammeter now reads the current through the lamp $L$ only, or 0.4927 ampere, and is therefore correct. But the voltmeter now reads the voltage across both the lamp and the ammeter, or 110 volts, and therefore, reads too high by the amount of the voltage drop in the ammeter.

Voltage drop in the ammeter $= 0.045 \times 0.4927 = 0.0222$ volts.
Then the voltage on the lamp $= 110 - 0.0222 = 109.98$ volts.

$$\text{Actual resistance of the lamp} = \frac{109.98}{0.4927} = 223.22 \text{ ohms.}$$

Resistance of the lamp computed from the instrument readings

$$= \frac{110}{0.4927} = 223.26 \text{ ohms}$$

$$\text{Per cent error} = \frac{223.26 - 223.22}{223.22} \times 100 = 0.0179 \text{ per cent — less}$$

than $\frac{1}{50}$ of 1 per cent.

This error is allowable in most grades of commercial work.

It is evident, then, that when measuring resistance where the voltage drop is high with a corresponding low value of current, the voltmeter terminals should be placed to measure the drop over both the resistance and the ammeter. This is because the voltage across the ammeter is too small to affect appreciably the reading of the voltmeter.

Now let us consider the conditions in the second case, where the resistance to be measured is small in value. Here a comparatively **large** current is used with a correspondingly **low** voltage drop, as shown in Examples 3 and 4 below.

**Example 3.** Suppose that we wish to measure the resistance of a low resistance armature, *R*. Assume the instruments are connected as in Fig. 3–5, which arrangement we found gave us the greater error in the previous case (Example 1).

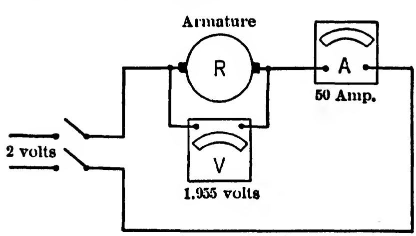



Fig. 3–5. The proper arrangement of instruments for measuring a low resistance.

*V* is a 3-volt Weston Voltmeter of 300 ohms resistance and *A* is a Weston Ammeter of 0.0009 ohm resistance. Assumed there are 2 volts on the circuit and that the ammeter reads 50.0 amperes. The ammeter reads the current through both *R* and the voltmeter and therefore reads too high (as it did in Example 1).

Voltage drop over the ammeter $= 0.0009 \times 50 = 0.045$ volts.
The voltmeter will read $2.00 - 0.045 = 1.955$ volts.

This reading is the actual voltage across the armature and is correct.

Current through the voltmeter $= \frac{1.955}{300} = 0.00652$ ampere.

Then the current through the armature $= 50 - 0.00652 = 49.9935$ amperes.

Actual resistance of the armature $= \frac{1.955}{49.9935} = 0.039105$ ohm.

Resistance as computed from the instrument readings

$$= \frac{1.955}{50} = 0.0391 \text{ ohm.}$$

Per cent error,

$$\frac{0.039105 - 0.0391}{0.039105} \times 100 = 0.0127 \text{ or about } \tfrac{1}{100} \text{ of 1 per cent.}$$

This is so small that it is allowable in all commercial work.

**Example 4.** Now suppose we arrange the instruments as in Fig. 4–5. The ammeter will read correctly the current through the armature, $R$, or 49.9935 amperes, but the voltmeter will read the voltage across both $R$ and the ammeter, or 2 volts.

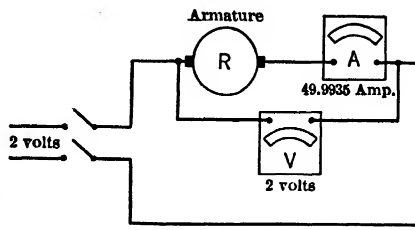



FIG. 4–5. A poor arrangement of instruments for measuring a low resistance.

Voltage drop over the ammeter $= 0.0009 \times 49.9935 = 0.045$ volt.

Voltage across the armature, $R$, $= 2 - 0.045 = 1.955$ volts.

$$\text{Actual resistance of the armature} = \frac{1.955}{49.9935} = 0.039105 \text{ ohm.}$$

Resistance as computed from the instrument readings

$$= \frac{2}{49.9935} = 0.04 \text{ ohm}$$

Per cent error,

$$\frac{0.04 - 0.039105}{0.039105} \times 100 = 2.28 \text{ per cent.}$$

This error is too high for most purposes.

It is thus evident that when measuring a low resistance through which a large current flows with a low voltage drop, the voltmeter should be connected immediately across the resistance under test, and not across the ammeter also. The reason for this is that the small amount of current flowing through the voltmeter does not affect appreciably the reading of the large current ammeter, while the voltage drop over the ammeter is appreciable in comparison with the drop over the resistance under test.

This method of measuring resistance has the advantage that it is very simple and is done with instruments usually available in all industrial plants. It has the disadvantage that the volt-

meter must be accurately calibrated to indicate volts and the ammeter to indicate amperes. Furthermore, in order to determine the probable error one must know the resistance of each instrument.

**Prob. 1–5.** It is desired to measure the resistance of an electric device. Instruments are arranged as in Fig. 1–5. The ammeter has a resistance of 0.009 ohm; the voltmeter has a resistance of 15,000 ohms. Ammeter reading 0.475 ampere. Voltmeter reads 120 volts. If instruments are correct, what is per cent error in resistance measurement due to the arrangement of the instruments?

**Prob. 2–5.** If above instruments were arranged as in Fig. 2–5. (*a*) What would each read? (*b*) What would be the per cent error in resistance measurement for this arrangement?

**2. Fall-of-Potential Method.** There are needed for this method: (1) A **known** or **standard** resistance *R*, (2) a voltmeter, (3) a source of power. See Fig. 5–5.

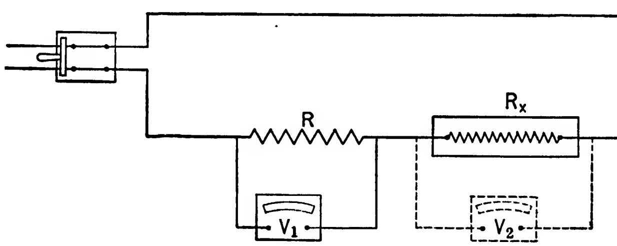



FIG. 5–5. The voltmeter is first connected as $V_1$, then as $V_2$. If the current remains constant the ratio of the resistances, R and $R_x$, is equal to the ratio of the voltmeter readings.

The known resistance ($R$), and the unknown ($R_x$) are connected to the power in series so that there will be the same current flowing through each. The voltmeter is placed to read the voltage ($V_1$) across $R$, and then to read the voltage ($V_2$) across $R_x$. Since the same current is flowing in each resistance, the voltage across each must be in the same ratio as the two resistances. That is, if the voltage $V_2$ is twice the voltage $V_1$, then the resistance $R_x$ must be twice $R$, for we have seen that it requires twice the voltage to force the same current through twice the resistance. We have seen in Chapter II that this relation may be expressed as an equation:

$$\frac{R_x}{R} = \frac{V_2}{V_1}.$$

Thus

$$R_x = \frac{V_2 R}{V_1}. \tag{1}$$

Since $R$, $V_2$ and $V_1$ are known quantities, the value of the unknown resistance ($R_x$) may be found.

The advantages of this method are:

(1) The voltmeter need not be accurately calibrated, providing the deflections are proportional to the voltage.

(2) It is an accurate method of measuring low resistances, if the voltmeter has a high resistance, or is replaced by a galvanometer. This shunts very little current around the series resistance.

(3) By means of a potentiometer in place of the voltmeter, no current is shunted around the resistance and thus very low resistances can be measured accurately. See Chapter XVII.

**3. The Wheatstone Bridge.** By far the most important appliance for accurately measuring resistances, is the well-known Wheatstone Bridge. It consists fundamentally of a loop of four resistances, $R$, $R_1$, $R_2$ and $R_3$, one of which is unknown. See Fig. 6–5. An electric current, generally from a battery, is sent

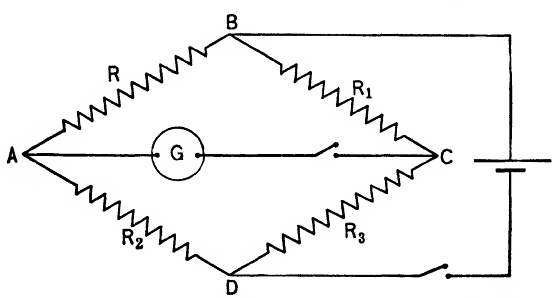



FIG. 6–5. Conventional diagram of the Wheatstone bridge circuit.

into the loop at $B$, and immediately divides into two parts, one part taking the branch composed of the resistances $R$ and $R_2$, the other part taking the branch made up of $R_1$ and $R_3$. Both branches come together again at $D$, and return the current to the battery. A galvanometer $G$ is now " bridged " between the two branches from $A$ to $C$.

Assume $R_3$ to be the unknown resistance. $R$, $R_1$, $R_2$ are variable resistances, and are now adjusted until, with the key in the circuit of the galvanometer closed, it gives no deflection. This is called " balancing " the bridge. The points $A$ and $C$ are then at the

same voltage level or potential. For we have seen that where there is any difference of potential between two points, a current always flows when they are joined by a conductor. Since the points $A$ and $C$ are joined through the galvanometer, and no current flows, they must be at the same potential.

Let us now consider the voltages across the separate resistances. We have said that the current divides into two branches, $BAD$ and $BCD$. Now the current in each branch need not be the same, so let us designate the current in the branch $BAD$, by the letter $I$, and the current in the branch $BCD$, by the letter $I_1$.

Then the voltage across $BA$ (that is, $R$) $= IR$;
Then the voltage across $BC$ (that is, $R_1$) $= I_1R_1$.

But we have said that the points $A$ and $C$ are at the same level (since no current flows through the galvanometer) and, therefore, the voltage drop from $B$ to $A$ $(IR)$ = the voltage drop from $B$ to $C$ $(I_1R_1)$. That is,

$$IR = I_1R_1. \tag{2}$$

For the same reasons, the voltage drop from $A$ to $D$, $(IR_2)$, must equal the voltage drop from $C$ to $D$, $(I_1R_3)$; that is,

$$IR_2 = I_1R_3. \tag{3}$$

Dividing equation (2) by equation (3) we get,

$$\frac{IR}{IR_2} = \frac{I_1R_1}{I_1R_3}.$$

Cancelling the $I$'s and $I_1$'s,

$$\frac{R}{R_2} = \frac{R_1}{R_3}. \tag{4}$$

**This is the fundamental equation of the Wheatstone bridge.**

Since $R_3$ is the only unknown, the equation can be solved and the value of $R_3$ found.

The ratio $\frac{R}{R_2}$ is generally made to equal some such fraction as $\frac{1}{10}$ and then it follows that $\frac{R_1}{R_3} = \frac{1}{10}$. $R_1$ is called the Rheostat arm. This makes $R_3$ 10 times the value of $R_1$. If $R_2$ is made the same as $R$, then $R_3$ will be the same as $R_1$.

In writing the equation for the bridge, notice that we started at

$B$ (a point where the battery line came in). For one side of our equation we read along one branch. That is, we said, " $R$ is to $R_2$ " or $\frac{R}{R_2}$.

For the other side of the equation we went **back to the point B again** and read along the other branch, saying, " $R_1$ is to $R_3$ " or $\frac{R_1}{R_3}$. We did **not continue around** the loop and make the second side of the equation " $R_3$ is to $R_1$." Had we done so, the two ratios would have been unequal and we could have formed no equation.

**It is to be noticed, then, that in making the fundamental equation for a Wheatstone bridge, we read along one branch for one side of the equation, then go back and read along the other branch for the other side of the equation.**

It is interesting, and of value, to know that it makes no difference at what point we start, providing we start at the **same point for each side of the equation.** Thus, suppose we start at $A$. Reading along the upper branch, we have, " $R$ is to $R_1$ " or $\frac{R}{R_1}$, for one side of the equation. Then, coming back to the point $A$ and reading the lower branch, we have, " $R_2$ is to $R_3$." The equation, then, $\frac{R}{R_1} = \frac{R_2}{R_3}$, is just as true as the first and as easy to solve.

If we start at the point $C$, we have " $R_3$ is to $R_2$ " or $\frac{R_3}{R_2}$, " $R_1$ is to $R$," or $\frac{R_1}{R}$, that is, $\frac{R_3}{R_2} = \frac{R_1}{R}$, which equation is also true.

**The important details about measuring resistance with a Wheatstone bridge are:**

**(1) Make a complete loop of four resistances.**

**(2) Connect battery terminals (through key) to two points that are not adjacent, that is, each branch of the current must always flow through two resistances before returning to the battery.**

**(3) Connect galvanometer through key across the two remaining points.**

**(4) In forming the equation, start at any point and read along two resistances for one side of the equation; then come back to same point, and read along the other two resistances for the other side of the equation.**

This method of resistance measurement has the following advantages:

The only parts which have to be accurately calibrated are the

spools of known resistance. Since these are not moving parts there is little to get out of order.

For the galvanometer, it is possible to use any instrument which is deflected by slight currents. It does not have to be calibrated.

Only a very small source of electrical energy is required.

It has the disadvantage of requiring considerable practice to operate it effectively.

**Prob. 3–5.** In a Wheatstone bridge, arranged as in Fig. 6–5, a balance is obtained when $R = 100$ ohms, $R_1 = 7.5$ ohms, $R_2 = 1000$. What is the value of $R_3$?

**Prob. 4–5.** In Fig. 6–5, if $\frac{R}{R_2} = 100$, and $R_3 = 6.89$, what is the value of $R_1$?

**Prob. 5–5.** What value is a convenient value for $R_2$, if $R$ is 1000 ohms and $R_1$ is 48 ohms and it is desired to measure $R_3$, which is known to be about 600 ohms?

### Instructions for Use of the Common, or Plug Type of Wheatstone Bridge

1. Never hold the bridge plugs in the hand or place them where they are likely to come in contact with oils or acids. They may become covered with oil and the "plug resistance" thus increased, or may be corroded so that they do not properly fit the sockets.
2. Insert all plugs firmly, with a twisting motion. Do not, however, insert them with force enough to strain the top, or caps of the plugs.
3. Always put a key in the battery circuit and one in the galvanometer circuit. Close the **battery circuit first,** then the circuit through the galvanometer. In breaking the circuit, open the key in the galvanometer circuit first.
4. In closing the galvanometer circuit, **make only slight contact** with the key until balance is nearly secured. The galvanometer may thus be protected from heavy currents. If the galvanometer is very sensitive, put a shunt across its terminals until the bridge is nearly balanced.
5. In making a resistance measurement, proceed in the same manner as in weighing with a chemical balance. First put in a coil estimated to be about correct for the resistance being measured. If this is too low, use one twice as great; if too high, one one-half as great. Fix in this way two limits between which the resistance lies. Then systematically bring these limiting values closer and closer together, until the nearest balance on the bridge is obtained. It will not, in general, be possible to secure an exact balance, but two values for the resistance in the rheostat arm may be found which will give steady deflections of the galvanometer needle in opposite directions. The correct value lies between. From the deflections of the galvanometer needle in the two cases, interpolate for the resistance which will give no deflection. Thus, suppose the small-

est coil on the bridge is 0.1 ohm. With this added, a steady deflection of the galvanometer of 2 divisions to the **right** is given. Without it, the deflection is 3 divisions to the **left.** 0.1 ohm makes a difference, therefore, of 5 divisions, and the resistance which would give no deflection is, therefore, $\frac{3}{5} \times 0.1 = 0.06$ ohm.

6. When balance has been obtained, reverse the current through the bridge and see if the balance is maintained.

**4. Slide-Wire Bridge.** Sometimes a single straight wire of uniform section and of high resistance takes the place of the resistances $R$ and $R_2$ as in Fig. 7–5. This makes a very convenient form of the "bridge." The point $A$ is then a sliding contact piece.

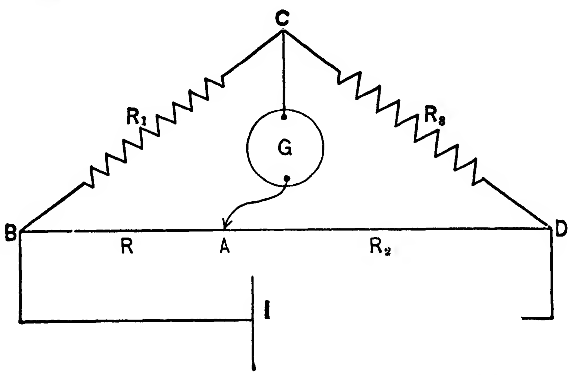



FIG. 7–5. The slide-wire Wheatstone bridge.

$R$ is the resistance of the straight wire from $B$ to $A$, and $R_2$ the resistance from $A$ to $D$. In this form the bridge is called the slide-wire bridge, a balance being obtained by sliding the contact maker $A$ along the wire $BD$, thus decreasing or increasing $R$ or $R_2$ as desired. The ratios are read as before, the lengths $BA$ and $AD$ being used instead of the actual resistances, since the resistance is proportional to the length.

This form has the advantage that a balance can be obtained very quickly. It is not so accurate as the other forms, however, because of the wearing of the wire, and of the necessarily low resistance of $R$ and $R_2$.

Sometimes $R_1$ is a permanent resistance coil, and the wire is so calibrated that the resistance of $R_3$ is read directly from the position of the slider $A$ on the wire. It is then called an "Ohmmeter." A telephone receiver in this case sometimes takes the place of the galvanometer.

Resistance coils are often placed in $R$ and $R_2$ to increase the accuracy of this form of bridge, though by so doing the range is lessened.

**Prob. 6–5.** In a bridge arranged as in Fig. 7–5, $BD$ is 100 centimeters long, $R_1$ is 15 ohms. If bridge balances when $BA$ is 47.6 centimeters, what value is $R_3$?

**Prob. 7–5.** If $R_1 = 34$ and $R_3 = 90$ (Fig. 7–5), what must the values of $BA$ and $AD$ be? $BD$ is 100 centimeters.

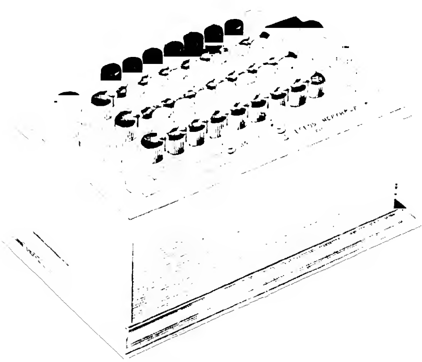


FIG. 8–5. The Post Office Wheatstone bridge. *Leeds & Northrup Co.*

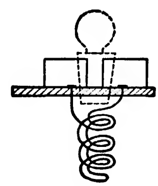


FIG. 9–5. A non-inductive resistance coil of a Wheatstone bridge.

**5. Construction of Various Types of Bridges.** The appearance of the regular " Post Office " form of Wheatstone bridge is shown in Fig. 8–5.

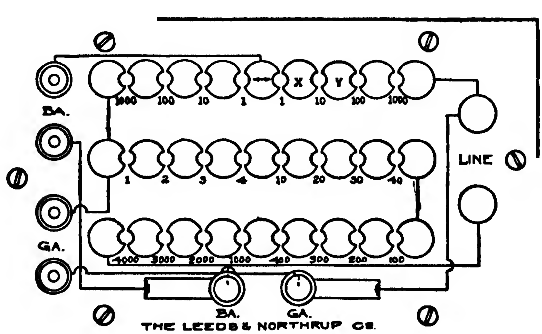



FIG. 10–5. Diagram of a Post Office Wheatstone bridge.

The resistance wire is wound non-inductively on spools as in Fig. 9–5. The ends of the wire are now connected across the gap between two adjacent brass blocks, as shown in Fig. 10–5. The

construction of one such coil and set of blocks is shown in Fig. 9–5. When the brass plug is inserted in the tapered hole between the blocks, it forms a short circuit around the coil and " cuts the coil out " of the circuit. In order to put a certain amount of resistance into a circuit, say between $x$ and $y$, Fig. 10–5, it is merely necessary to remove the plug, which short-circuits a coil of the desired resistance.

In Fig. 11–5 is shown the appearance of a " Decade " form of bridge. In this type, resistance is introduced into a circuit by **placing the plug in the hole,** instead of by removing the plug. The details of the construction are shown in Fig. 12–5.

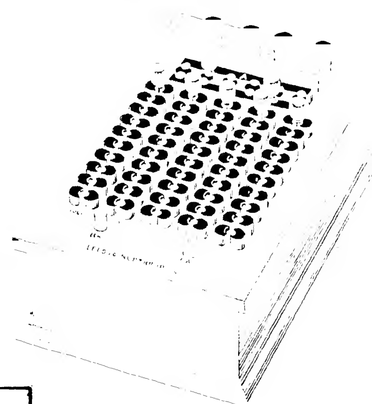


FIG. 11–5. A Decade Wheatstone bridge. *Leeds & Northrup Co.*

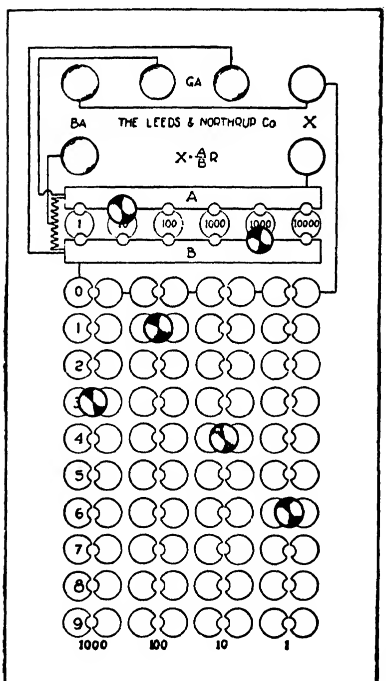



FIG. 12–5. Diagram of a Decade Wheatstone bridge.

The coils composing the variable resistance for balancing the bridge are arranged in such an ingenious way that 10 variations in value in each " decade " or row are made by the use of only 4 coils.

The ratio arms are made by " plugging in " between bar $A$ and any block for one arm, and between bar $B$ and any **other block** for the other arm.

The "Dial" form of bridge is shown in Fig. 13–5. The ratio arms are formed as in the "decade" style. The balancing resistance is adjusted by turning the brushes, by means of the knurled

head, from block to block on the dials. The resistance coils are connected between these blocks.

By means of this form of bridge, a balance may be obtained in the shortest possible time.

The galvanometers used in connection with these bridges are described in Chapter XVII.

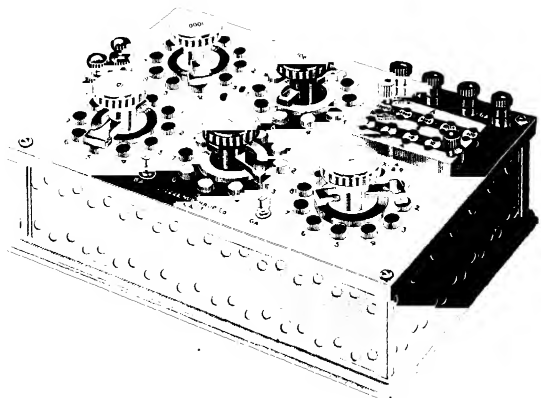


FIG. 13–5. A Dial Wheatstone bridge. *Leeds & Northrup Co.*

**6. Location of Faults in Cables, etc.** It sometimes happens that a telephone conductor becomes "grounded." It is then very desirable to locate the distance of this "fault" from one end or the other of the line before starting out from the office to repair it. Where the wire is laid underground, it would be too costly a proceeding to inspect any great length of cable, and thus, very refined methods have been worked out for locating such faults to a great degree of precision. There are two methods in common use, called respectively, the Murray loop and the Varley loop. They are both modifications of the Wheatstone bridge, and, therefore, the underlying principles stated above apply to them.

**7. Murray Loop.** This test ordinarily makes use of the slide-wire form of the bridge and is arranged as in Fig. 14–5.

There is a "ground" at *F* in the conductor *DC*, of a line between two cities. A good wire *BC*, which happens to lie between the two cities and which is exactly like *DC*, is joined to *DC* at *C*. By means of the slide-wire *BD*, a complete loop of resistances is formed. The battery is connected to the slider *A* and to the ground. This is the same as connecting in the battery at *A* and *F*, since the ground serves as a conductor. The galvanometer is placed across

*BD.* We now have a regular slide-wire Wheatstone bridge, as in Fig. 7–5; the resistance of the slide-wire, $R_2$ and $R$, forming one side of the bridge, while $R_3$ (the resistance out to the fault) and $R_1$ (the resistance through good wire to fault) form the other side.

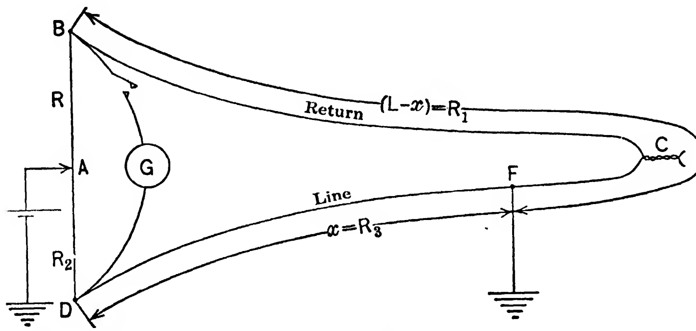



FIG. 14–5. The Murray loop, a special Wheatstone bridge arrangement for locating a " ground " in a cable.

The bridge is balanced by moving $A$ along the wire until there is no deflection of the galvanometer. Then the regular equation of the Wheatstone bridge holds true:

$$\frac{R_2}{R} = \frac{R_3}{R_1}.$$

If we let $L$ equal the length of entire loop, which would be known, and let $x$ equal distance out to fault, then the equation would become,

$$\frac{R_2}{R} = \frac{x}{L - x}.$$

As $x$ is the only unknown quantity, it can easily be found as follows:

$$x = \left(\frac{R_2}{R + R_2}\right)L.$$

But $R + R_2 =$ the entire length of the slide-wire $BD$, and we may write the equation:

$$\frac{x}{L} = \frac{R_2}{R + R_2} = \frac{AD}{BD}. \tag{5}$$

This shows that $x$ is the same fraction of the total length of the loop that $R_2$ is of the total length of the slide-wire.

If the "good" wire $BC$ is not of the same size and material as the "faulty" wire, the resistance of the loop $BCD$ is first found in the usual way by a bridge. It is then connected up as in Fig. 14–5, and worked out as above, with the exception that $L$ stands for the **resistance** of the entire loop instead of for the **length** of it, and $x$ for the resistance out to fault. If the size of the "faulty" wire is known, the distance out to the fault is readily computed.

In the place of the slide-wire $BD$, two sets of resistance coils are often substituted.

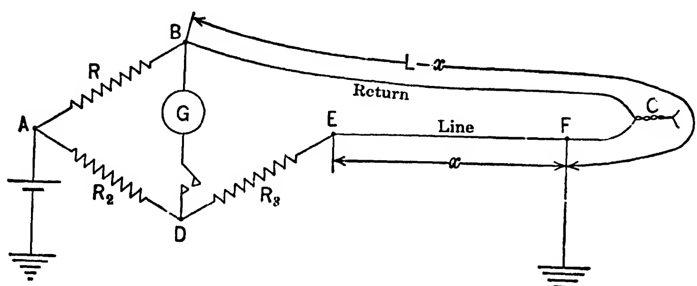



FIG. 15–5. The Varley loop, another bridge arrangement, for locating a "ground" in a cable.

**8. Varley Loop.** In the Varley loop, Fig. 15–5, instead of varying the ratio of $R_2$ to $R$, as in the Murray form of bridge, these resistances are left constant and a balance is obtained by means of a third set of resistance coils $R_3$, inserted in series with the faulted cable. The resistance of the total length of the good and bad cables $L$, is either known or first found as in Murray loop.

The value of $x$ (resistance out to fault) is then found by the equation of the Wheatstone bridge:

$$\frac{R_2}{R} = \frac{R_3 + x}{L - x};$$

$$x = \frac{R_2L - RR_3}{R + R_2}. \tag{6}$$

As all the values in the right-hand member of the equation are known, the value of $x$ may be found and the distance out to the fault $F$ be computed from the resistance per foot of the cable.

An interesting variation in the use of the Varley loop is shown in Fig. 16–5.

The two cables must here be " twin " wires. Then $N$ represents the resistance out to fault on the bad wire. Thus $N$ would also represent the resistance of an equal distance out on the good cable. Now let $x$ represent the resistance of the bad cable from fault $F$ to far end. Then $x$ will also represent the resistance of an equal

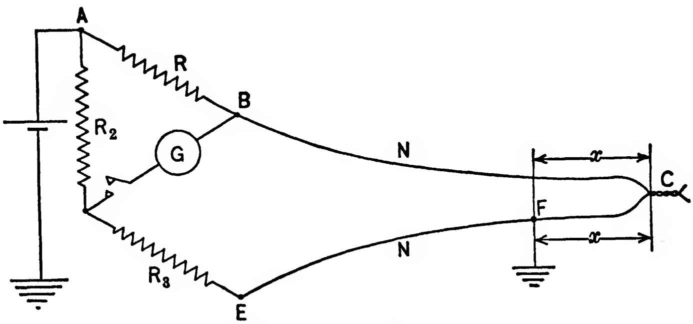



FIG. 16–5. The Varley loop using a " twin " return wire.

length of the good cable. The resistances $R_2$ and $R$ are made equal and the bridge is balanced by regulating $R_3$ as before. The equation then becomes:

$$\frac{R_2}{R} = \frac{R_3 + N}{2x + N}. \tag{7}$$

But as $R_2 = R$, we have,

$$R_3 = 2x. \tag{8}$$

Thus $x$ equals $\frac{1}{2}$ of whatever we have to make $R_3$ in order to balance the bridge. This gives a very rapid method of finding the location of a ground in a telephone circuit, and is as accurate as the resistances used in $R$, $R_2$ and $R_3$.

**Prob. 8–5.** A Murray loop is arranged as in Fig. 14–5 with a meter slide-wire. $BA = 56$ centimeters when the bridge is balanced. The length of the line between the two cities is 2 miles. The return wire is exactly like the faulted wire. How far from test end is fault $F$?

**Prob. 9–5.** In a Murray loop arranged as in Fig. 14–5, the resistance of the loop $= 37.8$ ohms. The line wire $DC$ is No. 18 B. & S. copper. If bridge balances with $AD$ equal to 28 centimeters, how far from test end is fault $F$?

**Prob. 10–5.** A Varley loop in which the resistance of the twin cables is 44.8 ohms is arranged as in Fig. 15–5. A balance is obtained when $R = 10$ ohms, $R_2 = 100$ ohms, $R_3 = 292$ ohms. If the faulted wire is copper, No. 14 B. & S., how far from $E$ is the fault?

**Prob. 11–5.** A Varley loop is arranged as in Fig. 16–5 with the return the same as the faulted line, No. 12 B. & S. copper. $R = R_2$; $R_3 = 5.12$ ohms. How far from $C$ is fault $F$?

**Prob. 12–5.** If wire in Prob. 11–5 is iron and 50 ohms to the mile, how far will $F$ be from $C$?

**9. Voltmeter Method for Insulation and Other High Resistances.** It is often necessary to know whether a certain resistance comes within or exceeds a given value; as, for instance, the insulation resistance between the commutator and the frame of a motor. A voltmeter of known resistance $R_v$* is placed across a constant source of power as $AB$, Fig. 17–5, and voltmeter reading $V$ is noted.

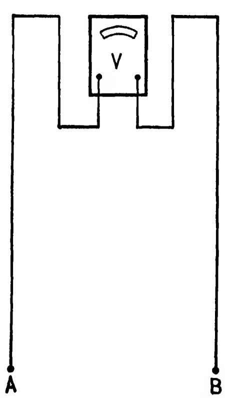



FIG. 17–5. In determining the insulation resistance of the windings on a motor, the voltage V, across the supply circuit AB, is first measured.

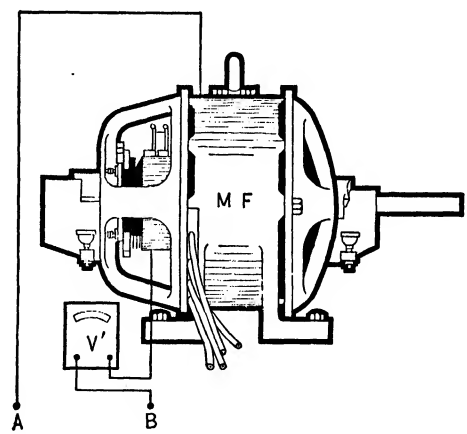



FIG. 18–5. The voltmeter measures the voltage across itself. Therefore, the voltage of the supply circuit, AB, minus the voltmeter reading V′ equals the voltage drop through the resistance of the insulation.

The frame of the motor $MF$ is then connected to terminal $A$, and commutator segment $C$ is connected through the voltmeter to the other terminal $B$, as in Fig. 18–5. The reading $V'$ of the voltmeter is then taken. Since the motor-insulation and voltmeter are in

* The resistance of most voltmeters is generally given on the case or accompanies the manufacturer's data on the instrument.

series, the voltage of $AB$ is distributed across the motor and voltmeter in direct proportion to their separate resistances. The voltmeter reads the voltage across itself as always, thus $V'$ = voltage across the voltmeter. The voltage across the motor-insulation then equals the voltage across $AB$, $(V)$ minus voltage across voltmeter, $V'$. That is,

$$\text{Voltage across voltmeter} = V';$$
$$\text{Voltage across motor-insulation} = V - V'.$$

Since the two are in series, the resistances of each are proportional to the voltage across each.

$$\frac{\text{Insulation Resistance of Motor}}{\text{Resistance of Voltmeter}} = \frac{V - V' \text{ (volts across motor-insulation)}}{V' \text{ (volts across voltmeter)}}.$$

Let $R$ = insulation resistance of motor; $R_v$ = resistance of voltmeter. The equation then becomes:

$$\frac{R}{R_v} = \frac{V - V'}{V'}. \tag{9}$$

This equation can be solved for $R$, since all other values are known.

$$R = R_v\left(\frac{V}{V'} - 1\right). \tag{10}$$

In commercial work a circuit of around 500 volts should be used in this test.

**Example 5.** In testing the insulation resistance of a motor with a 440-volt circuit, a 750-volt voltmeter, having 75,000 ohms resistance, was used. The voltmeter read 442 volts when put across the circuit, and 10 volts when put in series with the insulation between windings and frame. What is the insulation resistance of the motor?

$$R = R_v\left(\frac{V}{V'} - 1\right)$$
$$R = 75{,}000\left(\frac{442}{10} - 1\right) = 75{,}000 \times 43.2 = 3{,}240{,}000 \text{ ohms.}$$

If the voltmeter had shown a deflection of 1 volt, the insulation resistance would have been:

$$R = 75{,}000 \times (442 - 1) = 33{,}225{,}000 \text{ ohms.}$$

Had the voltmeter shown no deflection, it would mean that the resistance was greater than 33,225,000 ohms, but too great to be measured with the voltage used.

**10. Insulation Resistance of Covered Wire, etc.** It is required by fire insurance companies that insulated wire of various grades shall be up to a certain standard insulation resistance per mile after being soaked in salt water for a stated length of time. The following method, as shown in Example 6, is the common practice for finding the ohmic resistance of the insulation.

Note that the galvanometer constant is first found, and then used in determining the insulation resistance.

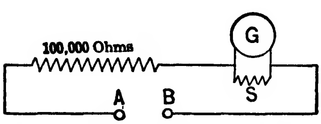



FIG. 19–5. A shunted galvanometer in series with a 100,000-ohm resistor, for determination of the galvanometer constant.

**Example 6.** In Fig. 19–5 a known high resistance, 100,000 ohms, is connected in series with a shunted galvanometer $G$ across a constant potential source of power $AB$. The resistance of the galvanometer and of the shunt $S$ are known. The deflection of the shunted galvanometer is noted, and by means of the resistances of the galvanometer and shunt, it is possible to compute what the deflection would be without the shunt.

For example, suppose the following data to be obtained:

$$\begin{aligned} \text{Deflection} &= \phantom{00}80 \text{ scale divisions.} \\ \text{Resistance of } G &= 2000 \text{ ohms.} \\ \text{Resistance of } S &= \phantom{00}10 \text{ ohms.} \end{aligned}$$

Since the $S$ and $G$ are in parallel, the current will divide between them in inverse proportion to the resistance; that is, the greater part of the current will go through the smaller resistance $S$.

Assume the current to be divided up into 2010 parts; then 2000 parts flow through the shunt and 10 parts through $G$. That is, $\frac{10}{2010}$ of the current goes through the galvanometer, and, therefore, the deflection of 80 is only $\frac{10}{2010}\left(\text{or } \frac{1}{201}\right)$, as large as it would be if it were unshunted.

An unshunted galvanometer would then deflect $201 \times 80 = 16{,}080$ divisions, when connected in series with 100,000 ohms, with the same current flowing in the line. If there were 1,000,000 ohms (1 megohm) in the line, the unshunted galvanometer would deflect only $\frac{1}{10}$ as much (the more resistance the less current). That is, the deflection would be $\frac{1}{10} \times 16{,}080 = 1608$ (the deflection for 1 megohm). This is called the **galvanometer constant.** The galvanometer constant is the computed deflection which this galvanometer (unshunted) would show if one

megohm were connected in series with it, and the combination placed across the same source of emf as that to be used in testing the cable insulation.

The shunt is now removed from the galvanometer as in Fig. 20–5. The coil $C$, immersed in salt water, is put in place of the 100,000 ohms, one connection being made to the end $Y$, of the coil, and the other to the

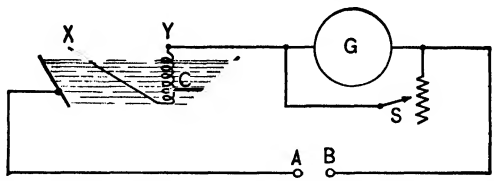



FIG. 20–5. The galvanometer is now unshunted and in series with the insulation of the cable, for determination of insulation resistance.

metal tub. The other end $X$ of the coil is kept above the water, so that any current which goes through $G$ must leak through the insulation of the coil of wire into the water.

The same potential is applied as in determining the galvanometer constant. Assume the galvanometer now reads 16. Since a deflection of 1608 means a megohm resistance in the line, a deflection of only 16 means a much greater resistance than 1 megohm in the line. In fact, the deflections are inversely proportional to the resistance of the line. Thus the unknown insulation resistance $x$ is as much greater than one megohm as 1608 is greater than 16.

$$x = \frac{1608}{16} = 100.5 \text{ megohms}$$

This is the insulation resistance of the coil.

If the coil in the above example were $\frac{1}{4}$ mile long the insulation resistance per mile would be only $\frac{1}{4}$ as great, or 25.1 megohms. This is because the current would have 4 times as much area to leak through in a mile as in $\frac{1}{4}$ mile.

Notice that the galvanometer constant 1608 is not used as a **multiplier** but as a **dividend,** which is **divided** by the deflections.

Note that in this test the resistance of the battery and galvanometer are neglected because they are too small in comparison with the resistance of the insulation and standard box to affect the results appreciably. When the standard 100,000-ohm box above is in the line, the galvanometer is shunted so that it has less than 10 ohms resistance, which is one ten-thousandth of the total resistance of the circuit. When unshunted, in the second part of the test, the resistance of the galvanometer is but one fifty-thousandth of the total line resistance. In fact, the standard resistance of 100,000 ohms might be left in the line in the second part as well,

and its resistance neglected, so small is it in comparison with the insulation resistance.*

**Prob. 13–5.** In a test, as in Paragraph 10, for the insulation of a coil of wire, the following data were taken:

Resistance of Galvanometer = 3000 ohms;
Resistance of Shunt = 20 ohms.

Deflection of shunted galvanometer with 200,000 ohms in series = 18.4.

Deflection of unshunted galvanometer with the resistance of the insulation in series = 4.3.

Length of coil = 500 feet.

(*a*) What is the galvanometer " constant "?

(*b*) What is the insulation resistance per mile of cable?

## SUMMARY OF CHAPTER V

**METHODS OF MEASURING RESISTANCE.**

**(1) Voltmeter-ammeter.**

**(a) Makes use of instruments in general use, but which must be accurately calibrated.**

**(2) Fall of potential.**

**(a) Standard resistance and voltmeter which need not be accurately calibrated to read volts.**

**(b) Accurate method for measuring low resistance if voltmeter has high resistance or if a potentiometer is used.**

**(3) Wheatstone bridge.**

**(a) Three standard variable resistances and galvanometer. Has advantage of being a " no-deflection " method, thus galvanometer need not be calibrated. A slide-wire may take the place of two of the resistances.**

**(b) Modifications of this bridge are used in the Murray and Varley " loop " methods of locating faults in cables.**

**(4) Voltmeter, for high resistance.**

**A simple method of ascertaining whether or not insulation resistance between different parts of a machine exceeds a certain minimum.**

**(5) Insulation resistance of covered wire.**

**Galvanometer " constant " found by noting deflection when a standard resistance takes the place of the coil. Galvanometer deflections must be proportional to current through it. Source of power must be of constant potential. Wire must be soaked for a stated period.**

* A portable instrument for rapidly determining insulation resistance, called the "Megger," is described in Chapter XVII.

## PROBLEMS ON CHAPTER V

**Prob. 14–5.** Certain types of Weston direct-current Ammeters are designed to have 45 millivolts drop across them when they are giving their full scale reading.

(*a*) What must be the resistance of a 5-ampere ammeter?
(*b*) Of a 50-ampere ammeter?
(*c*) Of a 15-ampere ammeter?
(*d*) How much current would a 5-ampere ammeter take if placed by mistake across 110 volts?

**Prob. 15–5.** Certain types of Weston direct-current Voltmeters have approximately 100 ohms for each volt of the scale. What current does a 150-volt voltmeter take when placed across a 112-volt circuit?

**Prob. 16–5.** It is desired to measure the resistance of an appliance by the ammeter-voltmeter method. A 15-ampere ammeter connected in series with the appliance reads 12.45 amperes. A 150-volt voltmeter connected across the appliance reads 101.4 volts. (*a*) Compute the resistance of the appliance from these readings, making allowance for the current taken by the voltmeter and the drop across the ammeter. (*b*) Is this the best arrangement of the instruments?

Assume the voltmeter has 100 ohms per volt of scale, and ammeter requires 0.045 volt for a full scale deflection.

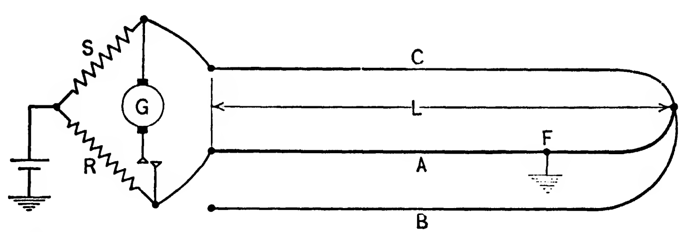



FIG. 21a–5. A Fisher loop, arranged to measure the resistance of the faulty line and return.

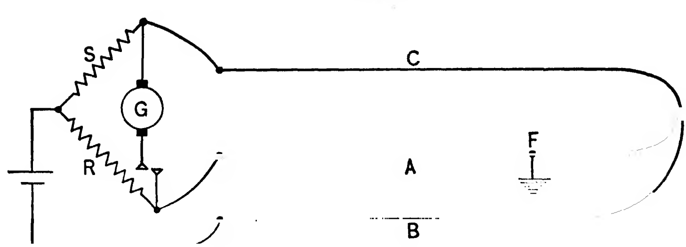



FIG. 21b–5. The battery of Fig. 21a–5 is now disconnected from the ground, and connected to the far end of the line through conductor B.

**Prob. 17–5.** Fig. 21*a*–5 and 21*b*–5 represent the Fisher modification of the Murray loop. Conductor *A* is grounded at *F*. There are two other conductors (*C* and *B*) which may be unlike each other and unlike *A*.

The far ends of conductors $A$, $B$ and $C$ are joined. Battery is grounded as in Fig. 21*a*–5 and resistances $R$ and $S$ are adjusted until galvanometer does not deflect. $R$ is found to be 24 ohms and $S$ to be 88 ohms.

The battery is then disconnected from the ground and connected to conductor $B$, as in Fig. 21*b*–5, and $R$ is readjusted until a balance is again obtained. Let this new value of $R$ be represented by $R_1$, which is found to be 62 ohms, while $S$ remains unchanged. The length ($L$) of the faulted conductor is 6520 feet. How far out is the fault ($F$)?

**Prob. 18–5.** One terminal of the field windings of a motor was connected to the negative terminal of a 112-volt circuit. A voltmeter, connected between the positive terminal of the circuit and the frame of the motor, indicated $\frac{1}{2}$ of a volt. The voltmeter had a 300-volt range and a resistance of 98.2 ohms per volt of scale. What was the value of the insulation resistance between the field coils and the frame of the motor?

**Prob. 19–5.** In Fig. 22–5, $R$, $R_1$ and $R_2$ represent adjustable resistances mounted on a wooden panel; $B$ represents a battery cell; $G$, a galvanometer; $K_1$ and $K_2$, keys. Show how you would connect this apparatus to the appliance $X$ in order to measure the resistance of $X$. Do not change the position of any piece of apparatus on the sheet.

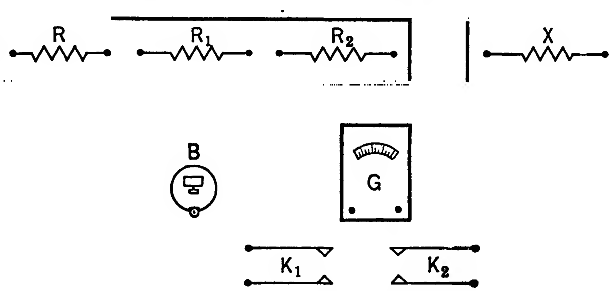



FIG. 22–5. The component parts of a Wheatstone bridge.

**Prob. 20–5.** The 5-ampere ammeter described in Prob. 14–5 is to be used with the 150-volt voltmeter of Prob. 15–5 to measure a resistance which was found to be 31.5 ohms. The source of power is a 110-volt line. Show how you would connect up instruments to measure this resistance to the greatest degree of precision. Give readings of instruments, and per cent error in the value of resistance, obtained by dividing the voltmeter reading by the ammeter reading.

**Prob. 21–5.** A 110-volt two-wire system has an average insulation resistance for each wire of 200 megohms per mile. What will the current leakage be on a 5-mile line?

**Prob. 22–5.** Insulation resistance should be high enough to prevent more than one-millionth of the rated current from leaking through the insulation. On this basis, what should be the insulation per mile of a half-mile line, transmitting 120 kilowatts at 550 volts?

# CHAPTER VI

## MAGNETS AND MAGNETISM

Practically all electrical apparatus, from the electric door-bell to the modern turbo generator, depends for its operation upon the action of magnetic fields produced by strong magnets. Before we can understand the action of motors and generators, we must become thoroughly familiar with the nature of these magnetic fields; how they act, and how they are produced.

**1. Magnets and Magnetic Substances.** A **magnet** may be defined as any material which has the property of attracting iron or steel to itself. A **magnetic substance** is any material which acquires this property when placed near a magnet, or near a conductor carrying an electric current. Any material which is given this property is said to be **magnetized.***

By far the best **magnetic substances** are **iron** or **steel.** They are practically the only magnetic materials used in the construction of motors, generators and all equipment where strong magnets are employed. Cobalt and nickel, and some of their alloys, are magnetic. One of these alloys, " Permalloy," developed by the Bell Telephone Laboratories, is used in telephones where an excellent weak magnet is desired. Salts of certain metals and liquid oxygen are attracted to a magnet.

Under the action of sufficient magnetizing force, it is believed all materials are magnetic to some degree at least. But iron and steel possess this property in so much greater degree than all other substances, that they are generally referred to as the **magnetic materials.**

**2. Types of Magnets.** The ancient Greeks discovered that certain stones found in Magnesia, Asia Minor, had the property of attracting particles of iron, and hence were given the name **magnets.** It was also discovered that if one of these stones were freely suspended, one end would take a position pointing north, and hence was called a **leading stone**, or "lodestone," by marine navigators. These we call **natural magnets.** Natural magnets consist of an iron ore, known to chemists as magnetite.

If a piece of hardened steel be rubbed with a lodestone, it will

* From Pender's Handbook for Electrical Engineers.

*become magnetized to some extent and will keep this magnetism* indefinitely. If soft iron is treated the same way it will become magnetized, but will not retain much of its magnetism. These magnets we call **artificial magnets.**

The iron and steel in the cases just mentioned are not magnetized sufficiently for use in electrical machinery. In practice, iron and steel are magnetized by the effect of an electric current, by which very much stronger magnets may be obtained. This process will be discussed in the next chapter.

**3. Bar Magnets: Poles; Lines of Force; Magnetic Field.** We are all familiar with the bar magnet which consists of a small magnetized bar of steel.

Although the bar magnet has no great commercial use, it is perhaps the simplest appliance from which to learn certain fundamental facts concerning magnetism.

If we place a glass plate over a bar magnet and scatter iron filings on it, the filings will arrange themselves as in Fig. 1–6.

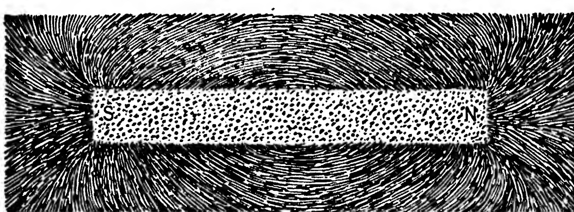



FIG. 1–6. The field of a bar magnet mapped with iron filings.

Note that the filings are attracted in great quantities to regions near the ends of the bar. These regions are called the **poles** of the magnet.

Note also that the filings arrange themselves in definite paths or lines from one pole, in space, around to the other. This shows us that magnetism manifests itself as a force, aligning the particles in streams or lines, which indicate the direction of the magnetic attraction. This gives us the conception of **magnetic lines of force**, and we think of magnetism as being of this nature. The stronger the magnet, the greater is the number of these **magnetic lines of force.**

If we place a small compass near each end of the magnet, we find it always points along a line of force; at one end, pointing toward the magnet; at the other, away from the magnet. This indicates that this force action is from one pole, through space or other

*material, around to the other pole in a definite direction; and it is* convenient to assume that this force is in the direction in which the compass needle points, and that these magnetic lines have the direction thus indicated. The end of the magnet **from which** the compass needle points is called the **north pole,** and the other end, the **south pole.** It is easy then to think of these magnetic lines as coming out of a north pole, going around the magnet and into the south pole. The entire region from which the lines come, we call the **north pole,** and the entire region into which the lines seem to enter, we call the **south pole.**

The entire space or region outside the magnet in which these lines exist, and in which other magnetic material is effected, is called the **magnetic field** of the magnet.

**4. Path of Magnetic Lines.** A map of the magnetic field of a magnet presents even a better idea of the lines than the iron filings. Figure 2–6 represents such a map. Note that here the lines come

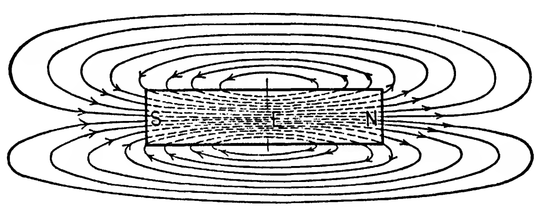



Fig. 2–6. The field of a bar magnet as mapped with an exploring compass.

out of the north pole, go around to the south pole, enter the magnet here and go through the magnet to the north pole.* **Each line is a complete loop.** In fact the whole magnetic field has the nature of a whirl, and is always thought of and spoken of as a flux of **magnetism** which goes around its **circuit** — out of the north pole around to the south pole, into the south pole and through the magnet to the north pole again. The **magnetic flux** through a **magnetic circuit** is not a flowing current, as an **electric current** flowing through an **electric circuit,** but, as will be shown later, in many respects can be treated in a similar way.**

It is of the utmost importance that the student grasp the idea

* In much of the literature on this subject, magnetism in the magnet itself is called "lines of magnetic induction," and that outside the magnet, "lines of force." In practical work we can consider them the same.

** It should be definitely understood that electricity and magnetism are two entirely different things. One should not be confused with the other.

of the **magnetic flux** always constituting a **complete magnetic circuit.** Do not think of the lines starting out from a north pole and ending at a south pole, but think of them as continuing on through the magnet to the north pole again.

The fact that the lines actually run through this magnet and form a complete loop, and do not merely start at the north pole and end at the south pole, is proved by breaking a magnet into several pieces. Each piece becomes a separate magnet with a north pole and south pole of its own. See Fig. 3–6.

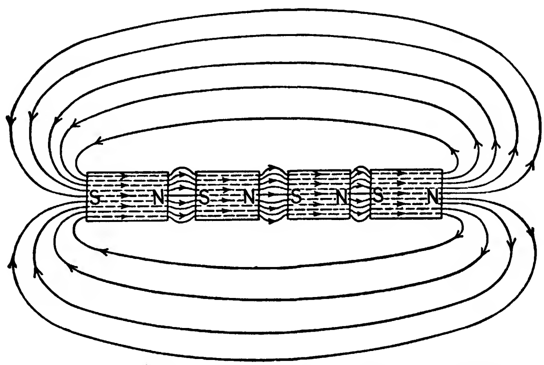



FIG. 3–6. A bar magnet broken to show that the magnetic lines extend through it.

**5. Nature of Magnetic Lines.** Magnetic lines act very much like a bundle of stretched rubber bands. Therefore, they have two distinct actions.

(1) They represent a **tension** along their length and are always tending to **shorten,** just as a stretched rubber band does.

(2) They exert a **sidewise crowding** effect on one another, each always tending to push the others away sidewise, just as each band in a close bundle of rubber bands would tend to crowd the others sidewise.

By means of these two properties, the tension, and the crowding, we can explain many magnetic phenomena, as well as those discussed below.

**6. Attraction and Repulsion of Magnets.** When we place the north pole of one magnet near the south pole of another, we note that the magnets attract each other. The tension of the magnetic lines explains this action. Figure 4–6 shows the magnetic field about these magnets as outlined by iron filings. Figure 5–6 shows the direction of the lines in different parts of the field. The lines

coming out of the north pole of magnet *B* enter the south pole of magnet *A*, go through this magnet, and emerging from the north pole of *A*, return to magnet *B* by way of its south pole. Each magnetic line thus threads the two magnets, and the tension in these lines tends to pull the magnets together, as so many loops of rubber bands would.

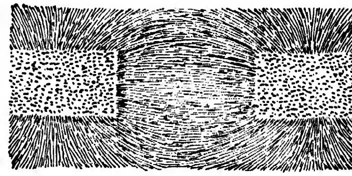


FIG. 4–6. The magnetic field between two unlike poles, as shown by iron filings.

Imagine two spools as in Fig. 6–6, threaded by rubber bands and then drawn apart. It is easy to see that the bands would tend to draw them together.

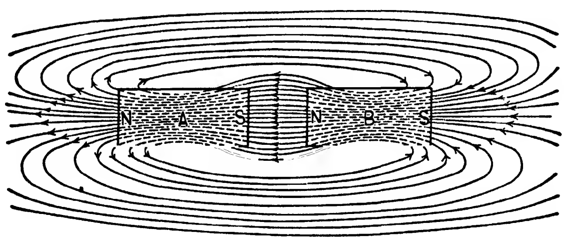



FIG. 5–6. The magnetic field between two unlike poles, as shown by an exploring compass.

If we place two like ends of magnets near each other, either two north ends or two south ends, the magnets tend to repel each other. The crowding effect of the lines explains this action. In

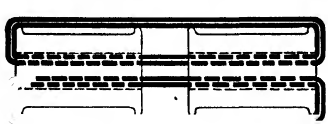


FIG. 6–6. The action of two unlike poles is similar to that of two spools threaded with stretched rubber bands.

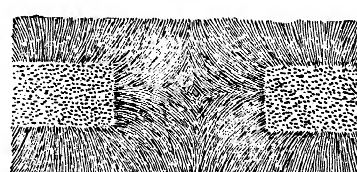


FIG. 7–6. The field between two like poles, as shown by iron filings.

Fig. 7–6, the shape of the magnetic field about two like poles is shown by iron filings. Fig. 8–6 shows the direction of these lines in various parts of the field when the two poles are north. Note that the lines coming out of each north pole crowd into the space between the poles and exert a sidewise pressure on one another.

Imagine two spools with rubber bands threaded through each, pushed hard against each other as in Fig. 9–6, and it is easy to see

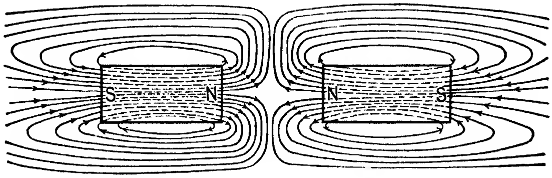



FIG. 8–6. The field between two like poles, as shown by a compass.

how the crowded rubber bands would tend to push the spools farther apart.

From the phenomena discussed above, it can be stated that "**Unlike poles attract, and like poles repel each other.**"

**7. Weber's Theory. Magnetic Molecules.** Weber's theory of magnetism is perhaps the best one advanced to explain the different magnetic phenomena.

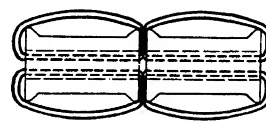


FIG. 9–6. The action of two like poles is similar to that of two spools, each threaded separately by rubber bands. When the two spools are held firmly together, the compression of the rubber between them tends to force them apart.

He supposes that all matter is made up of small molecules which are minute magnets. In iron and steel, these little magnets are strong; in all other materials they are weak. When a piece of material is not magnetized, these molecules lie in no regular position with regard to one another, as in Fig. 10–6. When the material is magnetized, the molecules all lie with their $N$ ends pointing the same way, as in Fig. 11–6.

A good illustration of this is to imagine a herd of cattle crowded into a pen. They will face in every direction. They thus fairly

FIG. 10–6. Weber's theory of magnetic molecules. The arrangement of the molecules in an unmagnetized bar.

represent the molecules of an unmagnetized piece of iron. Now suppose someone comes to one end of the pen with some fodder. The cattle will all head toward that end of the pen. They then represent the position of the molecules in a magnetized piece of iron.

As to whether or not a permanent magnet is formed depends

upon whether the molecules tend to retain this position of Fig. 11–6, or return to their original position of Fig. 10–6. The molecules of hard steel tend to retain this magnetic position, whereas the molecules of soft iron tend to return to the original position. Thus

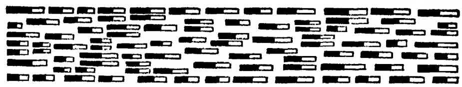


FIG. 11–6. The arrangement of the molecules in a magnetized bar.

hard steel forms a Permanent magnet, and iron or soft steel, a Temporary one.*

**8. Reluctance of a Magnetic Circuit.** We have likened the magnetic flux in a magnetic circuit to the electric current in an electric circuit. It will be remembered that the flow of an electric current is opposed by the resistance of the circuit. In a similar manner, the maintaining of the magnetic flux is opposed by the resistance of the magnetic circuit. This **magnetic resistance** is called **Reluctance** to distinguish it from Electric Resistance.

To carry an electric current we generally use copper or aluminum circuits because the resistance of these metals is low compared with the resistance of other materials. Similarly, to carry a magnetic flux, we generally use iron or soft steel circuits, because the reluctance of these materials is low compared with the reluctance of other materials. However, an electric circuit is almost always made up entirely of copper or aluminum. The electricity is rarely made to flow through the air, except in radio circuits. But unfortunately, it is often necessary to cause the magnetic flux to flow a certain distance through the air. It is fortunate, therefore, that the reluctance of the air is only a few thousand times as great as the reluctance of most iron, while the resistance of the air is several million times the resistance of copper. Accordingly, we can often use air as part of the magnetic circuit without having to overcome too great a reluctance in the circuit. If the reluctance of air were as much greater than the reluctance of iron, as the resistance of air is greater than the resistance of copper, it would be practically impossible to make the powerful electromagnets demanded in the creation of large generators and motors.

**9. Why Iron is Attracted to a Magnet.** Because the reluctance of iron and steel is lower than the reluctance of air, iron and steel are attracted to a magnet. When a piece of soft iron, for instance,

* To explain all magnetic phenomena Weber's theory must be considerably elaborated.

*is placed in the air near a magnet, the magnetic lines will be denser* in the piece of iron than in the surrounding air, because the reluctance of the iron is lower than that of the air. Fig. 12–6 shows this. A small piece of iron $A$, which was not magnetized, was placed in the magnetic field of the magnet $M$. The field is distorted by the lines apparently trying to go through the iron, instead of through the air, because the iron offers an easier path. The iron now has a

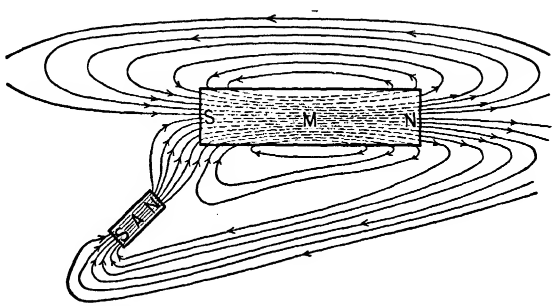



Fig. 12–6. A piece of iron, A, becomes a magnet by induction, when placed in a magnetic field.

north and a south pole of its own, since lines of force enter one end and leave the other. We have, then, two magnets with their unlike poles near each other, and they, therefore, attract each other. This explains why any piece of soft iron placed near a magnet is always attracted to it. If the piece of iron is placed near the magnet, of course more lines go through it than if it is placed farther away, and the iron becomes a stronger magnet and greater attraction exists between it and the magnet $M$.

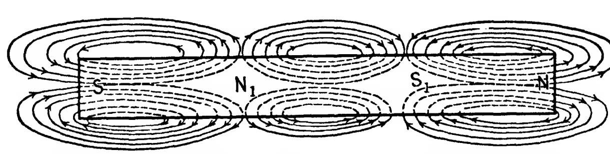



Fig. 13–6. A bar magnet with two consequent poles, $N_1$ and $S_1$.

**10. Consequent Poles.** It is possible to so magnetize a bar of steel that it has more than two poles. Thus in Fig. 13–6, there are four poles, of which $N_1$ and $S_1$ are called **consequent poles.** If the bar is sawed off across $N_1$, the larger piece would have three poles, two north, $N$ and $N_1$, and one south, $S_1$. But note that here again each line forms a complete loop and the path of the magnetic

*lines is in complete magnetic circuits, the lines always leaving by a* north pole and entering by a south pole.

**11. Ring Magnets.** There are cases where a piece of iron may be strongly magnetized and still possess no poles, as in the ring, Fig. 14*a*–6. Since the lines nowhere come out of the iron, they cannot produce any poles or external field. If, however, we break the ring, as in Fig. 14*b*–6, we have the usual two poles, a north

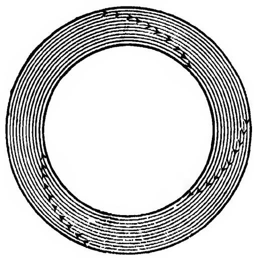


Fig. 14a–6. A ring magnet with no external field and thus, no poles.

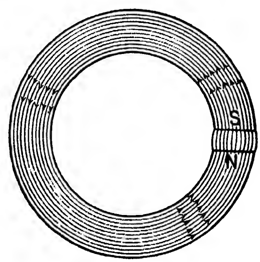



Fig. 14b–6. A ring magnet with an air gap, making a north and a south pole.

and a south, with an external field of great intensity, though small in area. The complete ring is used in some types of meters and also in transformers where no external field is desired. The broken ring, or horseshoe, is the fundamental form of the magnetic circuit of generators and motors, where an intense external field is desired in a limited air space.

**12. Magnetic Field of a Dynamo.** The magnetic field of the two-pole dynamo in Fig. 15–6 may be considered as merely the field of a bar magnet bent into a horseshoe. Note that here, too, the lines come out of the north pole and enter the south pole, go through the yoke and back again to the north pole. The armature revolves in the field between the north and south poles, and the wires on it cut the magnetic lines of the field and produce a voltage across the commutator. Figure 16–6 shows the field of a four-pole dynamo. Note that the lines come out of a north pole and go into a south pole, and that north poles and south poles **alternate** around the frame. In a four-pole machine, there are four magnetic paths; in a six-pole machine, six magnetic paths, etc.

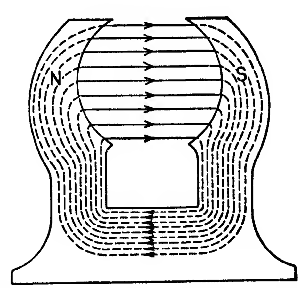



Fig. 15–6. The magnetic circuit of a two-pole dynamo.

In the operation of generators, motors, and most electric machinery, it is essential that the magnetism, or the number of magnetic lines of force, be readily increased or decreased. Therefore, soft steel or iron is always used for magnets in these machines.

**13. Permanent Magnets.** Many electrical measuring instruments, such as ammeters, voltmeters, watthour meters, etc., contain a permanent magnet. In order that the instrument retain its precision, the magnet must remain of the same strength from month to month. Magnets in magnetos used for electric ignition and other purposes are permanent magnets. These magnets are made of hardened tungsten steel or chromium steel, generally bent into the form of a horseshoe.

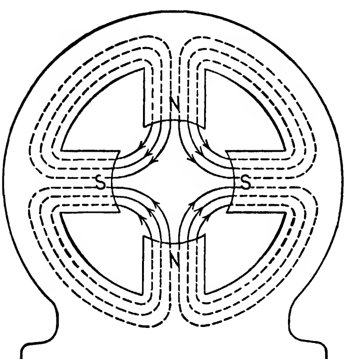


FIG. 16–6. The magnetic circuit of a four-pole dynamo.

In hardened steel, the molecules are more compact and are not as free to move as in the softer metals; therefore, on the basis of Weber's theory, it is easily seen why hardened steel keeps its magnetism better than soft steel or iron.

In making a permanent magnet, the steel is first bent into the desired shape, heated to a cherry red, and then immersed in water, oil or mercury, and agitated while cooling. It is then magnetized, as described later, and a soft-iron " keeper " placed across the poles of the magnet.

A magnet made of steel, treated in this way, will be very strong at first; but even hardened steel will gradually lose some of its magnetism, especially if it is handled roughly.

**14. Aging of Magnets.** It has been found that by every jar that a magnet receives, it loses some of its strength, though the amount lost by each successive jar becomes less and less. Thus all magnets which are to be of unvarying strength are put through a process of " aging," which has the effect of settling their strength at a definite point. With reasonable care, they will retain this strength indefinitely. By one method, the steel is steamed for 30 hours or longer, then magnetized and steamed for 4 or 5 hours more. This treatment demagnetizes it more or less, of course, but the remaining magnetism is very nearly permanent, especially if the steel forms a nearly closed loop.

**Heat and rough treatment will ruin the best of magnets, hence the need of careful handling of electrical instruments.**

**15. Laminated Magnets.** It has been found that thin magnets are stronger in proportion to their weight than thick ones. So magnets are often made of thin sheets of steel, bound with all north poles and all south poles together, as shown in Fig. 17–6. This is stronger than a magnet of the same total dimensions made of one solid piece of steel. The probable reason for this fact is that in tempering the steel with larger cross-sectional area all molecules are not affected alike.

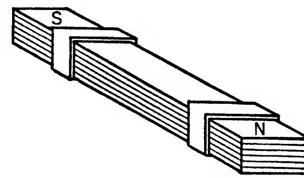


Fig. 17–6. A "laminated" magnet made up of a number of thin flat magnetized strips of iron or steel.

This arrangement is called a compound magnet.

**16. Magnetic Screens.** The low reluctance of iron is made use of whenever it is desired to cut the lines of force out of a magnetic field. For instance, it is desired to free the space *A*, Fig. 18–6, of magnetic lines. It would do no good to surround it with glass, porcelain, copper, brass, wood or any other non-magnetic material, because the reluctance of these is about the same as that of air. A shield of soft iron is put in the field. The iron screen, since it has a much lower reluctance than air, takes up nearly all the lines in the field *B* and conducts them around the space *A*. The space is thus not **insulated** from the magnetic flux, but the flux is **shunted** around it. Instruments and watches are sometimes thus shielded from the effects of a magnetic field. Neither the air nor any other material can be said to be a magnetic insulator, because, as we have seen, the reluctance of air and all other non-magnetic materials is not enough greater than that of iron and steel to prevent an appreciable magnetic current from being set up in them. On the other hand, the **resistance** of air, rubber, enamel, etc. is so much greater than the **resistance** of metals, that these substances act as very efficient insulators to electricity and only a slight electric current leaks through them. Still, we

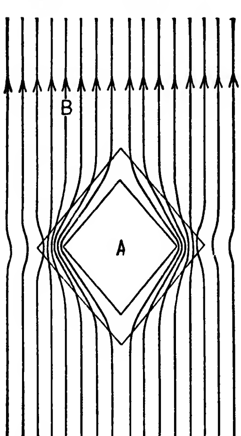


Fig. 18–6. A soft iron shell will practically shield the space inside from magnetic lines.

have seen in Chapter IV that, good as these substances are, they are not absolutely perfect insulators and some minute part of the current always leaks out through the best insulated wires. This quantity of electricity is not to be compared, however, with the comparatively large magnetic flux that leaks out of magnetic paths, even when they are composed almost entirely of iron and steel.

**17. The Compass.** A compass is merely a bar magnet suspended so that it can turn freely. We have seen that a magnetic line of force represents the direction in which a north pole is urged by the magnetic field. If a small magnet could be made with only one pole and that a north pole, and could be placed so that it was free to move near the north pole of a large magnet, it would be repelled from this north pole and travel around to the south pole, taking a path indicated by that line of force in which it happened to be placed. In fact, a magnetic line of force may be defined as a line along which a north pole is repelled and a south pole attracted. But every magnet which has a north pole has also a south pole. Thus, there is always a double-action on a compass; the north pole tends to move in the direction of the lines of force and the south pole tends to move in the opposite direction. This causes a compass to swing around until it assumes a position in which it lies along a line of force, with its north pole pointing in the direction of the magnetic flux.

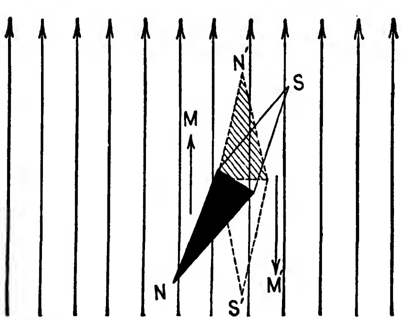


FIG. 19-6. A compass tends to lie along the magnetic lines in a magnetic field.

This is illustrated by Fig. 19-6. If we place the compass needle in the position $S$–$N$ in a magnetic field $M$, the north pole $N$ will be repelled along in the direction of the field and the south pole $S$ will be pulled along in the opposite direction. This will tend to swing the compass around on its axis until it takes the position $S'$–$N'$, in which it is parallel to the lines of force in the field.

This also explains why iron filings scattered on a glass plate above a magnet arrange themselves along the lines of force. The filings all become small magnets, or compasses, and when the plate is tapped gently they are free to move, and naturally, take positions along lines of force.

To understand why a compass at the surface of the earth always points approximately north, it is merely necessary to realize that the earth itself is a large magnet with the magnetic lines near its surface running from south to north. A compass thus takes a position along one of these lines. This is shown clearly in Fig. 20–6.

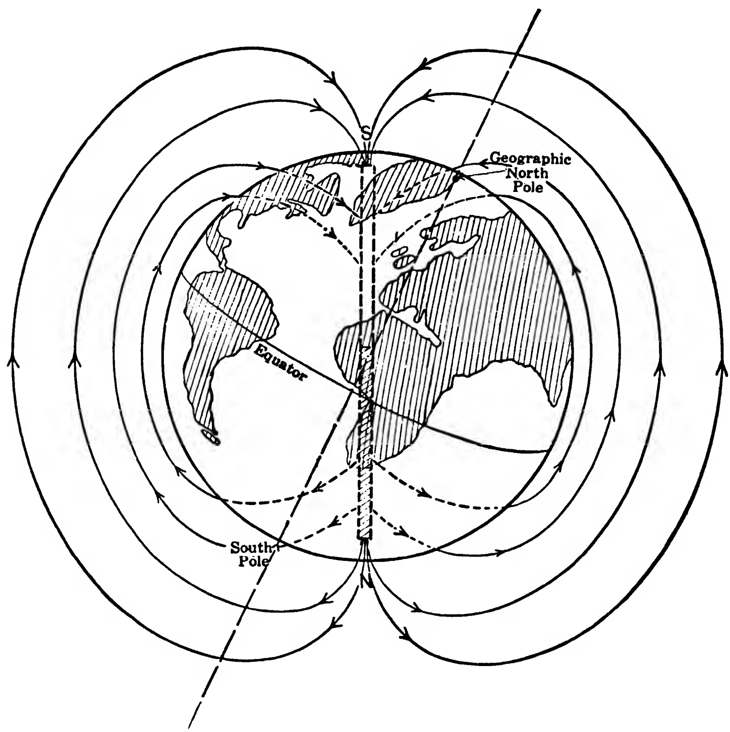



**Fig. 20–6.** The earth is a large magnet. The magnetic poles do not coincide with the Geographic Poles. Since the North Magnetic Pole attracts the **north** pointing end of a compass, it really is a south pole.

The place to which the north end of a compass points is not the geographical North Pole of the earth, but a spot in Boothia, a peninsula in the northern part of Canada. At this spot, the earth's magnetic lines enter the earth, run through the earth, emerge near the geographical South Pole, and flow north near the earth's surface. This gives us on the earth's surface a practically uniform magnetic field flowing from south to north, called the Earth's Magnetic Field.

Note also, that since the lines enter the earth in Boothia, this spot is really the South Magnetic Pole. So the north pole of a compass really points to the South Magnetic Pole. But since this South Magnetic Pole lies near the North Geographical Pole, it causes no confusion to say that the compass points north. It must always be kept clearly in mind, however, that **a compass always points away from a north magnetic pole,** as this is the surest test of polarity in dealing with electric machinery.

Practically, we are not so much interested in the North and South Poles as in the exact direction of the lines at different parts of the earth's surface; especially that part which is out of sight of land, and where the compass must be depended upon for direction. In order to map very completely the earth's magnetic field, The Carnegie Institute built a special ship called the "Carnegie." This vessel was remarkable in that it was constructed almost entirely of non-magnetic materials. With the delicate instruments which could be used on the "Carnegie," an accurate magnetic survey of the important portions of the earth's surface has been made.

For a discussion of other magnetic phenomena, see any encyclopedia. Some special topics of interest are: **History** of the compass; determination of the **inclination** and **declination** of the magnetic needle; **intensity** of the earth's field; **paramagnetic** and **diamagnetic** materials; magnetization of gases and liquids; effects of the medium in which the magnetic field lies, etc.

All magnets which we use are in the earth's magnetic field. Thus the field about a magnet is really the resultant field of the earth's field and the magnet's field.

## SUMMARY OF CHAPTER VI

**A MAGNET is a body which possesses the property of attracting iron and steel.**

**THE POLES of a magnet are those regions where the attraction is greatest.**

**It is usual to represent the strength of these poles by lines called MAGNETIC LINES OF FORCE. These magnetic lines are represented as coming out of a NORTH POLE, passing around the magnet, and going into a SOUTH POLE, passing through the magnet to a north pole.**

**Thus a MAGNETIC LINE OF FORCE forms a complete circuit.**

**A MAGNETIC FIELD is a space in which there are magnetic lines.**

**A RING may be magnetized so that all the lines lie inside the iron, thus producing no poles. Most electrical machines consist of magnets, the force lines of which lie partly in air and partly in iron or steel.**

**Magnetic lines act like stretched rubber bands and tend to SHORTEN. They also exert a LATERAL CROWDING effect upon one another. This explains the ATTRACTION OF UNLIKE POLES and the REPULSION OF LIKE POLES.**

**RELUCTANCE is the resistance which a substance offers to the magnetic flux.**

**The reluctance of iron and steel is low: thus a piece of iron in a magnetic field offers less reluctance than the air to the magnetic lines. The lines thus become denser in the iron, making it a magnet. This explains the attraction of iron and steel by a magnet.**

**NO MATERIAL is a good MAGNETIC INSULATOR, thus a space to be screened from magnetic lines must be surrounded by material of low reluctance, which will shunt most of the magnetic lines around the space.**

**A COMPASS is a magnet free to turn. It always takes a position lying along a line of force.**

**The EARTH is a large magnet with the magnetic South Pole lying near the geographic North Pole. The north pole of a compass, therefore, points to the magnetic South Pole which is near the geographic North Pole.**

## PROBLEMS ON CHAPTER VI

**Prob. 1–6.** Draw a 6-pole motor frame showing the complete paths of the magnetic lines.

**Prob. 2–6.** Draw an 8-pole generator frame showing the complete paths of magnetic force lines.

**Prob. 3–6.** Draw the magnetic field produced by two magnets placed side by side as in Fig. 21–6.

**Prob. 4–6.** Reverse one magnet in Fig. 21–6 and draw the magnetic field produced.

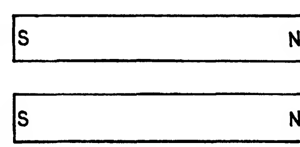



FIG. 21–6. Two bar magnets.

**Prob. 5–6.** A compass is placed at the point marked $X$ in Fig. 22–6 near a piece of soft iron. Make a drawing showing direction which compass needle will take, and direction of magnetic field on all sides of the soft iron piece.

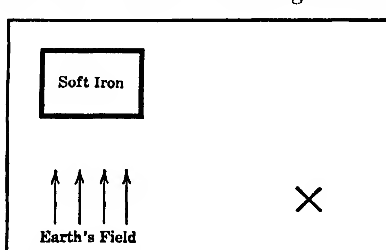



FIG. 22–6. How does the iron affect the earth's field at the point marked "X"?

**Prob. 6–6.** Draw the magnetic field for the motor in Fig. 23–6.

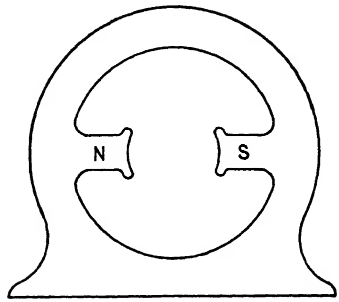



FIG. 23–6. The frame of a two-pole direct current dynamo.

**Prob. 7–6.** Draw the magnetic field of magnet shown in Fig. 24–6.

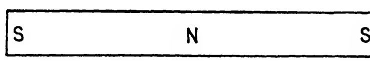



FIG. 24–6. A bar magnet with a consequent north pole.

# CHAPTER VII

## ELECTROMAGNETS

It is a fundamental law that wherever there is an **electric current** there is also a **magnetic flux. Electricity in motion** always produces a **magnetic field** at **right angles** to the **electric current.**

**1. Magnetic Field About a Straight Wire.** When a straight wire is carrying an electric current, a magnetic field exists around

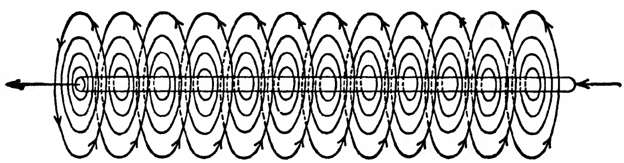


FIG. 1–7. The circular magnetic field about a straight wire carrying a current.

the wire in concentric circles, in planes 90° to the current, as shown in Fig. 1–7. Note that this field is not in spirals, but is in the form of closed circles. Looking at Fig. 1–7 from the left hand end, the

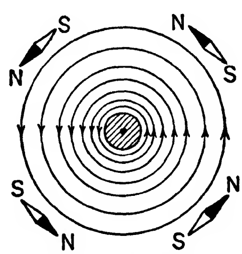



FIG. 2–7. The magnetic field about a conductor carrying a current which is flowing toward the observer — or out.

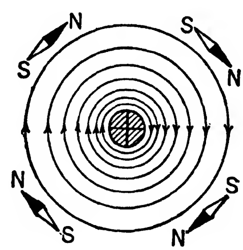



FIG. 3–7. The magnetic field about a conductor carrying a current which is flowing away from the observer — or in.

field would look like Fig. 2–7. From the right hand end of Fig. 1–7, the relation would look like Fig. 3–7.*

* Note that it is customary to denote the direction of current by an arrow; and that a dot in the center of a circle, representing a conductor, thus ⊙, indicates the point of the arrow or current out; and a cross in the center of the circle, thus ⊕, indicates the feathered end of the arrow, or current in.

If a compass needle (with north end shaded) is moved about the wire, it will point in the directions indicated in the above figures, (2–7 and 3–7). This fact shows that the magnetic flux exists in circular form about the wire. This fact also can be seen readily by passing a wire through a cardboard on which iron filings have been sprinkled, as in Fig. 4–7. The iron filings arrange themselves in

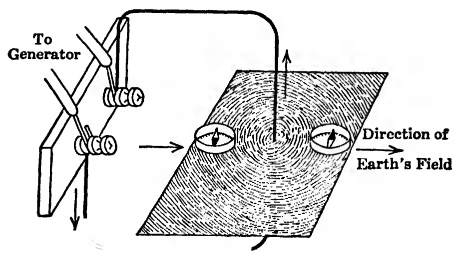



FIG. 4–7. Iron filings and small compasses show the shape and direction of the magnetic field about a straight wire carrying a current.

circles around the wire. The direction of these circular magnetic lines can be ascertained by compasses placed near the wire.

A considerable amount of current, say 50 to 100 amperes, is necessary to make a distinct figure with iron filings.

**2. The Relation of Direction of Current to Magnetic Field.** There is always a definite relation between the direction of current in a conductor and the direction of the magnetic field. This may be readily found by two rules.

(*a*) **Thumb Rule.** If we **grasp the wire** with the **right hand,** so that the **thumb** points in the **direction of the flow of current,** as indicated in Fig. 5-7, the **fingers** will **wrap around the wire in the direction** of the magnetic lines of force.

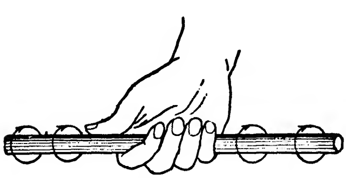


FIG. 5–7. Grasp the conductor with the right hand, pointing the thumb in the direction of the electric current, and the fingers will wrap around the wire in the direction of the magnetic flux.

(*b*) **Right-Hand-Screw Rule.** To drive a right-hand screw forward **in the direction of the current,** we **turn** it in the **direction** of the lines of force, or in the direction of the magnetic field. This is shown in Fig. 6–7. If the direction of current is reversed, the magnetic field will be in the direction we must turn the screw, to reverse the motion along its axis.

Note that the direction of the field, as indicated by the compasses in Figs. 2–7, 3–7 and 4–7, checks with the two rules just given.

**3. Magnetic Field of Two Parallel Conductors.** When two conductors are carrying current in the same direction, as in Fig. 7–7, a certain flux is set up around each wire, but it will be noticed the lines of force are in **opposite directions between the two wires** and

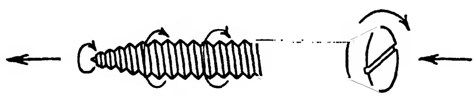


FIG. 6–7. A magnetic field has the same relation to the current as a right-hand screw has to its forward motion.

tend to oppose, or neutralize, each other. Therefore, most of the lines encircle both wires, and due to their tension there is a force tending to attract the wires and pull them together. In Fig. 8–7, the current in the two wires is in opposite directions and the flux is

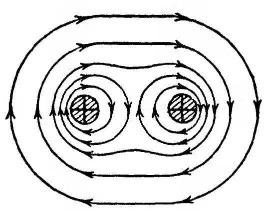


FIG. 7–7. The magnetic field about two conductors carrying current in the same direction.

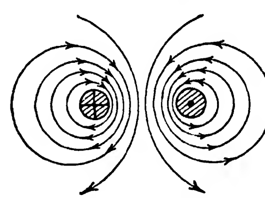


FIG. 8–7. Magnetic field about two conductors carrying current in opposite directions.

in the **same** direction between the wires. This crowding of the lines between the two wires exerts a repelling action tending to force the wires apart.

In large electrical machinery where the currents are great, these forces may be exceedingly large.

**4. Field About a Coil Carrying a Current.** If a wire is made into a loop as in Fig. 9–7, we find, by the Thumb Rule, that the lines of force, which everywhere whirl around the wire, all **enter** the same face of the loop and all come out of the other face.

If we now place several loops together into a loose coil, as in Fig. 10–7, most of the lines will thread the whole coil. If we make a close coil, practically **all** the lines will thread the whole coil, and return outside the coil to the other end.

The reason that practically no lines of force encircle the separate loops of a closely wound coil, but all thread the entire coil, is explained by referring to Fig. 11–7. This drawing represents an enlarged longitudinal section of the coil in Fig. 10–7. The current entering the ends of half loop at *A*, *B* and *C*, comes out again at *D*, *E* and *F*. If the wire ends, *A* and *B*, were pushed nearer one another, the field on the right side of *A*, being in the opposite direction, would neutralize the field on the left side of *B*. The space between the wires *A* and *B* would thus be neutral or free of lines of force. The lines now would be compelled to continue on through the whole length of the coil, and would not slip into the spaces between the loops and encircle each wire with a separate field.

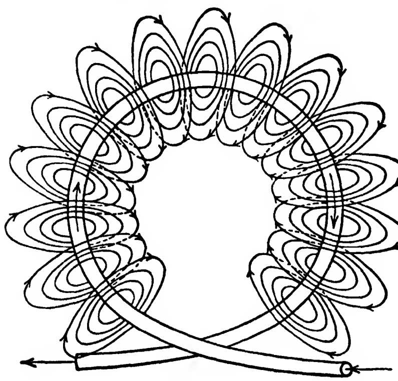


FIG. 9–7. The magnetic field about a single loop of wire carrying a current. The magnetic lines are everywhere at right angles to the current.

We thus have the same shaped field as in and about a bar magnet; one end being a north pole, since all the lines come out of it,

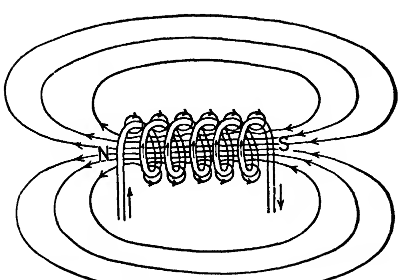



FIG. 10–7. The magnetic field about a loosely wound coil. Most of the lines thread the entire coil. Magnetic poles are thus established at the ends of the coil.

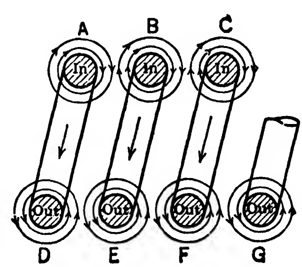



FIG. 11–7. A longitudinal cross-section of the coil in Fig. 10–7.

and the other end a south pole, since all the lines enter it. In order to find the polarity of such a coil, it is best to use a modified form of the Thumb Rule for a straight wire.

**5. Thumb Rule (for coil).** Grasp the coil with the right hand so that the **Fingers** point in the direction of the **current** in the coil, and the **Thumb** will point in the direction of the magnetic flux, or to the north pole.

Fig. 12–7. A lifting magnet handling scrap metal. *Cutler-Hammer Co.*

Note that in the Thumb Rule for a coil, the **fingers** point in the direction of the **electric current,** but that in the Thumb Rule for a straight wire, the **fingers** point in the direction of the **magnetic flux.**

**6. Coil Equivalent to a Bar Magnet.** A coil like that of Fig. 10–7 with a current flowing through it is then the equivalent of a bar magnet with two poles. Its magnetic field has the same shape as the field of a bar magnet and it obeys the laws of a bar

magnet, as stated in Chapter VI. That is, unlike poles of two coils attract each other and like poles repel; if the coil is free to turn and is placed in a magnetic field, it will tend to take such a position that the lines inside the coil are parallel to the lines in the field. Such coils are called solenoids.

**7. Electromagnets.** Very strong magnets are made by inserting a piece of iron or steel, called a core, in the coil. The reluctance of iron and steel is so much lower than air, as explained in Chapter VI, that the same current in the coil sets up thousands of times as many lines in the iron core as it would in the air alone.

**8. Magnetic Hoists.** The powerful poles of magnetic hoists are built in this way. Coils of wire are wound around a number of cores of soft iron. As long as there is no electric current in the coils the iron is not a magnet. The face of the iron cores is brought in contact with pieces of iron or steel castings, etc., and the current turned on. The iron cores now become such strong magnets that each square inch of their ends will lift from 100 to 200 pounds of iron.

To release the load of iron it is merely necessary to turn off the electric current, or reverse the direction of it.

Figure 12–7 shows a Cutler-Hammer lifting magnet.

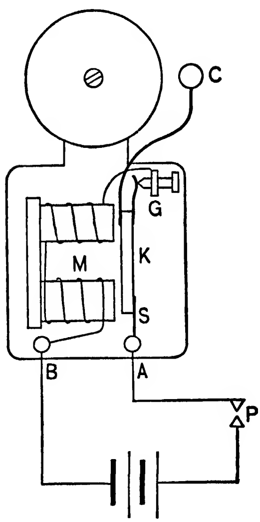


FIG. 13–7. A vibrating electric bell.

**9. The Electric Bell.** One of the commonest applications of electromagnets is found in the electric bell, a simple type of which is shown in Fig. 13–7.

When the button $P$ is pushed, the circuit is closed and a current flows through the coils on the electromagnet $M$. This causes the armature $K$ to be drawn toward the magnet, the clapper $C$ hitting the bell. But as the armature is drawn toward $M$, it moves away from the set screw $G$. This causes the circuit to be broken at that point, so the magnet $M$ loses its magnetizing force and no longer attracts the armature $K$. The spring $S$ then causes $K$ to fly back and again make contact at $G$. The current once more flows through the magnet and again $K$ is pulled over, and the bell struck. This vibrating action takes place very rapidly, as long as the button $P$ is held down. Bells so constructed are very cheap

and are in common use, battery cells, or low voltage transformers, being used as a source of power.

**10. The Telegraph.** The telegraph was one of the first important commercial developments of the electromagnet.

In Fig. 14–7 is shown a telegraph system with two stations. A large number of stations may be arranged in series on a single line. Note that there is a local circuit at each station and a main circuit between stations. The main circuit is closed when the line is not in use. Thus a current flows in coils $R$ and $R_1$ which holds down the armatures $N$ and $N_1$ of the local circuits. The batteries on the main line must be of the closed-circuit type to prevent their running down.

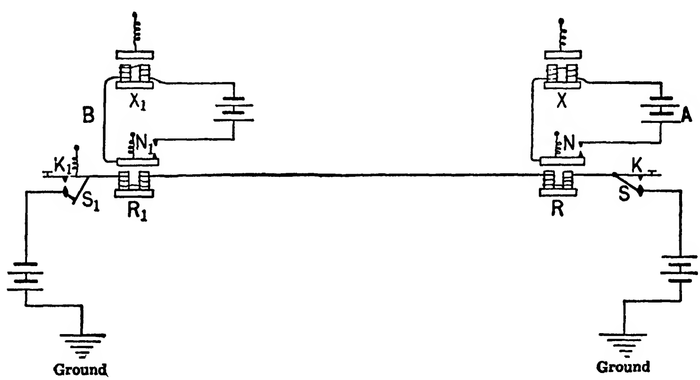



FIG. 14–7. Diagram of the circuits in a telegraph system with two stations.

Suppose station $A$ wishes to send a message to $B$. The short-circuiting switch $S$ is opened and the main line is thus opened. This releases the armature on the relays $R$ and $R_1$, because the main current is broken. These relays are similar to the magnet and vibrator in an electric bell, but they are much more delicate so that a minute current will operate them. They are used merely to open and close the local circuits. When any key, for example the one at $A$, is up, the spring pulls up the released relay armatures $N$ and $N_1$ and closes the local circuits so that the sounders $(x)$ and $(x_1)$ attract their armatures, making a click. Now, when $A$ presses down his key $K$ again, the armatures of both relays are drawn down, opening the local circuits. The springs on the armatures of the sounders $(x)$ and $(x_1)$ now draw these up and produce another click. This action produces the peculiar clicking sound which is heard in a telegraph office. Every time the key $K$ is released or pressed down, all the relays on the line move and actuate

the armatures of the sounders in all the local circuits. At most stations, cut-out switches are arranged in the local circuits so that only the sounder of the station called is allowed to click. The so-called "dots and dashes" are produced by varying the length of time between clicks; the time for the dashes being slightly longer than that for the "dots." When the line is not being used, all instruments are left in service in order to receive the "calling signal." The main line usually consists of a galvanized iron wire or copper, either 165 mils or 203 mils in diameter. The ground forms the "return."

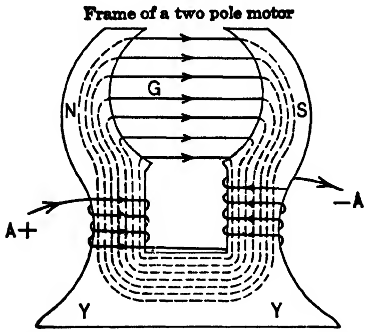



FIG. 15–7. The magnetic field of a two-pole dynamo.

**11. Generator and Motor Fields.** But the most important use of electromagnets is in generators and motors, where they are used to create the intense magnetic fields necessary in the economical generation of electric power. In Fig. 15–7, which represents a 2-pole motor, coils are wound on iron or soft steel cores in such a way as to make one pole face a north pole and the other a south pole. The steel yoke $Y$ joins the two cores together at one end, so that the magnetic circuit is composed entirely of iron except the space $G$ where the armature goes, a large part of which is also made of soft iron. The magnetic circuit is then a nearly closed iron circuit in the direction indicated.

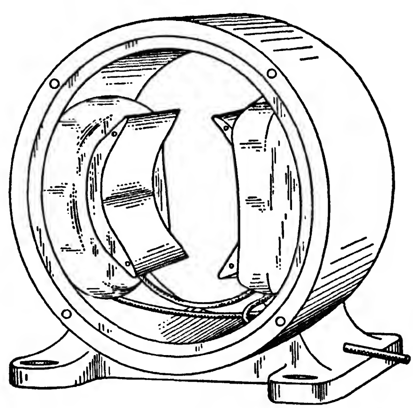


FIG. 16–7. A modern two-pole direct-current motor frame and poles.

In Fig. 16–7 is illustrated a form of a 2-pole motor. Figure 17–7 shows how the current in the field coils is made to set up magnetic fluxes in the proper directions. Figure 18–7 shows the appearance of one of these field coils removed from the frame of the motor.

**12. "Sucking" Coils.** Just as a bar magnet will attract a piece of iron, so a coil, since it has a field like a bar magnet, will attract a piece of iron. When the plunger of soft iron in Fig. 19-7 is placed in the field of the coil, it becomes magnetized as marked, the north pole being nearest the south pole of the coil. Thus, it is attracted into the coil, being drawn up until the centers of coil and plunger coincide. The strongest attraction exists when the center of the iron plunger **nearly** coincides with the center of the coil.

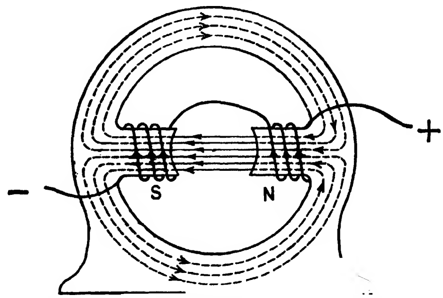


Fig. 17-7. The magnetic field of the motor of Fig. 16-7.

Extensive use is made commercially of this principle. It is the device used to operate the circuit breaker shown in Fig. 20-7. In this device, the plunger is sucked in only when the current in the coil becomes excessive. Then it automatically opens the breaker, shutting off the power.

**13. Non-Inductive Coils.** Since the direction of the magnetic field about a wire depends on the direction of the electric current, it is seen that the fields of two currents flowing in opposite directions must oppose each other, and will neutralize each other if the

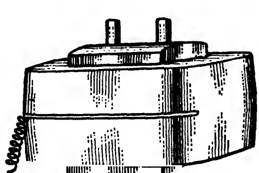


FIG. 18-7. A field coil mounted on a field pole.

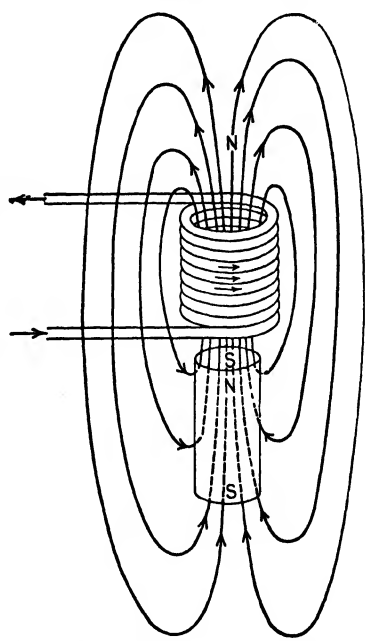


FIG. 19-7. A sucking coil. The flux set up by the current in the coil attracts the soft iron plunger.

wires are near enough together. Use is made of this fact whenever it is desired to have a current with a very weak magnetic field. Two wires are wound into a single coil so that the current in one wire flows in one direction, and the current in the other wire in the

Fig. 20–7. A circuit-breaker using the sucking coil principle.
*I. T. E. Circuit Breaker Co.*

other direction, around the coil. Such a coil has no perceptible magnetic field, and is said to be **Non-Inductively** wound. Resistance coils used in the measurement of resistance are wound in this way, as are almost all **resistance** coils in commercial ammeters and voltmeters. Note Fig. 9–5, page 130.

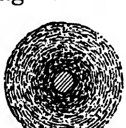


Fig. 21–7. The circular magnetic field about a wire carrying a current, as shown by iron filings.

**14. Combination of Parallel and Circular Fields.** When we combine the circular field about a wire with the parallel field at the pole face of a magnet, we secure a remarkable result which is the fundamental action underlying the construction and operation of electrical motors and many electrical measuring instruments. We have seen that the magnetic field about a wire carrying a current takes the form of circles as shown by the iron filings in Fig. 21–7. The field

between two unlike poles takes the form of nearly parallel lines as shown by the iron filings in Fig. 22–7.

If we now place a wire in the field of Fig. 22–7, perpendicular to these lines of force, and send an electric current through it, this

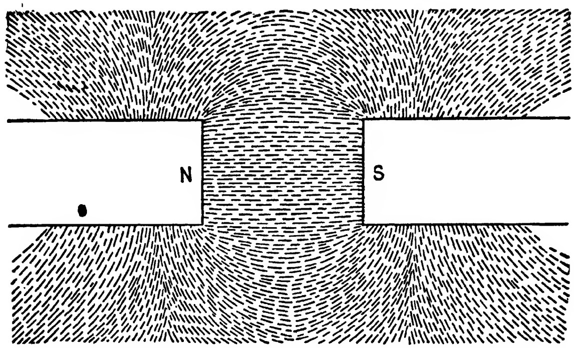



Fig. 22–7. The magnetic field between two unlike poles, as shown by iron filings.

current would tend to set up a circular field around the wire. But this circular field is affected by the parallel field already existing, and the result is the combination of the two fields shown in Fig. 23–7. Note that the lines have become denser on the upper

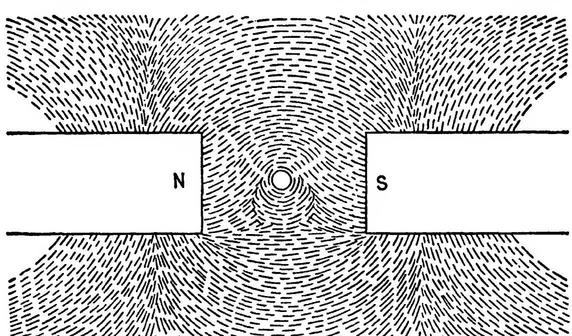



Fig. 23–7. The effect on the magnetic field of Fig. 22–7, when the wire of Fig. 21–7, carrying an electric current, is placed in it.

side of the wire and more scattered on the lower. Fig. 24–7 and 25–7, by giving the direction of the lines show this better.

The cause of the lines being more crowded on one side of the wire is due to the circular shape of the field about the wire, which

makes the direction of the force lines on one side of the wire exactly opposite to the direction of those lines on the other side. In Fig. 24–7, notice that the direction of the lines above the wire is to the right; below, to the left. When this wire is placed in a uniform field, as in Fig. 25–7, the lines above the wire are in the same direction as those of the field and thus strengthen the parallel field above. Those below the wire are in the opposite direction to the field and thus neutralize or weaken the parallel field below the wire. Thus a strong field is formed above the wire and a weak field below.

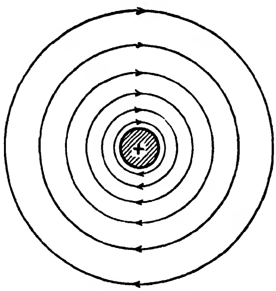


Fig. 24–7. A diagram of Fig. 21–7, showing the direction of the magnetic lines and the electric current.

Now imagine these lines to be rubber bands stretched into these shapes. It is easily seen how they would tend to force the wire down.

This is the fundamental principle of the operation of electric motors, certain voltmeters and ammeters, D'Arsonval galvanometers, etc. A large force is obtained by having a very strong field with many wires in it, carrying heavy

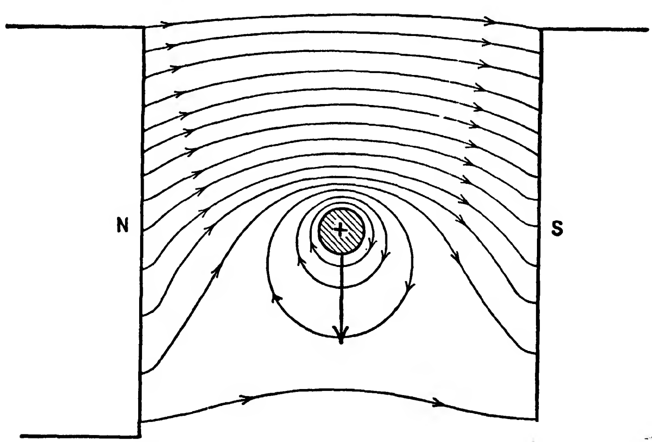



Fig. 25–7. A diagram of Fig. 23–7, showing the direction of the magnetic field and the electric current. There is a force "down" on the wire.

currents. The sum of the pressures on all the wires can thus be made to amount to a very great force. It will be seen later, however, that by far the greater part of the force which turns the

armature of a motor is produced on the iron core and not on the windings.

**15. Ammeters and Voltmeters.** The practical application of this force existing between a parallel field and a wire carrying a current is well illustrated by the action of Ammeters and Voltmeters, which, by the way, are really small motors. The only difference is that the poles are usually permanent magnets, and the armature is not allowed to rotate, but turns against a spring.

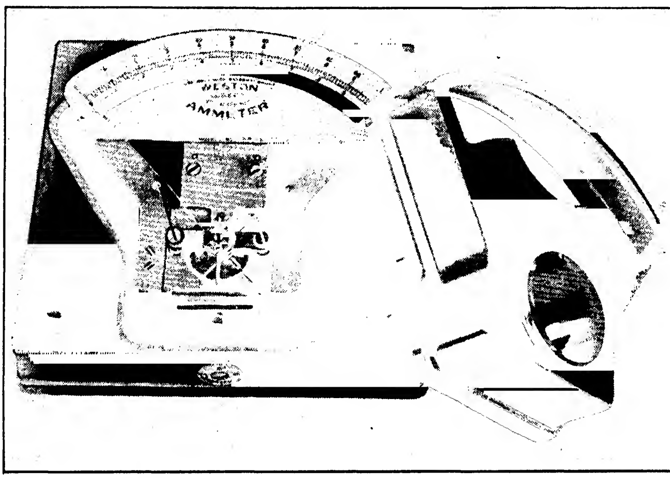



Fig. 26-7. An interior view of an electric meter.
*Weston Electrical Instrument Co.*

Figure 26-7 shows a view of the moving coil of a Weston ammeter.

Figure 27-7 is a detailed drawing of this coil and Fig. 28-7 shows the action of the coil as a current is sent through it.

The coil $AB$ is carefully set on jeweled bearings between the poles of a permanent aged magnet $N$ and $S$. Watch-springs $W$ hold the coil in place, so that the pointer is held at zero on the scale, when no current is flowing through the coil. Suppose a current is led into the moving coil so that it goes down along the side $B$ and up the side $A$. On a top view, Fig. 28-7, the current would go **in at** $B$ and **out** at $A$. The clockwise circular field around $B$ would strengthen the field of the permanent magnet, $NS$, **above** the wires at $B$ and weaken it **below.** This would urge the side $B$ **downward.** In the same way the counter-clockwise field about $A$ would

strengthen the magnet's field below $A$ and weaken it above. Thus $A$ would be urged **upward.**

These two actions would cause the coil to turn against the tension of the springs $W$. The stronger the current flowing through the coil, the stronger the combination field causing the coil to turn. The pointer could then indicate strength of the current as it moved over a scale graduated in amperes. The instrument is thus an ammeter. As the current in any coil is proportional to the voltage across its terminals, the amount the coil turns must also be proportional to the voltage. The scale accordingly might be graduated to read volts. It would then be a voltmeter and would have much greater resistance, as explained in Chapter V.

These instruments are described fully in Chapter XVII. Fig. 12–17 shows more complete details of the usual type. For the similar action between the armature and field of a motor, see Fig. 8–11 and description relating to it.

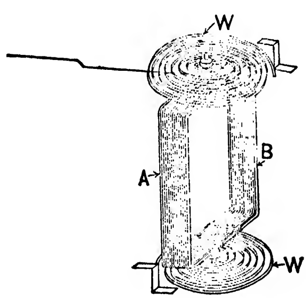



FIG. 27–7. The moving coil of the electric meter of Fig. 26–7.

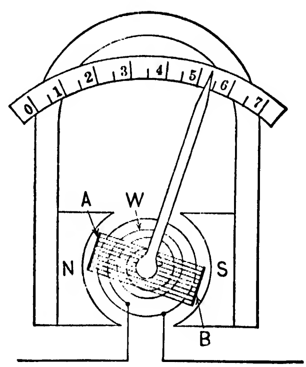



FIG. 28–7. Diagram of the meter of Fig. 26–7.

## SUMMARY OF CHAPTER VII

**There is ALWAYS A MAGNETIC FIELD about a wire carrying a current of electricity.**

**THUMB RULE FOR STRAIGHT WIRE. Grasp the wire with the RIGHT hand so that the thumb points in the direction of the current, the fingers then will point in the direction of the magnetic field around the wire.**

**RIGHT-HAND-SCREW RULE. In driving a right-hand screw forward in the direction of the current, the screw is turned in the direction of the magnetic field.**

**CURRENTS IN TWO PARALLEL WIRES set up a magnetic field, which either attracts or repels the wires.**

**An electric current flowing in a coil of wire makes an ELECTROMAGNET of the coil, one face becoming a north pole and the other a south pole.**

**THUMB RULE FOR A COIL. Grasp the coil so that the Fingers point in the direction of currents around coil, and the Thumb points to the north pole. This is another form of the rule for straight wire.**

**The magnetic field about a wire, when combined with a uniform field, causes a crowding of the lines on one side of the wire, and a thinning out of the lines on the other side. The wire tends to move toward the thinner part of the field. This is the basis of the moving force of an electric motor.**

**A coil made thus, and free to turn, is placed between the poles of a permanent magnet. On sending a current through it, the coil turns proportionately to the current. This is the principle of many Ammeters and Voltmeters.**

**The powerful magnets composing the magnetic field of generators and motors are made by coiling insulated wire around soft iron cores.**

## PROBLEMS ON CHAPTER VII

**Prob. 1-7.** Draw the magnetic field about the wire *A*, Fig. 29–7, when the current is flowing as indicated.

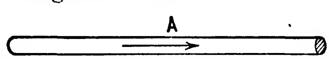



FIG. 29–7. A straight wire carrying a current.

**Prob. 2-7.** If *A*, Fig. 30–7, represents a cross section of wire with the current coming out, and the poles of the motor are as marked, draw resulting field between north and south pole. In what direction will wire *A* tend to move?

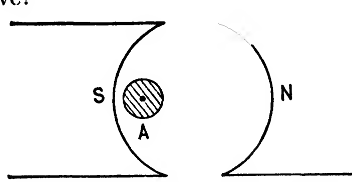



FIG. 30–7. A wire carrying a current in a magnetic field.

**Prob. 3-7.** *A* and *B*, Fig. 31–7, represent the cross section of a loop of wire on an armature in the magnetic field of a motor. If the current

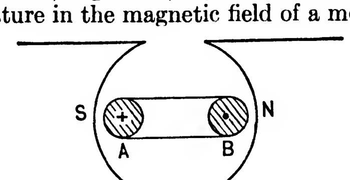



FIG. 31–7. A coil carrying a current in a magnetic field.

is as marked in *A* and *B* and poles of motor are as marked, in which direction will armature rotate?

**Prob. 4–7.** Draw the magnetic internal and external field for iron core with electric current flowing around it as indicated in Fig. 32–7.

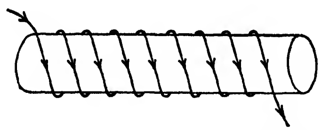


FIG. 32–7. A simple electromagnet.

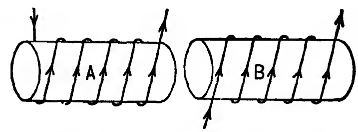



FIG. 33–7. Two electromagnets.

**Prob. 5–7.** Draw the field between coils *A* and *B*, in Fig. 33–7. State whether attraction or repulsion exists between them.

**Prob. 6–7.** Show windings on cores *A* and *B* of an electric motor, with direction of electric current indicated, and draw flux for the magnetic circuit, with direction of flux indicated in Fig. 34–7.

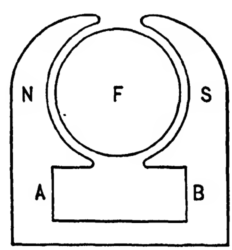



FIG. 34–7. The magnetic circuit of a two-pole direct-current dynamo.

**Prob. 7–7.** Draw a 6-pole dynamo field, showing circuits and direction of flux, and of electric current.

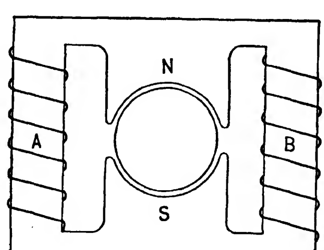



FIG. 35–7. The magnetic circuits of a two-pole, old type "consequent pole," direct-current dynamo.

**Prob. 8–7.** Show direction of current in windings on *A* and *B* to produce poles *N* and *S*, as marked in Fig. 35–7. Show magnetic flux as usual.

**Prob. 9–7.** Show current in windings on the 4-pole motor frame to produce north and south poles as marked in Fig. 36–7. Indicate magnetic flux as usual.

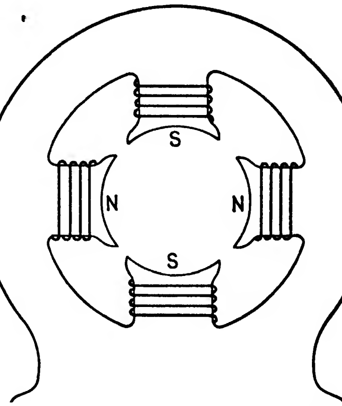



FIG. 36–7. The magnetic circuits of a four-pole direct-current dynamo.

**Prob. 10–7.** A standard telegraph sounder has 5 ohms resistance. The local circuit is wired with 120 feet of No. 22 B. & S. copper wire. The local battery cell has an emf of 1.08 volts and an internal resistance of 3 ohms. What current passes through the local circuit every time the sounder acts?

**Prob. 11–7.** A 15-mile telegraph line, using the ground of negligible resistance for a return, has 3 relays of 20 ohms each connected in it. The line is operated by 10 cells in series, each having an emf of 1.07 volts and 2.8 ohms internal resistance. The line wire is the best grade of galvanized iron, 165 mils in diameter, with a resistance per mil-foot of 69 ohms. What current flows through this line when no message is being sent?

**Prob. 12–7.** What current flows through a 40-mile telegraph line when no message is being sent, if the line wire is of second-grade galvanized iron, 203 mils in diameter and with a mil-foot resistance of 81 ohms? The battery consists of 20 cells in series, each cell having an emf of 1.06 volts and an internal resistance of 3 ohms. There are 4 relays of 50 ohms each on the line, and the return is through the ground of negligible resistance.

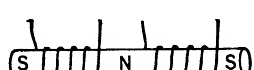



FIG. 37–7. A differentially wound electromagnet.

**Prob. 13–7.** Draw magnetic field of Fig. 37–7, and put current arrows on windings to produce field as marked.

# CHAPTER VIII

## THE MAGNETIC CIRCUIT

The operation of practically all electric power machines depends upon the mutual action of electricity and magnetism. To understand the performance of these machines, one must be familiar with the laws governing the magnetic circuit as well as those of the electric circuit.

In the previous chapters, the general nature of magnetism was considered. In this chapter, we will consider the laws governing the magnetic circuit, the relations of the quantities involved, and the calculation of such circuits.

**1. Similarity of Electric and Magnetic Circuits.** Just as electricity or an electric current can be made to flow in an electric circuit, so magnetism, magnetic lines, or magnetic flux, can be set up in a magnetic circuit.

In Fig. 1–8, the battery acts as a source of electromotive force and forces a current through the electric circuit. Similarly, in Fig. 2–8, an electric current flowing through the turns of the coil acts as a source of magnetic pressure, or magnetomotive-force; this sets up the magnetic flux in the iron core, which constitutes a magnetic circuit.

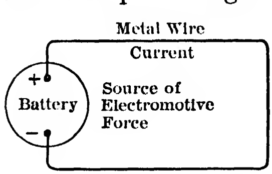



FIG. 1–8. The electromotive-force of the battery sets up an electric current in the metallic circuit.

It will be seen that in many ways these two circuits are similar; and it is found that the laws governing them can be written in much the same form.

However, there are many differences in the two circuits. In Fig. 1–8, the battery itself is part of the electric circuit, and its resistance is a part of the **resistance** of the circuit. In Fig. 2–8, the coil carrying the electric current is not itself a part of the magnetic circuit, nor does it affect the **reluctance** of the circuit — the reluctance of a magnetic circuit corresponding to the resistance of an electric circuit.

Again, the electric current is confined to the wire of known cross section, and only a negligible amount leaks through the insulation and supports. In the magnetic circuit, the magnetic flux is not so confined, as the reluctance of the surrounding air is relatively not so great compared to the circuit itself. We know that there is no known insulator for magnetic flux. In actual machines, 10 or 15 per cent, or more, of the flux may leak across air paths and cannot be utilized. Also the dimensions of flux paths are not always easy to calculate. Therefore, magnetic circuits cannot be calculated with the accuracy that is possible in electric circuits.

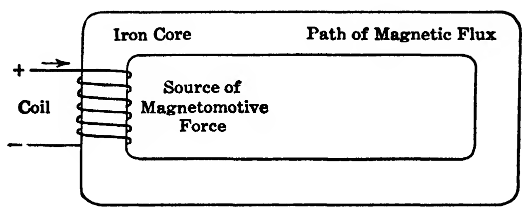



FIG. 2–8. The magnetomotive-force of the coil sets up a magnetic flux in the iron circuit.

It requires power to maintain a current in an electric circuit. In Fig. 1–8, the battery is supplying energy to the circuit so long as the current is flowing. This current produces heat in the circuit, even when it is steady.

In the magnetic circuit, after the flux is established, no heat is produced in the circuit itself, and therefore no energy is used in keeping it magnetized, if the electric current in the coil is steady. All the power used in supplying current to the coil in Fig. 2–8 can be calculated as $I^2R$ losses in the coil itself. Since no energy is used in the magnetic circuit, it indicates that there is **no movement** of the flux or flux lines, although we speak of magnetic lines as constituting a flux.

There is no apparent effect when the current in the coil is steady, but a varying or changing current shows a very pronounced effect on the circuit. It requires a momentary expenditure of energy to magnetize the iron circuit and turn the molecules, as indicated in Weber's Theory, but this takes place only when the current is rising, after the electric circuit to the coil is closed. This momentary expenditure of energy, due to the changing value of the flux, is called the energy stored in the magnetic field. See Chap. IX, paragraph 9.

In the electric circuit,

$$\text{Current} = \frac{\text{Electromotive-force}}{\text{Resistance}};$$

and in the magnetic circuit a similar relation holds,

$$\text{Magnetic flux} = \frac{\text{Magnetomotive-force}}{\text{Reluctance}}.$$

Magnetic units are based on the cgs or centimeter-gram-second system of measurement.*

**2. Magnetic Current or Flux (Lines of Force).** Let us consider the simplest sort of magnetic circuit possible. Such a circuit can be represented by Fig. 3–8, which consists of a wooden ring 6.25 centimeters mean diameter, made of 1.25 centimeters round stock. On this ring are wound 730 turns of insulated wire.

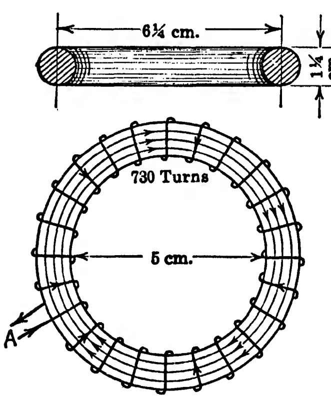



FIG. 3–8. An electromagnet consisting of a ring, about which is wound a coil of wire.

If we send an electric current through this wire in the direction marked, a magnetic flux will be set up in the wooden ring in the direction marked (clockwise).

We have seen that this magnetic flux consists of magnetic lines of force, which in this case exist in the circuit composed of the wooden ring. The amount of this flux is measured by the number of **lines** (called **maxwells**) which exist in the magnetic circuit, just as the amount of the electric current is measured by the number of **amperes** which flow in the electric circuit. We represent the number of amperes in the electric circuit by the letter ($I$). Similarly, we represent the number of lines in a magnetic circuit by the Greek letter ($\Phi$), pronounced **phi.**

In the iron circuits of electrical machinery, this flux may amount to several million lines.

**3. Flux Density, The Gauss.** In Chapter IV, the idea of current density has been suggested, and the rule has been given that the current in switch blades, etc., should not exceed 1000 amperes

* A system of measurement called the mks or meter-kilogram-second system has recently been approved.

per square inch of cross-section area. On this basis, the blades of a switch to carry 500 amperes should have a cross-section area of $\frac{1}{2}$ a square inch. Similarly, for convenience in the calculation of the iron circuits in electrical machinery, as we shall presently see, the lines per square centimeter, or per square inch, are used. This is called the **flux density** in the iron, and the letter ($B$) is used to denote this. The total flux in an iron path divided by the cross-section area equals the flux density, or

$$\frac{\Phi}{A} = B \tag{1}$$

where $\Phi$ = total flux, and $A$ = cross-section area at right angles to the path of the flux.

**Flux density** is also called the **degree of magnetization.** One line per square centimeter is called the **Gauss.**

**Example 1.** In Fig. 4–8, suppose there are 100,000 lines in the iron core, which has a cross-section area of $5 \times 5$ or 25 square centimeters. It is assumed that the lines are distributed uniformly throughout the iron path. Then the lines through every square centimeter $= \dfrac{100{,}000}{25}$ or 4000 gausses; that is, the flux density $B$ = 4000 gausses.*

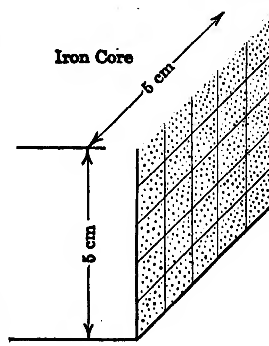



FIG. 4–8. The total number of magnetic lines in the iron core divided by the cross-sectional area in square centimeters, at right angles to the lines, is equal to the lines per square centimeter, or to the flux density in gausses.

Flux densities of 10,000 gausses (lines per square centimeter), or 64,500 lines per square inch, and above, are common in many electrical machines.

**4. Reluctance.** Reluctance in a magnetic circuit corresponds to resistance in an electric circuit. The reluctance of magnetic paths follows the same laws as does the resistance of electric circuits. The reluctance of a magnetic path will increase with the length, and decrease as the area of the path increases.

* The distinction between flux, $\Phi$, and flux density, $B$, is somewhat the same as the difference between the total population and the density of population for any area. For instance, the estimated population for the state of Massachusetts (1934) is 4,355,000. The land area of the state is 8039 square miles. The average density of population for the state is, then, $\dfrac{4{,}355{,}000}{8039}$ or 541.7 persons per square mile.

To calculate the resistance of a copper wire we use the resistance of a unit wire, or the resistance of a mil-ft, 10.4 ohms. We multiply this by the length in feet, and divide by the area in circular mils. Similarly, to calculate the reluctance of a magnetic circuit, we take a unit piece of the circuit and follow the same process. We use as a unit piece, the centimeter cube. The reluctance between parallel surfaces of a **centimeter cube** of air is taken as **one**; and all other magnetic units are based on this value. This is very convenient in our calculations as all materials, excepting iron and steel, have practically the same reluctance as air. (The reluctivity of iron and steel varies so widely that we do not attempt to learn its value, as will be seen later.)

The reluctance of a centimeter cube of air is 1. There is no generally accepted name for the unit of reluctance.*

We represent the resistance of an electric circuit by the letter ($R$). Similarly, we represent the reluctance of a magnetic circuit by the script letter ($\mathcal{R}$).

The resistance of a copper wire can be written:

$$R = \frac{10.4\, l}{d^2}$$

where $l$ = length in feet and $d^2$ = area in circular mils.

Similarly, the reluctance of a path of non-magnetic material of uniform cross section can be written:

$$\mathcal{R} = \frac{1 \times l}{A} \tag{2}$$

where $l$ = length in centimeters;
$A$ = area in square centimeters;
$1$ = reluctance of a centimeter cube (or the reluctivity) of air.

We can then calculate the reluctance of air (or other non-magnetic material) of any length and cross section. For instance, the reluctance of an **inch cube** of air $= \dfrac{1 \times 2.54}{2.54^2} = 0.394$ unit of reluctance.

* In America, the unit of reluctance was commonly called the Oersted. This term, however, has recently been adopted for the unit of magnetizing force, and hence, can no longer be used as the unit of reluctance.

If a magnetic circuit consists of two or more parts in series, the total reluctance is the sum of the individual reluctances, or

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \cdots, \text{ etc.} \tag{3}$$

Reluctances in parallel combine just as resistances in parallel, or

$$\mathcal{R} = \frac{1}{\dfrac{1}{\mathcal{R}_1} + \dfrac{1}{\mathcal{R}_2} + \cdots} \cdot \tag{4}$$

**Example 2.** Compute the reluctance of the wooden ring in Fig. 3–8, made of 1.25 centimeter round stock; mean diameter of the ring is 6.25 centimeters.

**Solution.**

$$\mathcal{R} = 1 \times \frac{l}{A} \cdot$$

Length of magnetic circuit $= 3.1416 \times 6.25 = 19.65$ centimeters.

Cross-section area $= 1.25^2 \times 0.7854 = 1.227$ square centimeters.

$$\text{Reluctance} = \frac{1 \times l}{A} = \frac{1 \times 19.65}{1.227} = 16 \text{ units.}$$

**Example 3.** In Fig. 5–8, the magnetic circuit is composed of an iron ring made of 2.13 centimeter round stock, and an air gap. What is the reluctance of the air gap, if it is 1.6 centimeters long?

**Solution.**

$$\mathcal{R} = \frac{1 \times l}{A}$$

$$= \frac{1 \times 1.6}{2.13^2 \times 0.7854} = 0.45 \text{ unit.}$$

This equation is particularly useful in finding the reluctance of air gaps in magnetic circuits of electrical machines. The method of calculating the iron parts of magnetic circuits is explained in succeeding paragraphs.

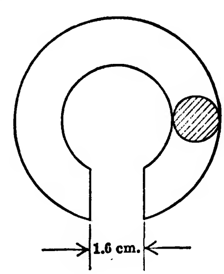



FIG. 5–8. A ring electromagnet with an air gap.

**Prob. 1–8.** What is the reluctance of a brass bar 40 centimeters long, made of rectangular stock 2 centimeters $\times$ 3 centimeters?

**Prob. 2–8.** Compute the reluctance of the air gap in Fig. 6–8. The air gap is 1.4 centimeters $\times$ 1.4 centimeters in area and 1.2 centimeters long.

**Prob. 3–8.** The length of an air gap is 0.875 centimeter. What cross-section must it have in order that the reluctance may not exceed 0.37 unit?

**Prob. 4–8.** What is the reluctance of an air path 2 inches long and 4.5 square inches in cross section?

**Prob. 5–8.** If the ring in Example 2 was made of 1.5-inch round stock with a mean diameter of 6 inches, what would be its reluctance?

**5. Ampere-Turns.** The number of turns of a coil of wire encircling or linking a magnetic circuit, multiplied by the current flowing in the coil, is called the **ampere-turns** of the coil. For instance, a coil of 1000 turns carrying 5 amperes has 1000 × 5 or 5000 ampere-turns. Or a coil of 500 turns carrying 10 amperes would give the same effect, and have 500 × 10 or 5000 ampere-turns. Ampere-turns are represented by the letters ($NI$) where $N$ is the number of turns and $I$ the number of amperes.

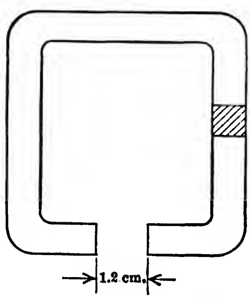



FIG. 6–8. A rectangular shaped electromagnet with an air gap.

**6. Magnetic Pressure. Magnetomotive-Force. The Gilbert.** A current of electricity cannot exist unless there is an electric pressure causing the current to flow. And neither can a magnetic flux exist unless there is a magnetic pressure to set up the flux. The current in the electric circuit flows as a result of the electromotive-force, or emf, of a battery, or generator. The magnetic flux is set up by the electric current flowing through the turns of a coil — the ampere-turns; and is called the **magnetomotive-force (mmf)**. The unit of electromotive-force is the **volt** and is represented by the letter ($E$). Similarly, the unit of magnetomotive-force is the **gilbert** and is represented by the script letter $\mathfrak{F}$.

On the basis that the reluctance of a centimeter cube of air is **one,** it can be shown that magnetomotive-force in gilberts equals $0.4\pi$ or 1.26 times the ampere-turns; or expressed as an equation:

$$\mathfrak{F} = 1.26\,NI. \tag{5}$$

**Example 4.** If a current of 2 amperes were put through the 730 turns of the coil on the wooden ring of Fig. 3–8, a magnetic pressure of 1.26 × 2 × 730 or 1840 gilberts would be produced. A current of 3 amperes in this coil would produce 1.26 × 3 × 730 or 2760 gilberts.

**7. Ohm's Law for the Magnetic Circuit.** Thus, in the magnetic circuit, we have magnetic pressure setting up a magnetic flux against the opposition offered by the magnetic reluctance. This is similar to the condition in the electric circuit, in which an electric pressure sets up an electric current against the opposition offered by the electrical resistance. The amount of the magnetic flux set up can be determined from Ohm's law for the magnetic circuit:

$$\text{Magnetic flux} = \frac{\text{Magnetomotive-force}}{\text{Magnetic reluctance}}$$

$$\text{Magnetic lines} = \frac{\text{Gilberts}}{\text{Units (reluctance)}}, \quad \text{or} \quad \Phi = \frac{\mathfrak{F}}{\mathfrak{R}}.$$

Like Ohm's law for the electric circuit, the equation can be transposed and written in three forms:

$$\text{Magnetic lines} = \frac{\text{Gilberts}}{\text{Units (reluctance)}}, \quad \text{or} \quad \Phi = \frac{\mathfrak{F}}{\mathfrak{R}}. \tag{6}$$

$$\text{Units (reluctance)} = \frac{\text{Gilberts}}{\text{Magnetic lines}}, \quad \text{or} \quad \mathfrak{R} = \frac{\mathfrak{F}}{\Phi}. \tag{7}$$

$$\text{Gilberts} = \text{Magnetic lines} \times \text{Units (reluctance)}, \quad \text{or} \quad \mathfrak{F} = \Phi\mathfrak{R}. \tag{8}$$

Remember that in the above equations, $\mathfrak{F} = 1.26\,NI$.

**Example 5.** The reluctance of the wooden ring in Fig. 3–8 has been found, in Example 2, to be 16 units. What magnetic flux would be set up in this ring if 2 amperes were sent through the 730 turns of this coil?

**Solution.**

$$\Phi = \frac{\mathfrak{F}}{\mathfrak{R}} = \frac{1.26 \times 2 \times 730}{16} = 115 \text{ lines.}$$

**Example 6.** The magnetic circuit of Fig. 7–8 is composed of iron. The coil has 280 turns. When 3.6 amperes are sent through the coil, a flux of 3000 lines is set up in the iron circuit. What is the reluctance of the circuit?

**Solution.**

$$\mathfrak{R} = \frac{\mathfrak{F}}{\Phi} = \frac{1.26 \times 3.6 \times 280}{3000} = 0.423 \text{ unit.}$$

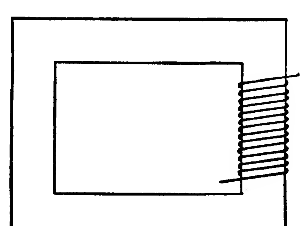


FIG. 7–8. An electromagnet with a closed iron path.

**Example 7.** How many amperes must be sent through the coil on the wooden ring of Fig. 3–8 in order to set up a magnetic flux of 400 lines? The reluctance of the ring from Example 2 is 16 units.

**Solution.**

$$\mathfrak{F} = \Phi \times \mathcal{R} = 400 \times 16 = 6400 \text{ gilberts.}$$
$$\mathfrak{F}\text{ (gilberts)} = 1.26\, NI.$$
$$\text{Amperes} = \frac{\text{Gilberts}}{1.26 \times N} = \frac{6400}{1.26 \times 730} = 6.96 \text{ amperes.}$$

**Prob. 6–8.** What magnetic flux will be set up in a magnetic circuit of 2.5 units reluctance, if the magnetomotive-force is supplied by a coil of 1236 turns carrying 0.28 ampere?

**Prob. 7–8.** The magnetic circuit of an electromagnet has 0.4 unit reluctance. How great a magnetomotive-force is necessary to set up a flux of 100,000 lines?

**Prob. 8–8.** A certain electromagnet has a flux of 200,000 lines. The coil has 450 turns carrying 1.4 amperes. What is the reluctance of the magnetic circuit?

**Prob. 9–8.** A coil consists of 980 turns. The reluctance of the magnetic circuit is .008 unit. How many amperes must flow through coil to set up a flux of 1,000,000 lines?

**Prob. 10–8.** How many turns must be wound on a brass ring, the reluctance of which is 1.73 units, if a current of 3.84 amperes is to set up 6000 lines?

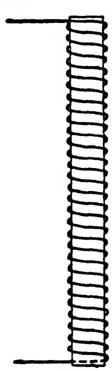


FIG. 8–8. A solenoid with a core of non-magnetic material.

**Prob. 11–8.** How many turns would be required to set up the same flux in Prob. 10–8, if the electric current were made twice as large?

**8. Magnetic Field Within a Long Coil or Solenoid.** It has been found in the case of a long coil, with air core, the length of which is at least ten (10) times the diameter, that practically all the reluctance offered to the magnetic flux is that offered by the air (or non-magnetic material) within the coil.

This is equivalent to saying that once the lines emerge from one end of the coil, they apparently take a path of zero reluctance through the air to the other end of the coil.

This seems reasonable, when one considers that the magnetic lines have an area with practically an infinite cross-section area, through which they may return to the coil. So in computing the reluctance of the magnetic circuit of a long coil, we have merely to find the reluctance of that part of the path which is within the coil.

Thus to find the flux in the center of a closely wound long coil

with a core of non-magnetic material, as in Fig. 8–8, we merely have to divide the gilberts by the reluctance of the path inside the coil.

As before, $$\mathcal{R} = \frac{1 \times l}{A},$$

Therefore, $$\Phi = \frac{1.26\,NI}{\dfrac{l}{A}} = \frac{1.26\,NIA}{l}, \qquad (9)$$

where $l$ = length of path in centimeters,
$A$ = area of path in square centimeters,
$N$ = number of turns in coil,
$I$ = current in amperes.

This equation gives the flux at the center only of a long coil. Only half this flux exists at the ends of the coil with air core, because the other half always leaks out through the sides of the coil.

**Example 8.** What magnetic flux is set up in an air-core coil, 100 centimeters long and 2 centimeters in diameter, if it consists of 4000 turns in which 0.24 ampere is flowing?

**Solution.**

$$\Phi = \frac{1.26\,NIA}{l} \qquad A = 0.7854 \times 2^2 = 3.14$$

$$\Phi = \frac{1.26 \times 4000 \times 0.24 \times 3.14}{100} = 38 \text{ lines.}$$

**Example 9.** What magnetic flux is set up in a coil, 30 inches long and 2.5 inches in diameter, wound on a brass rod? Coil has 2500 turns and carries a current of 0.5 ampere.

**Solution.**

Length of coil in centimeters = $30 \times 2.54 = 76.2$ centimeters.
Area of coil in square centimeters ($A$) = $0.7854 \times (2.5 \times 2.54)^2$
= 40.3 square centimeters.

$$\Phi = \frac{1.26 \times 2500 \times 0.5 \times 40.3}{76.2} = 836 \text{ lines.}$$

**Prob. 12–8.** How many amperes must be sent through an air-core coil, consisting of 400 turns, in order to set up 2000 lines at the center of the coil? The coil is 30 centimeters long and has a cross section of 3.52 square centimeters.

**Prob. 13–8.** How much current must be sent through the coil of Prob. 12–8 in order to set up a flux of 2000 at the ends of the coil?

**Prob. 14–8.** What diameter must a long air-core coil have if a current of 0.40 ampere is to set up a flux of 3200 lines at the center of the coil? The length of the coil is 22 centimeters, and it is wound with 1500 turns.

**Prob. 15–8.** It is desired to set up a flux of 1500 lines at the end of an air-core coil having 1200 turns. The coil is 15 inches long and 1.5 inches in diameter. How much current is needed?

**9. Magnetizing Force. Field Intensity.** It was stated in Art. 3, that for greater convenience in practical calculations, the flux density in lines per square centimeter was used. For the same reason, in the making of tables, as will be seen later, the **magnetomotive-force per centimeter** length of circuit is used. This is called the **magnetizing force** and is expressed in **gilberts per centimeter.** The term one gilbert per centimeter is now called the **Oersted.**

In the electric circuit, we speak of the voltage drop per foot or per mile of wire. Similarly, in the magnetic circuit we might call the **gilberts per centimeter,** or the oersteds, the magnetic pressure drop per centimeter length of circuit. It is really the magnetic pressure required to set up a given amount of flux (number of lines) through a centimeter length of circuit. **Magnetizing force** in gilberts per centimeter, or oersteds, is represented by the letter $H$. Expressed as an equation:

$$H = \frac{1.26\,NI}{l}; \tag{10}$$

where $l$ = length of the magnetic circuit in centimeters;

then, $Hl = 1.26\,NI.$

But from equation (5), $\mathfrak{F} = 1.26\,NI.$

Therefore, $\mathfrak{F} = Hl,$

or

$$\mathfrak{F} = Hl = 1.26\,NI. \tag{11}$$

Thus, if we send 2 amperes through the coil on an iron ring, as in Fig. 3–8, the mmf or $\mathfrak{F} = 1.26 \times 730 \times 2 = 1840$ gilberts magnetizing a magnetic circuit $6.25 \times 3.1416$ or 19.6 centimeters long. To magnetize one centimeter of this circuit would require $\frac{1840}{19.6}$

or 93.75 gilberts per centimeter, or oersteds $\left(H = \frac{\mathfrak{F}}{l}\right)$. From this value, we may determine the number of gilberts necessary to magnetize any length of this iron to the same degree. For instance, to magnetize a piece of this iron 40 centimeters long and the same cross-section to the same flux density, would require $40 \times 93.7$ or 3750 gilberts.

The student should get the distinction between $\mathfrak{F}$ and $H$ clearly in mind. $H$ is the number of gilberts necessary to send a given number of lines through **one** centimeter of a given circuit and, therefore, represents **gilberts per centimeter,** or oersteds. $\mathfrak{F}$ is the number of gilberts necessary to send a given number of lines through **any given length** of circuit, and therefore represents **gilberts.**

Gilberts per centimeter, or oersteds, $\left(\frac{1.26\ NI}{l}\right)$, is also called the **intensity** of the **magnetic force.***

**10. Permeability.** The permeability of a magnetic material is the ratio of the magnetic flux in that material, under the action of a mmf, to the magnetic flux which would exist under the action of the same mmf, were that material replaced by air. Permeability in a magnetic circuit is analogous to conductivity in an electric circuit, or, it is the reciprocal of the reluctivity of a magnetic circuit.

We already know that the reluctance of iron and steel is less than that of air. Therefore, it is easier to set up magnetic lines in the magnetic materials than in air. Permeability, then, is really a comparison of the readiness with which magnetic flux is set up in a magnetic material, to the readiness with which it may be set up in air.

Since the reluctivity is the ratio of the oersteds to the flux density set up in a centimeter cube of the material; so the permeability would be the inverse ratio, or the ratio of the flux density set up to the oersteds, for a centimeter cube of the material. It represents the number of lines set up in one square centimeter by one gilbert per centimeter length.

The reluctance of non-magnetic materials is constant and

* Great care must be exercised to distinguish between $B$, the **density** of the **magnetic lines,** which is measured in **gausses;** and $H$, the **intensity** of the **magnetizing force,** which is measured in **oersteds.**

equals 1. Therefore, the permeability of these materials is $\frac{1}{1}$ or 1, and is constant. Or we may state: one oersted will set up one line of force in a centimeter cube of a non-magnetic material. Thus the permeability of a non-magnetic material equals 1.

The permeability of iron and steel is not constant but depends upon the flux density, or to the degree to which it is magnetized. This will be discussed later.

Permeability is represented by the Greek letter ($\mu$), pronounced **mu.**

**To illustrate the above discussion,** consider that the magnetic circuit in Fig. 3–8 is of iron of the same dimensions as in Example 2, and that the reluctance of this iron path is 0.2 unit, if 2 amperes are put through the 730 turns of the coil. The mmf will be $1.26 \times 730 \times 2$ or 1840 gilberts. The flux $\Phi = \frac{1840}{0.2} = 9200$ lines.

Now suppose the iron ring is removed and the air core substituted. The reluctance is now 16 units, as figured in Example 2. With the same current in the coil and, therefore, the same mmf, the flux is now $\frac{1840}{16}$ or 115 lines. The ratio of flux in the iron to that in the air path is $\frac{9200}{115}$ or 80; or the permeability of the iron core is 80.

The area of the magnetic path in both cases above $= 1.25^2 \times 0.7854$ or 1.23 square centimeters. The flux density in the iron $\left(B = \frac{\Phi}{A}\right) = \frac{9200}{1.23} = 7500$ lines per square centimeter; and in the air core $= \frac{115}{1.23} = 93.75$ lines per square centimeter. The permeability of the iron core can again be figured as $\frac{7500}{93.75} = 80.$

Note here that the flux density, $B$, in the air core, 93.75 lines per square centimeter, exactly agrees in value with the oersteds or magnetizing force, $H$, as computed for this same circuit in Art. 9. That is, $H$ is numerically equal to $B$ in air.

This fact is confusing on first thought, but it must be remembered that $H$ and $B$ are entirely different things (quantities).

In an electric circuit of 1 ohm resistance, 1 volt will force 1 ampere of current to flow. In this case, the volts and amperes

are numerically equal, but they are two entirely different things. We think of volts, the pressure, as the driving force which sends the current through the electric circuit.

Similarly, we consider the gilberts per centimeter, or the oersteds, as the driving force which sets up the flux (flux density) in a magnetic circuit of unit length and cross section. Therefore, $H$ (1.26 ampere-turns per centimeter) is analagous to pressure. This is the **cause**; and the flux density thus set up, either in air or in a magnetic material, is the **effect.**

The permeability of any material can then be stated as the ratio of the flux density in that material to the flux density in air under the action of the same mmf; or as the ratio of the lines set up to the gilberts for a centimeter cube of the material.

Expressed as an equation

$$\mu = \frac{B}{H}; \tag{12}$$

where $B$ = lines per square centimeter and $H$ = oersteds, magnetizing force.

From the above illustrations,

$$\mu \text{ for air} = \frac{B}{H} = \frac{93.75}{93.75} = 1;$$

and for the iron coil with 2 amperes flowing through the 730 turns,

$$\mu = \frac{B}{H} = \frac{7500}{93.75} = 80.$$

**11. Reluctance of Iron and Steel.** In the previous article it was stated that the permeability of a magnetic circuit corresponds to conductivity in an electrical circuit. Therefore, $\frac{1}{\text{permeability}}$ or $\frac{1}{\mu}$ corresponds to the reluctivity, and we can express the reluctance of a magnetic material by the equation:

$$\mathcal{R} = \frac{l}{\mu A} \tag{13}$$

where $l$ = length in centimeters.

$A$ = area in square centimeters.

$\mu$ = the permeability at some definite flux density.

Thus the reluctance can be determined if the dimensions of the circuit and the permeability are known.

The reluctivity of iron and steel, as used in the electrical industries, is much lower than that of air or any of the non-magnetic materials, ranging roughly from 15 to 250 thousandths that of air for the various grades. But it is impractical to state the reluctivity of any particular grade of iron or steel; because, even in a given sample of known composition and treatment, the reluctance will vary with the degree to which it is magnetized, since the permeability changes with the flux density.

In this important respect, the reluctance of the magnetic materials differs from the resistance of electrical conductors, and in fact, from the reluctance of non-magnetic materials. The **resistance** of copper of a certain grade is always 10.4 ohms per mil-foot, if the temperature is standard, regardless of how many amperes are flowing through it; and the reluctance of all non-magnetic materials is one unit per centimeter cube, regardless of how many magnetic lines are set up in them.

But the reluctance of a piece of standard annealed sheet steel may be 0.0001 unit per centimeter cube when the flux density is 20,000 lines per square centimeter, and yet be only 0.00003 unit per centimeter cube when the flux density is reduced to 12,500 lines per square centimeter.

Therefore, it is impossible to use a standard value for the reluctivity of each grade of iron or steel as we can for non-magnetic materials; and we are compelled to use tables or curves showing the reluctivity for different degrees of magnetization. Since this involves a large amount of calculation, we avoid the use of reluctance in calculating such circuits by methods shown in the following paragraphs.

**12. Magnetization Curves.** For practical work, we experimentally obtain data to make tables and plot curves showing the number of **gilberts per centimeter** (oersteds) necessary to set up a given **number of lines per square centimeter** in any given sample of iron or steel. These are called magnetization curves or *B–H* curves.

In using these curves, we are really making use of Ohm's law for the magnetic circuit. Since the expression,

$$\text{Flux} = \frac{\text{Gilberts}}{\text{Reluctance}},$$

is true for a circuit of any dimensions, then

$$\text{Flux density} = \frac{\text{Gilberts per cm (oersteds)}}{\text{Reluctivity}}$$

expresses Ohm's law for a centimeter cube of material. Since gilberts per centimeter (oersteds) and flux density are found from our tables and curves, it is not necessary to know the reluctivity. The magnetic quantities for tables and curves of iron and steel of the grades commonly used in the electrical industry can be measured as follows.

A specimen is made into a ring and carefully measured, as in Fig. 9–8. It is closely wound with a magnetizing coil, *A*, and a test coil, *B*, and the turns of wire counted. Coil *B* has no electrical connection with coil *A*. When the current in the magnetizing coil *A* is suddenly changed by the rheostat, it produces an effect in the magnetic circuit of the ring (as stated at the beginning of the

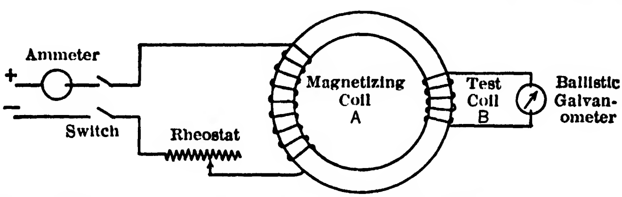



FIG. 9–8. Experimental determination of a magnetization curve by means of a test coil and a calibrated galvanometer. A change of current in the magnetizing coil A, will set up a current in the test coil B, and cause a deflection of the galvanometer.

chapter), and sets up a momentary voltage in the test coil *B*, which is registered by the deflection of the galvanometer. The galvanometer has been previously calibrated by an instrument known as a permeameter. Thus, a given change in the ampere-turns, or mmf of the coil, *A*, produces a change in the flux which is indicated by the galvanometer.

These curves are then plotted between oersteds (gilberts per centimeter) and flux per square centimeter. Since gilberts per centimeter = *H*, and flux density = *B*, these curves are called *B–H* curves. Figure 10–8 shows a typical *B–H* curve for annealed sheet steel.

**13. Practical Magnetization Curves.** In the calculation of any magnetic circuit, such, for instance, as that in Fig. 2–8, the practical information we desire to know is — **how many ampere-turns do**

**we need in the coil to set up a given amount of flux through the iron core.** The dimensions of the core must, of course, be known. In the United States the inch is the unit of length, not the centimeter. Therefore, the manufacturers of electrical ma-

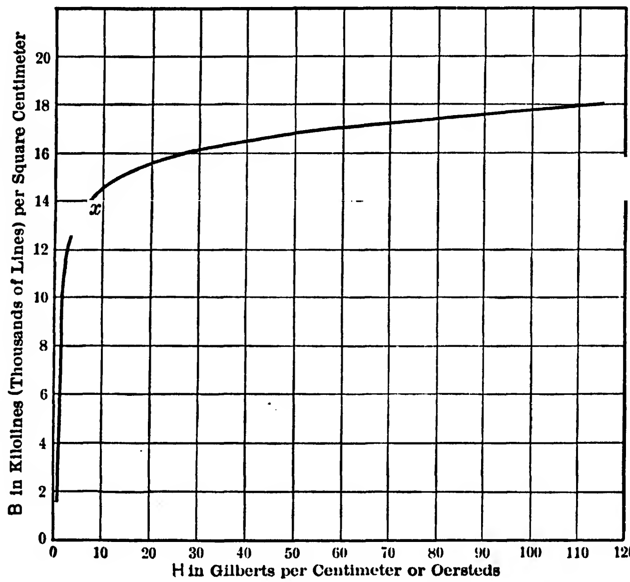



FIG. 10–8. The magnetization, or B–H, curve for a sample of annealed sheet steel, plotted between kilolines per square centimeter and H in gilberts per centimeter, or oersteds.

chinery prefer to work with ampere-turns per inch, instead of oersteds (gilberts per centimeter) or $H$; and resulting flux density, $B$, in lines per square inch, instead of in lines per square centimeter.

$$\text{Now, gausses, (lines per square centimeter)} \times 2.54^2 = \text{lines per square inch;}$$

$$\text{and oersteds, } \left(\frac{1.26\,NI}{l}\right) \times \frac{2.54}{1.26} = NI \text{ per inch.}$$

Thus, the $B$–$H$ curve in Fig. 10–8 can be plotted between lines per square inch, and $NI$ per inch.

Figure 11$a$–8 is a set of magnetization curves, plotted between kilolines per square inch and ampere-turns per inch, for soft steel castings, cast iron, wrought iron and standard annealed sheet steel.

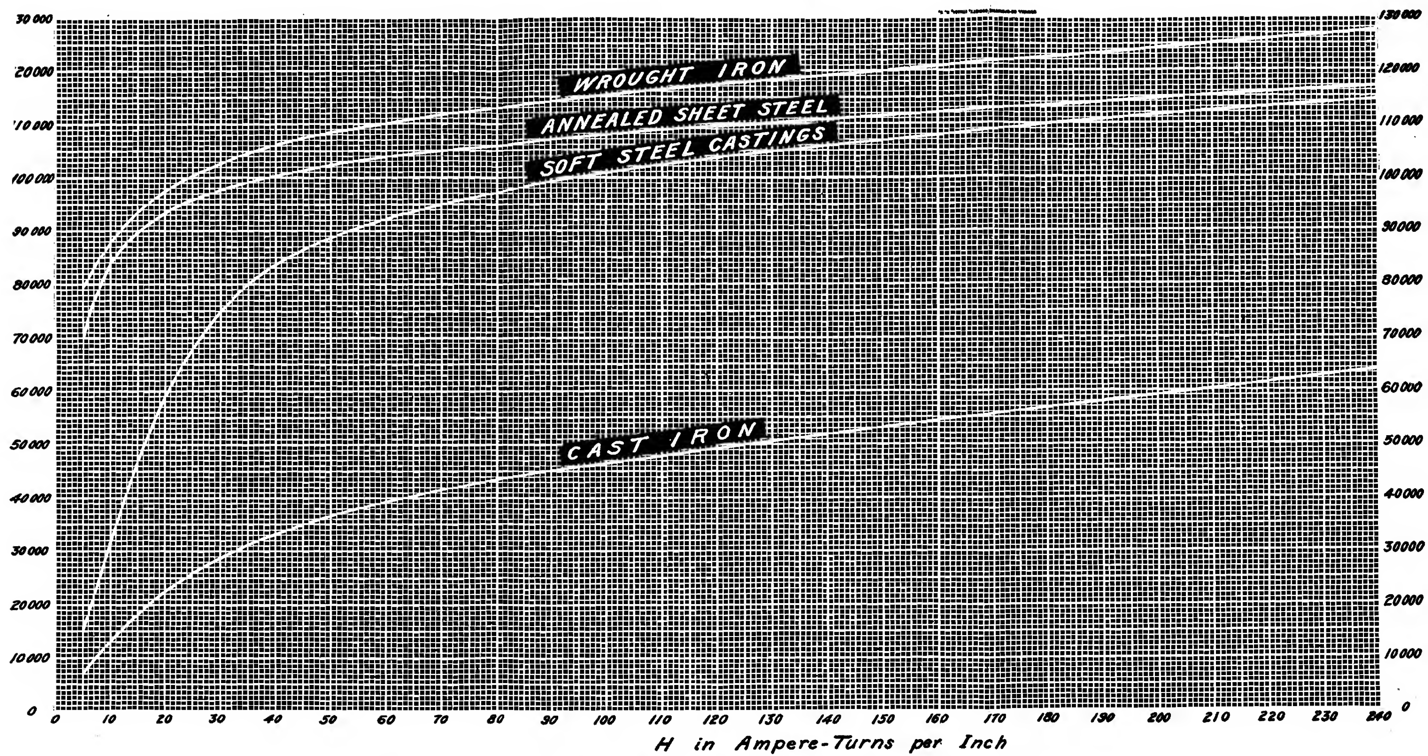



FIG. 11a-8. Magnetization curves of commercial irons and steels plotted between lines per square inch and ampere-turns per inch.

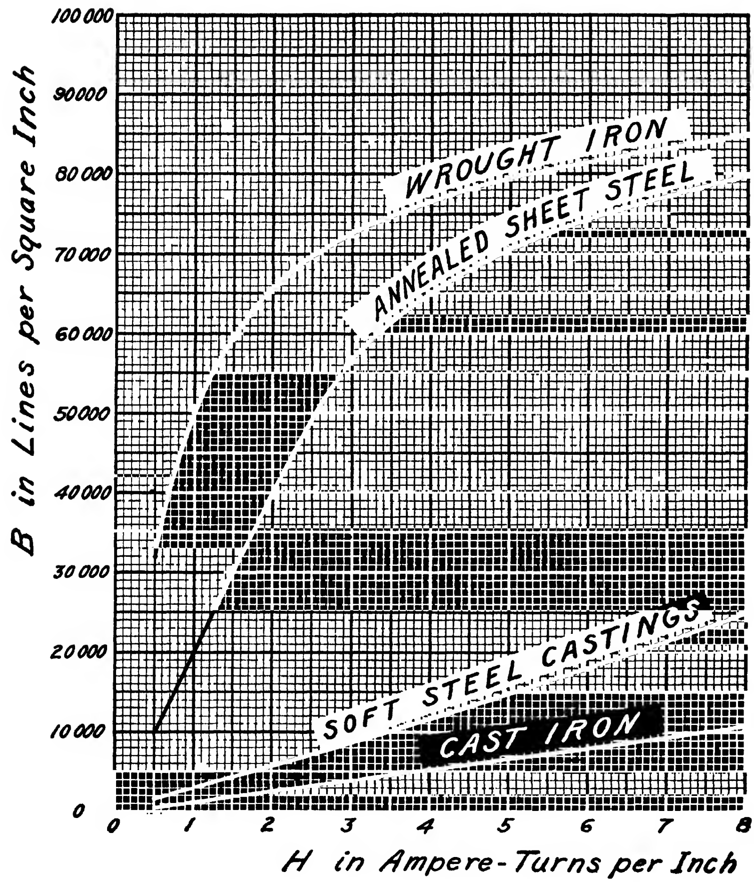



FIG. 11b-8. Enlargement of the lower part of the magnetization curves of Fig. 11a-8.

These curves represent the average for materials used by large manufacturing companies, and are values commonly met with in commercial materials. Figure 11*b*–8 is a set of the lower part of the curves of Fig. 11*a*–8, plotted to a larger scale, for more precise work.

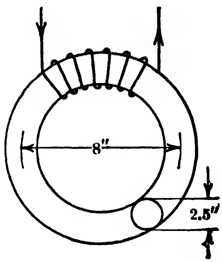



FIG. 12–8. A cast iron ring.

**14. Calculation of Simple Magnetic Circuits.** The facility which the use of these curves lends to magnetic computations is illustrated by the following examples.

**Example 10.** What electric current must be sent through a coil of 500 turns, wound on the cast-iron ring of Fig. 12–8, to set up a magnetic flux of 160,000 lines?

**Solution.**

$$\text{Area of magnetic circuit} = 2.5^2 \times 0.7854 = 4.9 \text{ square inches.}$$

$$\text{Flux density, } B, = \frac{160{,}000}{4.92} = 32{,}500 \text{ lines per square inch.}$$

From the curve, Fig. 11*a*–8 for cast iron, to magnetize cast iron to a density of 32,500 lines per square inch requires 39 ampere-turns per inch.

$$\text{Length of circuit} = 8 \times 3.1416 = 25.15 \text{ inches.}$$

To magnetize 25.15 inches of cast iron to a density of 32,500 lines per square inch requires 25.15 × 39 or 985 ampere-turns.

But

$$\frac{\text{ampere-turns}}{\text{turns}} = \text{amperes.}$$

Thus

$$\frac{985}{500} = 1.97 \text{ amperes.}$$

**Example 11.** What electric current must be sent through the coil in Fig. 3–8 in order to produce a magnetic flux of 19,350 lines in the ring, if a wrought-iron ring is used?

**Solution.**

$$\text{Area of ring} = \frac{1.25 \times 1.25 \times 0.7854}{2.54^2} = 0.19 \text{ square inch.}$$

$$\text{Flux density, } B, = \frac{19{,}350}{0.19} = 101{,}800 \text{ lines per square inch.}$$

From curve, Fig. 11*a*–8, to magnetize wrought iron to a flux density of 101,800 lines requires 28 ampere-turns per inch.

$$\text{Length of circuit} = \frac{6.25 \times 3.1416}{2.54} = 7.7 \text{ inches.}$$

To magnetize 7.7 inches of wrought iron to a flux density of 101,800 lines per square inch require 7.7 × 28 or 216 ampere-turns.

There are 730 turns in the coil on the ring. Thus $\frac{216}{730} = 0.295$ ampere is needed.

**Example 12.** If the ring in Example 10 were of annealed sheet steel, how many ampere-turns would be necessary to magnetize it to the same flux density?

**Solution.** From the curve, to magnetize sheet steel to a flux density of 32,500 lines per square inch requires 1.6 ampere-turns per inch.

To magnetize 25.15 inches of sheet steel requires 25.15 $\times$ 1.6 or 40.2 ampere-turns.

**Prob. 16–8.** The magnetic circuit of Fig. 13–8 was made up of annealed sheet steel pieces, having an average length of long sides of 9 inches and of short sides of 7 inches. Each side is $2\frac{3}{4}$ inches wide. The pile of sheets is $2\frac{1}{2}$ inches deep. If 625,000 lines are to be set up in this core, how many ampere-turns are needed in the coil? Subtract 8% from the depth to allow for the insulation between the sheets.

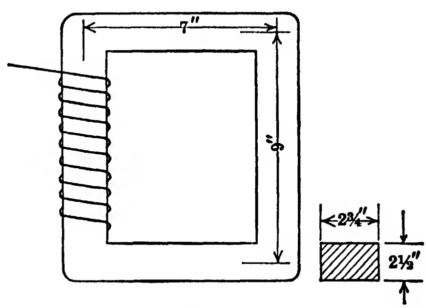


FIG. 13–8. A closed iron electromagnet made of rectangular stock.

**Prob. 17–8.** If the core of Fig. 13–8 were a solid soft steel casting, how many ampere-turns would be needed to set up 625,000 lines?

**Prob. 18–8.** How many ampere-turns would be needed to set up 625,000 lines in a core of brass sheets of same size as that of Fig. 13–8?

**Prob. 19–8.** If a core is made of standard annealed steel sheets only nine-tenths as deep as that of Prob. 16–8, but otherwise of the same dimensions, how many ampere-turns would be needed to set up a flux of 625,000 lines?

**Prob. 20–8.** If the core in Prob. 19–8 were to have the same flux density as the core in Prob. 16–8, how many lines would be set up?

**Prob. 21–8.** How many ampere-turns would be needed to set up the flux of Prob. 20–8?

**Prob. 22–8.** Each of two electromagnets is made of wrought iron. Each has a length of 2 feet. The first has a circular cross-section area of 2.4 square inches and is to be magnetized to carry 192,000 lines. The second has a circular cross-section area of 1.2 square inches and is to carry 96,000 lines.

(*a*) How many ampere-turns must be applied to the first?

(*b*) How many ampere-turns must be applied to the second?

(*c*) If the same size wire were used in both magnets, about how would the lengths of the amounts wound on the two compare?

**Prob. 23–8.** How many ampere-turns would be needed for the first magnet of Prob. 22–8 if it were to carry only the same number of lines as the second magnet?

**15. Series Magnetic Circuits.** The same laws apply to series magnetic circuits as to series electric circuits.

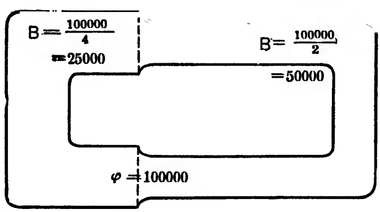



FIG. 14–8. A magnetic circuit, one part having a larger cross section than the other.

(*a*) The magnetic flux through all parts of a series circuit is the same. Note, however, that the flux density of different parts may be different. Thus, if 100,000 lines is the total flux in the circuit in Fig. 14–8, one part of which has an area of 2 square inches and the other an area of 4 square inches, there will be a flux density of 50,000 lines per square inch in one part, and a flux density of 25,000 lines in the other. In all parts of the circuit, however, there are 100,000 lines.

(*b*) The reluctance of a series circuit is the sum of the reluctances of the several parts.

(*c*) The magnetic pressure (in ampere-turns*) necessary to send a certain magnetic flux through a series circuit is the sum of the pressures necessary to send it through the several parts.

Note that the ampere-turns for each particular part need not be applied to that part of the circuit. In Fig. 14–8, we calculate the $NI$ necessary to send the flux through that part of the circuit where the flux density is 50,000 lines, and then, the $NI$ to send the flux through the part where the density is 25,000 lines. Then we add up the ampere-turns for both parts. A coil with the necessary ampere-turns may then be wound on **any** part of the circuit where it is most convenient.

**Example 13.** The U-shaped magnet of Fig. 15–8 is made of round wrought iron 1.40 inch in diameter. Its average length is 14 inches. The ends are machined flat and make a close fit on the machined surface of cast iron, which has a 1.85 inch by 1.85 inch cross-section. The average length of the path through the cast iron is 7 inches. Assuming that there is no air space between the wrought iron and the cast iron, how many ampere-turns must be used to set up a magnetic flux of 130,000 lines through the magnetic circuit?

* Ampere-turns $= \dfrac{\text{gilberts}}{1.26}$, since gilberts = 1.26 ampere-turns.

**Solution.**

Cross section of the wrought iron $= 1.40 \times 1.40 \times 0.785$
$= 1.54$ square inches.

Flux density ($B$) for wrought iron $= \dfrac{130{,}000}{1.54}$
$= 84{,}400$ lines per square inch.

Ampere-turns per inch necessary to set up 84,400 lines per square inch in wrought iron (from curve) $= 8$

Ampere-turns necessary to set up 84,400 lines per square inch in 14 inches of wrought iron $= 14 \times 8$
$= 112$ ampere-turns.

Cross section of cast iron $= 1.85 \times 1.85$
$= 3.42$ square inches.

Flux density ($B$) for cast iron $= \dfrac{130{,}000}{3.42}$
$= 38{,}000$ lines per square inch.

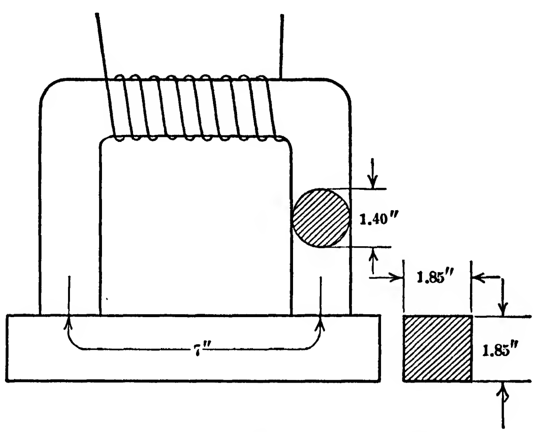



Fig. 15–8. A horseshoe-shaped electro lifting magnet.

Ampere-turns per inch to set up 38,000 lines per square inch in cast iron (from curve) $= 55$ ampere-turns.

Ampere-turns to set up 38,000 lines per square inch in 7 inches of cast iron $= 55 \times 7$
$= 385$ ampere-turns.

Ampere-turns necessary for entire magnetic circuit
$= 112 + 385$
$= 497$ ampere-turns.

The coil can be wound all on the U-shaped piece of wrought iron, or all on the cast-iron bar, or part on each — whichever is most convenient.

**Example 14.** Assuming that there is 1/1000 of an inch air space where each end of the U-shaped piece joins the flat bar, how many ampere-turns are necessary to set up the 130,000 lines?

**Solution.**

$$\text{The reluctance of an inch cube of air} = \frac{1 \times 2.54}{2.54^2} = 0.394 \text{ unit.}$$

$$\text{The reluctance of the two air gaps} = \frac{0.394 \times 0.002}{1.54} = 0.000512 \text{ unit.}$$

$$\text{The mmf for air gaps} = \mathfrak{F} = 1.26\, NI = \Phi\mathfrak{R}.$$

$$\text{Ampere-turns for air gaps} = NI = \frac{\Phi\mathfrak{R}}{1.26} = \frac{130{,}000 \times 0.000512}{1.26} = 53.$$

Total ampere-turns = 497 + 53 = 550 ampere-turns — **answer.**

We might, also, compute the ampere-turns necessary for the air gaps as follows:

$$\text{Flux density for air gaps} = \frac{130{,}000}{1.54}$$
$$= 84{,}500 \text{ lines per square inch.}$$

Ampere-turns per inch length of air gap, or ampere-turns necessary to send 84,500 lines per square inch through one inch of air

$$= \frac{84{,}500 \times 0.394}{1.26}$$
$$= 26{,}500 \text{ ampere-turns per inch.}$$

Ampere-turns necessary to send 84,500 lines per square inch through 0.002 inches of air = $26{,}500 \times 0.002$
= 53 ampere-turns.

Note that by this method, we first find the number of lines through a square inch of air, and compute the number of ampere-turns necessary to send this number of lines per square inch through one inch. This is really finding the number of ampere-turns necessary to send a certain magnetic flux through a path one inch long and one square inch cross section. The reluctance of such a path we know to be 0.394 units.

$$1.26 \text{ ampere-turns} = \text{flux} \times \text{reluctance}$$
$$= \text{lines per square inch} \times \text{reluctance per inch cube}$$

$$\text{ampere-turns} = \frac{\text{lines per square inch} \times \text{reluctance per inch cube}}{1.26}.$$

We, thus, have merely to divide the number of lines per square inch by 1.26 and multiply by the reluctance of one inch of air with one square inch cross section (0.394) to obtain the ampere-turns per inch length of air path.

Then, multiplying this value by the actual length of air path, we obtain **the number of ampere-turns necessary to set up the lines in the actual air gap.**

**Prob. 24–8.** Compute the number of ampere-turns necessary to magnetize the circuit of Fig. 15–8 with a flux of 140,000 lines. Air gap 2.5 thousandths inch at each end.

**Prob. 25–8.** If the U-shaped piece in Fig. 15–8 were cast steel, what number of ampere-turns would be necessary? Other data as in Prob. 24–8.

**Prob. 26–8.** The magnetic circuit of a certain 2-pole generator is made up of the following parts in series. Two cast-steel cores, each 13.2 square inches in cross-section and 6 inches long; one cast-steel yoke, 8 inches long, 14 square inches cross-section; two standard annealed sheet steel pole pieces, average length $2\frac{1}{4}$ inches, average cross-section 12.5 square inches; one armature core of standard annealed sheet steel, average cross-section 13 square inches; average length of magnetic path $2\frac{3}{4}$ inches; two air gaps, each with cross-section area 18 square inches and $\frac{3}{32}$ inch long. How many ampere-turns are necessary, if the flux is to be 1,100,000 lines? Assume all the flux flows through the whole of the above circuit.

**Prob. 27–8.** If the maximum current allowed in the coils of Prob. 26–8 is 0.8 ampere, how many turns are necessary?

**16. Parallel Magnetic Circuits.** In all multipolar and in many bipolar dynamos, the magnetic paths through the machines are in parallel. In dealing with these parallel magnetic paths, it is necessary to keep in mind only that the same general laws for the flow of electric currents through parallel electric circuits apply equally well to the setting up of magnetic flux through parallel magnetic paths. Remember that the same voltage which forces an electric current through one path in a parallel circuit also forces electric currents through all the other parallel branches of the circuit.

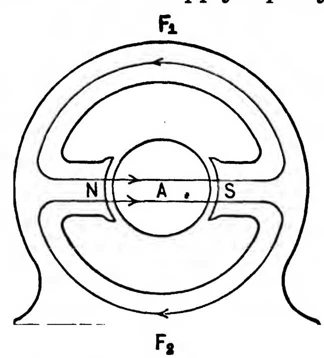


Fig. 16–8. The two parallel magnetic paths, $F_1$ and $F_2$, in a two-pole motor are in series with the part marked A.

Note how this same law applies to the ampere-turns needed to force magnetic currents through parallel circuits. In Fig. 16–8, representing the magnetic circuit of a bipolar machine, those parts of the circuit composed of the north pole, $N$, the armature core, $A$, the south pole, $S$, and the two air gaps, are in series and carry the whole magnetic flux (disregarding any leakage). As this magnetic flux enters the frame, it divides into two paths, $F_1$ and $F_2$, which we will assume have the same reluctance, as they will (approximately) in a well-designed machine.

Such a circuit is very similar to the electric circuit in Fig. 17–8. Here the lamp, $L$, carries the full electric current, and corresponds to the series part of the magnetic circuit made up of the two poles, armature and air gaps. Each of the two paralleled lamps, $L_1$ and $L_2$ (assumed to have the same resistance), carries one-half the current of the lamp, $L$. The voltage necessary to force an electric current through this series-parallel combination equals the sum of the voltage necessary to force it through the lamp $L$, plus the voltage to force half this current through either lamp $L_1$ or $L_2$. The same voltage which will force this half current through either one of $L_1$ or $L_2$ will force the other half through the other parallel lamp.

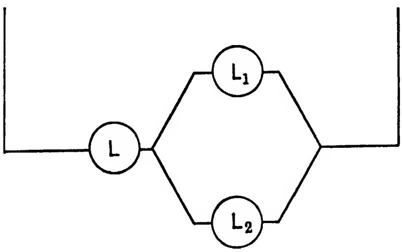


FIG. 17–8. The two parallel electric paths, $L_1$ and $L_2$, are in series with the part marked L.

Similarly, the ampere-turns necessary to force a magnetic flux through the combination of Fig. 16–8 equal the sum of the ampere-turns necessary to force the full flux through the two poles, armature and air gaps, plus the ampere-turns necessary to force half this flux through either $F_1$ or $F_2$. The same ampere-turns which force this half flux through either $F_1$ or $F_2$, force the other half of the flux through the other parallel path.

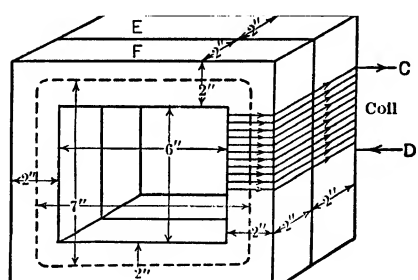


FIG. 18–8. A rectangular magnetic circuit composed of two soft steel castings. A coil is wound on one leg of both castings.

**Example 15.** (*a*) A soft cast-steel core is made of two rectangular pieces, $E$ and $F$, fitted together, and having the dimensions given in Fig. 18–8. A coil of 200 turns is wound around one of the legs of the core. How many ampere-turns are required to set up 800,000 lines in the magnetic circuit?

**Solution.**

Area of magnetic circuit $= 2 \times (2 + 2) = 8$ square inches.

$$\text{Flux density} = \frac{800{,}000}{8} = 100{,}000 \text{ lines per square inch.}$$

From curve for cast steel, there are required 95 ampere-turns per inch.

Mean length of magnetic circuit = 4 × 7 or 28 inches.

Total ampere-turns needed = 28 × 95 = 2660.

$$\text{And the current required in the coil} = \frac{2660}{200} = 13.3 \text{ amperes.}$$

Note that if the two castings are alike, 400,000 lines would be set up in each. The flux density in each would still be 100,000, and it would take 2660 ampere-turns to set up 400,000 lines through one casting alone.

We have here two magnetic paths in parallel.

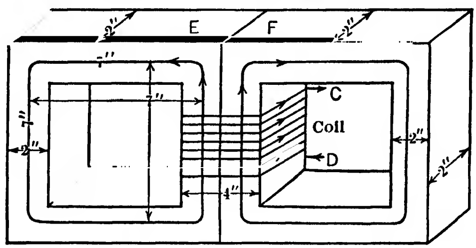



FIG. 19–8. The two castings of Fig. 18–8 are rearranged. The same coil is wound on the middle leg and sets up the same flux throughout the magnetic circuit.

(b) Now let the position of the two castings be changed, as indicated in Fig. 19–8. The same coil is wound on the middle leg and it is desired to again set up a flux of 800,000 lines.

Flux density in the middle leg is again

$$\frac{800{,}000}{8} = 100{,}000 \text{ lines per square inch.}$$

Ampere-turns per inch = 95.

Approximate length of middle leg = 7 inches.

Ampere-turns required = 7 × 95 or 665.

Now if 800,000 lines are set up in the middle leg, the flux will divide and 400,000 lines will be set up through each outside leg.

$$\text{Flux density again} = \frac{400{,}000}{4} \text{ or } 100{,}000 \text{ lines per square inch.}$$

Ampere-turns per inch = 95.

Ampere-turns to send the flux through the remaining parts of the core,

$F$, = 95 × (7 × 3) or 1995.

Total $NI$ = 665 + 1995 = 2660 ampere-turns.

This checks with (a) above and it is not necessary to calculate the rest of the circuit. In other words, the mmf, or ampere-turns, which will set up 800,000 in the middle leg and 400,000 lines in one outside leg will also set up the same flux in the other. The outside legs are in parallel.

**Example 16.** In Fig. 16–8, the length of cast-steel pole pieces $N$ and $S$ is 4 inches each, average cross-section is 15 square inches. Average length of magnetic path through annealed sheet steel armature is 3 inches, average cross-section is 20 square inches. Length of air gaps is $\frac{1}{8}$ inch each, cross-section is 30 square inches. Average length of each path through cast-steel frame is 18 inches, area is 8 square inches. How many ampere-turns must be put in the field coils to set up a flux of 1,500,000 lines in the armature?

**Solution.**

Pole pieces, cast steel:

$$\begin{aligned} B \text{ (lines per square inch)} &= \frac{1{,}500{,}000}{15} \\ &= 100{,}000 \text{ lines per square inch.} \\ NI \text{ per in. (from curve)} &= 95 \text{ ampere-turns.} \\ NI \text{ (for two-pole pieces)} &= 95 \times 8 \\ &= 760 \text{ ampere-turns.} \end{aligned}$$

Armature, annealed sheet steel:

$$\begin{aligned} B \text{ (flux density)} &= \frac{1{,}500{,}000}{20} \\ &= 75{,}000 \text{ lines per square inch.} \\ NI \text{ per in. (from curve)} &= 7 \\ NI \text{ (for armature)} &= 7 \times 3 \\ &= 21 \text{ ampere-turns.} \end{aligned}$$

Air Gaps:

$$\begin{aligned} \text{Flux density} &= \frac{1{,}500{,}000}{30} \\ &= 50{,}000 \text{ lines per square inch.} \\ NI \text{ per inch} &= \frac{50{,}000 \times 0.394}{1.26} \\ &= 15{,}650 \text{ ampere-turns.} \\ NI \text{ for both air gaps} &= 15{,}650 \times 0.25 \\ &= 3913 \text{ ampere-turns.} \end{aligned}$$

Total $NI$ for pole pieces, armature and air gaps — the series part of the circuit = 760 + 21 + 3913 = 4694 ampere-turns.

Frame, cast steel (only one need be considered):

$$\begin{aligned} B \text{ (flux density)} &= \frac{750{,}000}{8} \\ &= 93{,}750 \text{ lines per square inch.} \\ NI \text{ per inch (from curve)} &= 66 \text{ ampere-turns per inch.} \\ NI \text{ (for either path)} &= 18 \times 66 \\ &= 1188 \text{ ampere-turns.} \\ \text{Ampere-turns for total circuit} &= 4694 + 1188 \\ &= 5882 \text{ ampere-turns.} \end{aligned}$$

**Prob. 28–8.** The hoisting magnet of Fig. 20–8 is made of cast steel and has the following dimensions: length of $A$, 5 inches; cross-section, 20 square inches; average length of $B_1$ and $B_2$, 12 inches each; cross-section, 12 square inches; piece $C$ is of wrought iron; total average length of $C$, 14 inches; cross-section, 18 square inches; air gaps each 0.03 inch. How many ampere-turns must be applied to $A$ to set up a flux of 2,200,000 lines?

**Prob. 29–8.** If the air gaps in Prob. 28–8 are increased to 0.05 inch, how many ampere-turns will be needed?

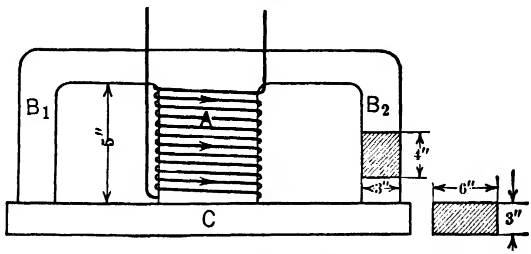



Fig. 20–8. A three-legged electro lifting magnet.

**Prob. 30–8.** If the piece $C$, Fig. 20–8, is cast iron, with an average length of 14 inches, and cross-section of 30 square inches, and air gaps are $\frac{1}{16}$ inch, how many ampere-turns must be used? Other data as in Prob. 28–8.

**Prob. 31–8.** If the air gaps in Prob. 28–8 are increased to $\frac{3}{32}$ inch, and the flux changed to 1,250,000 lines, how many ampere-turns are necessary?

**Prob. 32–8.** It is desired to set up a flux of 1,260,000 lines in the center leg of the cast-steel core of Fig. 21–8, with a coil of 200 turns wound on that leg. Cross-section area of center leg is 12 square inches and that of the remainder of the core is 6 square inches. How many amperes will be required?

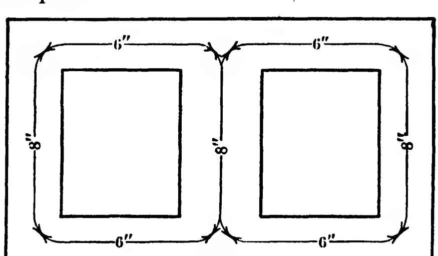



Fig. 21–8. A parallel magnetic circuit.

**Prob. 33–8.** The same flux is to be set up in the core of Prob. 32–8, but with two coils, one wound on each outside leg. If the same current as determined in Prob. 32–8 is used in each of these coils, how many turns must each have?

**Prob. 34–8.** (*a*) If one coil is wound on each of the three legs of the core in Prob. 32–8, and the same flux is to be set up as in that problem, with the same current per coil, how many turns must there be in each coil?

(*b*) How many ampere-turns in the opposite direction must be supplied by a coil on one of the outside legs in (*a*) above, in order that there may be no flux in that leg?

**17. To Compute Total Flux for Given Conditions.** Thus far in problems on series magnetic circuits, parts of which contain iron and steel, we have known the flux density to which the materials were magnetized, and have been able to determine the reluctance and the ampere-turns necessary to set up the flux. **If, however, we wish to find out how many lines are set up in a certain path of iron or steel by a given number of ampere-turns, we cannot divide the ampere-turns by the reluctance in order to find the number of lines, because the reluctance itself, as we have seen, depends upon the number of lines per square inch. Since we do not know the number of lines per square inch, we do not know the reluctance.**

It is usual, therefore, in this kind of problem, to make an estimate of the approximate number of lines per square inch, and note from the curves how many ampere-turns are necessary. From the information obtained by this estimate, we can make closer estimates and obtain more definite information, until we finally estimate very closely the number of lines set up by a given number of ampere-turns. This method is illustrated in the following example. It is called the " trial and error " method.

**Example 17.** How many lines are set up in the wrought-iron ring of Fig. 22–8, when 3060 ampere-turns are applied to it? The ring has a cross-section of 3 square inches and an average length of 20 inches. The air gap is $\frac{1}{8}$ inch long with an average cross-section of 4 square inches.

**Solution.**

Since air has so much greater reluctance than iron, we know that the greater part of the ampere-turns will be used in setting up the magnetic flux across the air gap.

Assume that **nearly all** the reluctance of the circuit is in the air gap and estimate the number of lines set up. We know, of course, that this value for the reluctance of the circuit is too low, since the iron has some reluctance, and therefore the number of lines computed on this basis will be too large. We can, however, correct this error, when we know **about** how many lines there are.

$$\text{Reluctance of air gap} = \frac{0.394 \times 0.125}{4} = 0.0123 \text{ unit.}$$

Since the iron part of the circuit has some reluctance, let us assume that the reluctance of the entire circuit is 0.013.

Then $\mathfrak{F} = 1.26\,NI = \Phi\mathfrak{R}$ or $\Phi = \dfrac{1.26\,NI}{\mathfrak{R}}$.

$$\text{Then, the estimated number of lines} = \frac{1.26 \times 3060}{0.013} = 296{,}000 \text{ lines.}$$

$$\text{Flux density in the iron, } B, = \frac{296{,}000}{3} = 98{,}800 \text{ lines per square inch.}$$

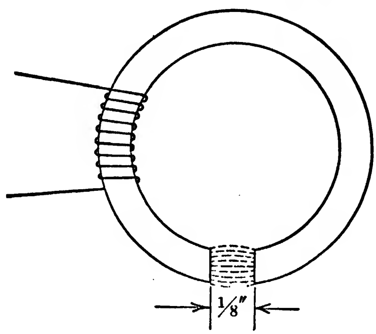



FIG. 22–8. A ring electromagnet with an air gap.

Ampere-turns per inch required to set up 98,800 lines per square inch in wrought iron (from curve) = 22.5 $NI$.

Ampere-turns for 20 inches of iron = 22.5 × 20 = 450 $NI$.
Ampere-turns left for air gap = 3060 − 450 = 2610 $NI$.

It is merely necessary to see if 2610 ampere-turns are sufficient to set up 296,000 lines in the air gap.

**Check.** $\text{Ampere-turns} = \dfrac{296{,}000 \times 0.0123}{1.26} = 2895\,NI$ necessary.

Since we have only 2610 $NI$ left after setting up 296,000 lines, or 98,800 lines per square inch in the iron, we see that we cannot set up that much flux and still magnetize the air gap.

We now know the flux density in the iron must be less than 98,800 lines per square inch.

But instead of assuming another value for the reluctance, let us assume some flux density less than 98,800.

For our next trial, let us assume a flux density of 90,000 lines per square inch, or a total flux of 90,000 × 3 or 270,000 lines.

For this flux density,

Ampere-turns per inch (from curve) = 12 $NI$.
Ampere-turns for 20 inches of iron = 12 × 20 = 240 $NI$.
Ampere-turns left for air gap = 3060 − 240 = 2820 $NI$.

Now, we can **again check** whether or not 2820 ampere-turns are enough to set up 270,000 lines in the air gap.

$$\text{Ampere-turns} = \frac{270{,}000 \times 0.0123}{1.26} = 2640\ NI.$$

But we have 2820 ampere-turns left for the air gap when we set up only 90,000 lines per square inch in the iron; so we know we can set up more than this amount of flux.

We now know, from the two trials above, that the flux density, due to the 3060 ampere-turns on the core, must be between 98,800 and 90,000 lines per square inch.

For our third attempt, let us try 95,000 lines per square inch or 285,000 lines in the iron.

$$\text{Ampere-turns per inch (from curve)} = 17\ NI.$$
$$\text{Ampere-turns for 20 inches of iron} = 17 \times 20 = 340\ NI.$$
$$\text{Ampere-turns left for the air gap} = 3060 - 340 = 2720\ NI.$$

Now let us determine whether 2720 ampere-turns are necessary to set up 285,000 lines in the air gap.

$$\text{Ampere-turns} = \frac{285{,}000 \times 0.0123}{1.26} = 2785\ NI$$

This is 2785 − 2720, or 65, more ampere-turns than we have available; so that the flux density will be slightly less than this, or approximately 94,000 lines per square inch, or a total flux of 94,000 × 3 or 282,000 lines.

Thus, we have determined that 94,000 lines per square inch are set up in the iron, or 282,000 lines throughout the circuit, by 3060 ampere-turns applied to the wrought-iron ring.

**Prob. 35–8.** How many lines of force will 2750 ampere-turns force through the magnetic circuit of the hoisting magnet of Fig. 15–8? The U-shaped part is cast steel, 13 inches long and 1.54 square inch in cross-section. The flat piece is cast iron, 7 inches long and 3.42 square inches in cross-section. The air gaps are 0.02 inch each and have 1.54 square inches area.

**Prob. 36–8.** If the flat piece in Prob. 35–8 were wrought iron of 3 square inches cross-section, how many lines would be set up?

**Prob. 37–8.** An electromagnet is made up of three sections in series as follows: cast iron 20 inches long, 22 square inches cross-section; two air gaps each $\frac{1}{4}$-inch long, 22 square inches cross-section; a section 7.5 inches long, area 11 square inches, made up of fine particles of magnetite iron, the reluctance of which is $\frac{1}{10}$ that of air. What will be the total flux, if the magnet is wound with 4000 turns carrying 2 amperes?

**Prob. 38–8.** What flux will be produced in the pole pieces of the dynamo in Example 16, if it is wound with 2 coils of 2500 turns each and carrying 1.4 amperes? Other data as in Example 16.

**Prob. 39–8.** If the flat piece in Fig. 20–8 is made of cast iron, how much flux will 4200 ampere-turns on $A$ produce in it? Other data as in Prob. 30–8.

**18. Saturation Point.** In the *B–H* curve for annealed sheet steel, in Fig. 10–8, we have a typical shape of the magnetization curve of magnetic materials. The peculiar shape of this curve should be noted in detail, as it is typical of all such magnetization curves of iron and steel.

In general, there are two stages of magnetization. In order to set up a few lines of force, a relatively small number of gilberts per centimeter is needed. For instance, to set up 8000 lines per square centimeter only about 1.5 gilberts per centimeter are needed, but it soon becomes increasingly difficult to set up further lines. This is shown by the fact that about 29 gilberts per centimeter are required to set up 16,000 lines per square centimeter. In other words, to **double** the flux, twenty times the magnetizing force is needed. To increase the flux density even slightly beyond this point, say to 17,000 lines (an increase of only 1000 lines per square centimeter), would require about 60 gilberts per centimeter.

The two stages of magnetization may be defined as follows. The first stage is that during which it is comparatively easy to add lines of force to the steel. This would take in the part of the curve up to the part marked "*x*," Fig. 10–8.

The second stage comprises all that part of the curve beyond the point, "*x*," where it is much more difficult to add lines of force. The point "*x*" is commonly called the Saturation Point. Note particularly that the saturation point is not that point at which the steel is so full of lines that no more can be added, but is that point beyond which it is extremely difficult to add more lines. In fact, there is practically no point beyond which it is impossible to set up more lines. It is probably true that if we continued to add ampere-turns indefinitely to a cast-steel magnetic circuit, we should continue to increase slightly the number of lines in the circuit.

The saturation point, then, really means that point beyond which it is usually unprofitable to magnetize the steel, because of the greatly increased difficulty in setting up lines of force. The point lies at the knee of the curve, where it bends into a nearly straight line running almost horizontally.

The behavior of magnetic material in this respect fits very nicely into our magnetic molecule theory which explains the facts somewhat in this manner. It is comparatively easy to turn the majority of the molecules into the magnetic position; therefore, a very few magnetic turns are needed up to a certain point. However, after the saturation point is reached, it becomes extremely difficult to

turn the rest of the molecules into the magnetic position. An ever increasing magnetizing force is required as the unmagnetized particles become fewer and fewer. If we carry the magnetization far enough, the steel will probably behave like a non-magnetic material, and the same number of ampere-turns would be needed to set up additional lines that would be required to set up the same number of lines in so much air.

In actual practice, the magnetization is rarely carried much beyond the saturation point.

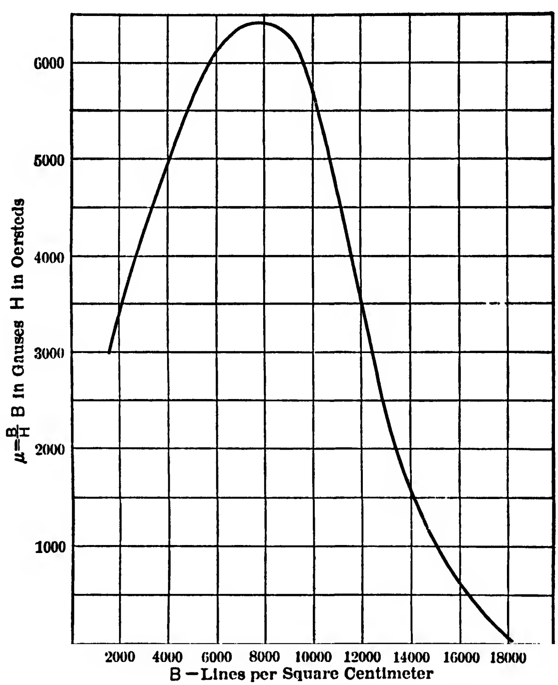



Fig. 23–8. Permeability curve for the annealed sheet steel of Fig. 10–8, showing the relation between flux density, B, in gausses, and permeability $\mu = \frac{B}{H}$.

**19. Permeability and its Relation to Flux Density.** Another way of stating the facts, in the above section, would be to say that in the first stage of magnetization, that is, as far as the saturation point, the permeability of the steel is comparatively high. That is, it is comparatively easy to magnetize it. Beyond this point the permeability of the steel decreases rapidly, until finally it probably

would reach that of air. Thus, from Fig. 10–8, the saturation curve for cast steel, the permeability, when there are 6000 gausses, equals 6000/.95 or 6300. When the flux density is 10,000 gausses the permeability equals 10,000/1.75 = 5700. At 12,000 gausses, the permeability equals 12,000/3.4 = 3500. At 16,000 gausses, the permeability equals 16,000/27 = 590. Note that the permeability decreases very rapidly after the knee of the saturation curve is reached. Note also, as discussed in Art. 11, that the permeability really shows how many gausses are set up by each oersted. Or permeability, $\mu$, shows the ratio of $B$ to $H$ and we can again write,

$$\mu = \frac{B}{H}.$$

Curves plotted between the corresponding values of $B$ and $\mu$ for the different magnetic materials are called permeability curves. Figure 23–8 is the permeability curve for cast steel. Note that the permeability is high up to a certain degree of magnetization and then drops off very quickly as the flux density increases.

**20. Tractive Force of Electromagnets.** Suppose we wind a coil, the area of the hole being very slightly over ½ square inch, on a paper tube and send sufficient electric current through the coil to produce a field in the air inside the tube. Suppose we then thrust into the tube a small cylinder of annealed steel having a cross-section area of ½ square inch. It will become magnetized to a flux density of, say, 50,000 lines per square inch. The steel magnet tends to stick in the tube by a force equal in pounds to one seventy-two millionth of the product of the field strength inside the coil times the number of lines in the steel magnet. This force is called the **tractive** force of the magnet and can be expressed in an equation:

$$F = \frac{\Phi B_a}{72{,}000{,}000}, \tag{14}$$

where

$F$ = force in pounds tending to hold magnet in tube,
$\Phi$ = lines of force in steel,
$B_a$ = lines per square inch in air at the end of the steel. This is practically the same as the flux density in the steel.

$$F = \frac{25{,}000 \times 50{,}000}{72{,}000{,}000}$$
$$= 17.4 \text{ pounds.}$$

But when a steel bar is thrust into a coil carrying a current, it is difficult to compute with any degree of precision the number of lines in a steel magnet or the field strength at the face of the magnet. So that while the above example illustrates the meaning of the rule, the practical application of it is better illustrated in connection with magnets in which these values can be computed; that is, in magnets, the magnetic circuit of which is almost entirely in the iron or steel.

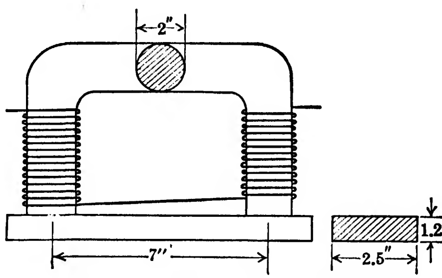



FIG. 24-8. A horseshoe shaped lifting magnet with flat ends.

**Example 18.** For instance, it is possible to compute satisfactorily the tractive force with which the wrought iron bar of Fig. 24-8 is held to the U-shaped magnet from the following data:*

| | |
|---|---|
| Length of U-shaped cast-steel magnet | = 18 inches. |
| Area of U-shaped magnet = $2 \times 2 \times 0.785$ | = 3.14 square inches. |
| Average length of magnetic path in wrought-iron bar | = 8 inches. |
| Area of wrought-iron bar ($2.5 \times 1.2$) | = 3.0 square inches. |
| Length of each air gap | = 0.002 inch. |
| Turns in each coil | = 1500. |
| Current through coils | = 0.8 ampere. |

**Solution.**

First, compute number of lines in magnetic circuit.

Ampere-turns $= 2 \times 1500 \times 0.8 = 2400$ ampere-turns.

Assume greatest reluctance to be in U-shaped piece, and try allowing 1800 ampere-turns for this part.

$$\text{Ampere-turns per inch for U-shape} = \frac{1800}{18} = 100 \text{ ampere-turns per inch.}$$

* For practical data on the construction of electromagnets, see C. R. Underhill's, "Solenoids, Electromagnets and Electromotive Windings," D. Van Nostrand Co.; Standard Handbook for Electrical Engineers; Pender's Handbook for Electrical Engineers; Proceedings of American Institute of Electrical Engineers, etc.

From curve for cast steel, 100 ampere-turns per inch will set up 101,000 lines per square inch.

Total flux = $101{,}000 \times 3.14 = 317{,}000$ lines.
Flux density in air gap = 101,000 lines per square inch.
Ampere-turns per inch of air gap for 101,000 lines per inch

$$= \frac{101{,}000 \times 0.394}{1.26}$$
$$= 31{,}600 \; NI \text{ per inch.}$$

Ampere-turns for 0.004 inch air gap $= 31{,}600 \times 0.004.$
$= 126 \; NI.$

$$\text{Flux density in wrought iron} = \frac{317{,}000}{3}$$
$$= 105{,}700 \text{ lines per square inch.}$$

From curve for wrought iron,

Ampere-turns per inch to set up 105,700 lines per square inch = 39 ampere-turns per inch.

Ampere-turns for 8 inches $= 8 \times 39$
$= 312$ ampere-turns.

Check:

| | | |
|---|---|---|
| Ampere-turns for cast steel | = | 1800 |
| Ampere-turns for air gaps | = | 126 |
| Ampere-turns for wrought iron | = | 312 |
| Total ampere-turns | = | 2238 |

Since we have 2400 ampere-turns available in the coils, the flux density in the steel will be somewhat more than 101,000 lines per square inch, say 102,000 lines per square inch, which is a close approximation.

$$\text{Total lines} = 102{,}000 \times 3.14$$
$$= 320{,}000 \text{ lines.}$$

The wrought-iron piece is therefore a magnet with 320,000 lines threading it. Each end is in a field of 102,000 lines per square inch, because this is practically the flux density at the ends of the U-shaped magnet.

Thus each end of the wrought iron is held to the U-shaped piece with a force equal to $\frac{320{,}000 \times 102{,}000}{72{,}000{,}000} = 453$ pounds.

Since both ends are attracted to the U-shaped magnet,

$$\text{Total tractive force} = 2 \times 453$$
$$= 906 \text{ pounds.}$$

**A peculiar fact** concerning the tractive force of magnets is brought out by considering the effect of chamfering the edges of the U-shaped piece in Fig. 24–8, so that it has the appearance shown in Fig. 25–8.

Assume that we chamfer off $\frac{1}{8}$ inch all around.

The area of each end will be

$$1.75 \times 1.75 \times 0.785 = 2.40 \text{ square inches.}$$

This will not materially affect the number of lines in the magnetic circuit, but the flux density in the air at the magnet tips will be

$$\frac{320{,}000}{2.40} = 133{,}000 \text{ lines per square inch.}$$

The tractive force at each end therefore equals:

$$\frac{320{,}000 \times 133{,}000}{72{,}000{,}000} = 592 \text{ pounds.}$$

$$\text{Total tractive force} = 592 \times 2 = 1184 \text{ pounds.}$$

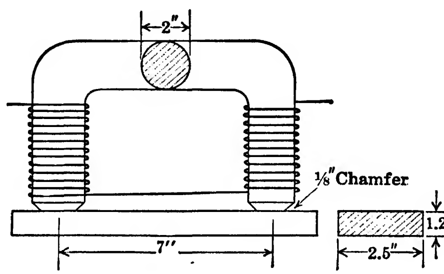



FIG. 25–8. The lifting magnet of Fig. 24–8, with the ends chamfered.

Note here the surprising fact that by **decreasing** the area of the ends of the magnet about $\frac{8}{10}$ of a square inch, without materially decreasing the total flux, we have **increased** its tractive force by more than 275 pounds or about 30 per cent. This seems unreasonable, until we examine more carefully the equation for tractive force,

$$F = \frac{\Phi B}{72{,}000{,}000}.$$

Then the cause is apparent.

Since $$\Phi = BA,$$

the equation can be written:

$$F = \frac{(BA)\,B}{72{,}000{,}000} = \frac{B^2 A}{72{,}000{,}000}. \qquad (15)$$

This shows that the tractive force varies directly, not only as the area, but also as the square of the flux density. Thus, when we decrease the area, we increase the flux density proportionally. Therefore, by decreasing the area, while we tend to decrease the tractive force in the same ratio, we also increase the tractive force by the square of this ratio because of the corresponding increase in the flux density.

A striking example of the practical application of this fact exists in the design of the powerful electromagnets used in hospitals for extracting steel and iron particles from the eye. These magnets are constructed with conical shaped ends in order to provide the maximum flux density and, therefore, the maximum tractive force.

**Prob. 40–8.** Calculate the tractive force in the magnet of Fig. 24–8 if 0.9 ampere is sent through the coils.

**Prob. 41–8.** What would be the tractive force of the magnet in Prob. 40–8, if the ends were chamfered $\frac{3}{16}$ inch all around?

**Prob. 42–8.** Fig. 26–8 represents the core of a transformer. The cross-section area of the core is 4.2 sq. inches. A saw cut 0.032 inch in width has been made at $A$. The flux density in the saw cut is 100,000 lines per square inch. Find the force tending to draw the core together at $A$.

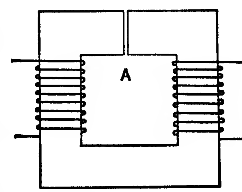



FIG. 26–8. A transformer core with a small air gap.

**21. Hysteresis.** In Fig. 10–8, we saw how the flux density $B$ increases when the magnetizing force $H$ increases, even when the curve was carried to a point considerably beyond saturation. If now we gradually take away the magnetizing force, let us see what happens.

In Fig. 27–8, the line $Axc$ is the magnetization curve of soft cast steel. It is a reproduction of the curve of cast steel in Fig. 10–8, plotted between $B$ in gausses and $H$ in oersteds. At point $c$ the flux density is 18,000 gausses, and the magnetizing force 125 oersteds.

When, however, the magnetizing force $H$ is decreased, the flux density $B$ does not decrease along the same line by which it had increased, but follows the curve *cdef*. At the point $e$, although the magnetizing force $H$ has become zero, there still remains a flux density, $B$, of 10,200 gausses. The flux density $B$, therefore, is said to lag behind the force $H$. This lagging is called Hysteresis, and is the cause of a certain loss in every alternating-current machine and in the armatures of direct-current machines. The reason for this is seen if we consider the rest of the curve.

At the point $e$, as has been said, although $H$ has become zero, the value of $B$ has only decreased to 10,200, represented by the line $Ae$. This value is called the **remanence.** To get this magnetic remanence out of the iron, it is necessary to set up a magnetizing force, $H$, in the opposite direction, of 10 oersteds. This force is represented by the line $Af$ and is called the **coercive** force.

If now this magnetizing force in the opposite direction, $-H$, is increased, a flux density, $-B$, will be set up in the opposite direction, and a curve of magnetization in this direction can be drawn as *fg*, until *B* has as large a negative value at *g* as it had a positive value at *c*.

The magnetizing force is again gradually decreased, and again the flux " lags " behind the force, until when *H* has again become zero, the value of *B* has decreased to *R* only. There is thus a remanence of about 10,200 gausses, *AR*, left in this direction, to remove which again requires a coercive force *AK*.

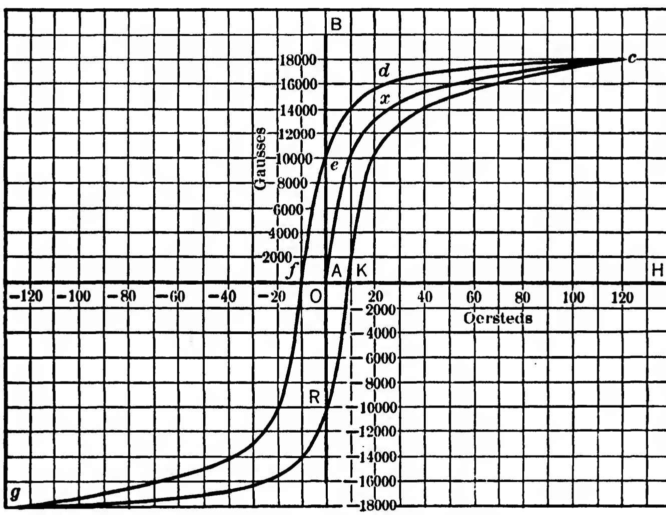



FIG. 27–8. A hysteresis loop.

To bring the flux density up to the value *c* again, we merely have to increase this force to its original value for the density at this point. The cycle thus completed is called the " Hysteresis Loop."

It can be shown that the area of this loop represents the work done in overcoming the hysteresis effect on a cubic centimeter of this specimen of iron, just as the area of a steam indicator card represents the work done in overcoming the resistance to motion offered by the piston. Only, in this case, all work done, as represented by the area of the loop, is wasted.

This can be illustrated as follows:

Since the value of *B*, *i.e.*, the magnetic density of the iron, always lags a little behind the magnetizing force, some extra force must be exerted continually to urge on the change in the magnetization.

When the flux density is being increased, the value of $H$, or magnetizing force, has to be a little greater than would be necessary, were there no hysteresis effect. Similarly, as the iron is being demagnetized, the magnetizing force must be larger in the opposite direction in order to overcome the drag of the magnetic lines which seem to persist in the iron. Thus there is a constant action taking place between the magnetized iron and the magnetic field of the coil.

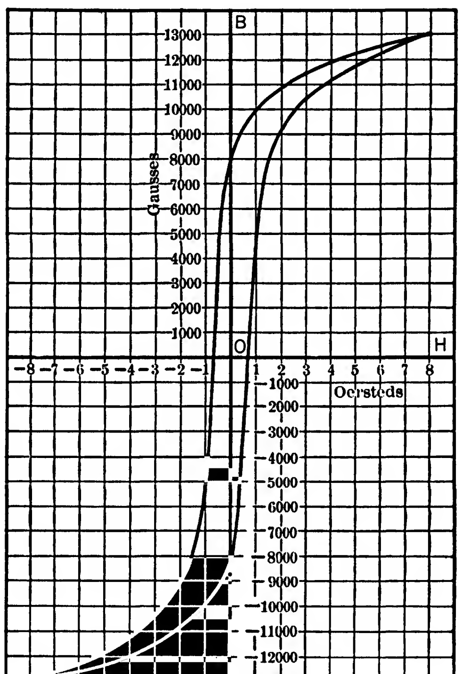



FIG. 28–8. The hysteresis loop for a sample of annealed sheet steel.

Applying Weber's Theory of magnetism: When an mmf is set up in the iron, the molecules (the small magnets of which the material consists) do not immediately respond, and some extra magnetizing force must be continually exerted to get the molecules to change position. When the mmf is decreased, a considerable number of these molecules do not return to their former position, even though the mmf is reduced to zero. An mmf in the opposite direction must be applied to bring these small magnets to their

original state. Thus, there is a constant drag of these molecules against the mmf applied to the iron.

It is easy to see, that in taking the iron from the flux density $C$, in Fig. 27–8, to that in the opposite direction at $g$, and then back again to $C$, that the molecules have reversed their position, or turned over, twice in the cycle. This requires an expenditure of energy, which is a loss, and which shows itself as heat.

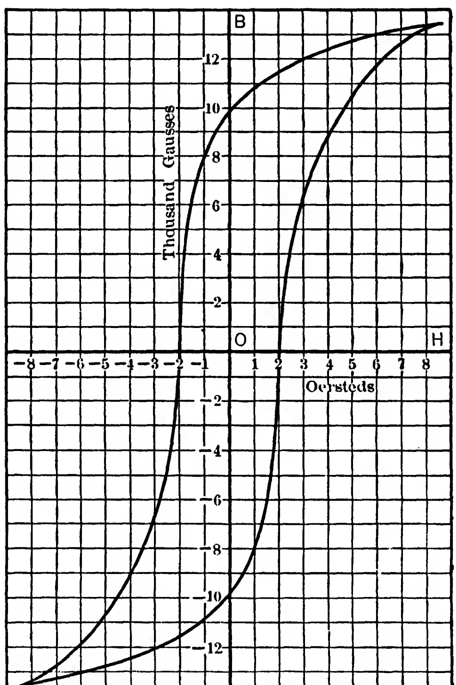



FIG. 29–8. The hysteresis loop of steel which has not been so well annealed as that of Fig. 28–8.

An iron or steel armature core, revolving in a two-pole dynamo, is taken through a complete magnetic cycle (double alternation) in one revolution. In a four-pole machine it would be taken through two cycles in one revolution. So there is always this hysteresis loss in the iron cores of all electrical machinery. The greater the number of alternations a second, the greater the total loss. For this reason, the iron to be used in alternating-current machines, and in armatures in general, has to be selected with proper care as to its hysteresis qualities.

If a piece of annealed steel or annealed sheet steel is put to this test, the loops plotted will be very narrow and contain little area (Fig. 28–8), showing the loss to be small.

In the case of steel, which has not been so well annealed, the loop will widen out (Fig. 29–8) and contain a larger area. For this reason, either standard annealed sheet steel or annealed silicon steel is used in transformer cores, armatures, etc.

Hysteresis loss can be expressed by an equation as follows:

$$W_h = \frac{A \times B \times H}{4\pi} = \text{ergs per cubic centimeter per cycle,} \tag{16}$$

or as

$$W_h = \frac{0.0796 \times A \times B \times H}{10^7} = \text{watt-seconds per cubic centimeter per cycle;} \tag{17}$$

where $A$ = the area of the hysteresis loop in square inches;
$B$ = flux density in lines per square centimeter;
$H$ = magnetizing force in gilberts per centimeter.

It is necessary that $B$ and $H$ be plotted to scale.

The following examples will illustrate the idea.

**Example 19.** Find the hysteresis loss in watt-seconds per cubic centimeter per cycle, in the steel used to obtain the loop in Fig. 27–8.

We will assume the loop has an area of 12 square inches and that:

On the horizontal axis,
1 inch = 20 oersteds; magnetizing force, $H$.
On the vertical axis,
1 inch = 4000 lines per square centimeter; flux density, $B$.

Then 1 square inch of loop = $20 \times 4000 = 80{,}000$ ($BH$ units).
Area of curve = $12 \times 80{,}000 = 960{,}000$ ($BH$ units).
Hysteresis loss for one cycle,

$$P = \frac{0.0796 \times 960{,}000}{10^7} = 0.0076 \text{ watt-second.}$$

**Example 20.** If 20 pounds of the steel in Example 19 go through the cycle of magnetization 60 times per second, what power would be lost on account of hysteresis?

**Solution.**

1 cubic foot weighs 460 pounds;
1 cubic foot = 1728 cubic inches;

$$20 \text{ pounds} = \frac{20 \times 1728}{460} = 75 \text{ cubic inches;}$$

$$= 75 \times 16.4 = 1230 \text{ cubic centimeters.}$$

From Example 19:

1 cubic centimeter loses 0.0076 watt-second per cycle.
Then 1230 cubic centimeters would lose:
$0.0076 \times 60 \times 1230$ watt-seconds in 60 cycles = 561 watt-seconds.

Since this loss occurred in one second, the power lost = 561 watts.

**22. Computation of Hysteresis Loss.** Dr. Steinmetz found by a series of tests that, where the variation of flux is sinusoidal, the hysteresis loss can be expressed approximately as a formula, which can be written as:

$$P = \frac{KB_{max}^{1.6}}{10^7} \tag{18}$$

in which

$P$ = loss in watts per cubic centimeter of magnetic material for 1 cycle per second,
$B_{max}$ = greatest flux density in lines per square centimeter during the cycle,
$K$ = Steinmetz's hysteresis constant, depending upon the quality of the material used.

For ordinary so-called "annealed electrical sheet steel," such as is commonly used in the cores of armatures, etc., this constant ($K$) may be taken as 0.003. For silicon steel, such as is used in transformers, a fair value of $K$ is 0.00065. For its value in connection with other materials, see any electrical handbook.

**Example 21.** An armature core of standard annealed sheet steel weighs 42 pounds. Armature turns 1800 revolutions per minute in a 2-pole field. Maximum flux density = 10,100 gausses. What is the hysteresis loss in the armature core? Sheet steel weighs 0.283 pounds per cubic inch.

**Solution.**

Loss per cubic centimeter per cycle per second

$$= \frac{0.003 \times 10{,}100^{1.6}}{10^7} = 0.000766 \text{ watt.}$$

1800 rpm = 30 rev per second, which would take the magnetization through 30 cycles per second.

Loss per cubic centimeter for 30 cycles per second
$= 0.000766 \times 30 = 0.0230$ watt.

Armature contains $\frac{42}{0.283} = 148.3$ cubic inches
$= 148.3 \times 16.4 = 2430$ cubic centimeters.

Loss in armature $= 2430 \times 0.0230 = 55.9$ watts.

**Prob. 43–8.** The armature of a motor contains 70 pounds of sheet steel (specific gravity, 7.8). The magnetic flux is 17,000 lines per square centimeter maximum, and goes through 40 cycles per second. What is the loss in kilowatts? (A cubic foot of water weighs 62.5 pounds.)

**23. Magnetic Qualities of Nickel, Cobalt and Alloys.** While iron and steel are usually spoken of as the magnetic materials, it is true that both nickel and cobalt are magnetic to some extent, though far less than iron and steel. Nickel reaches its highest permeability, about 225, at a flux density of about 1.55 gausses; and cobalt has its highest permeability, about 195, at approximately 2320 gausses. In order to compare nickel and cobalt with other materials, it is only necessary to remember that non-magnetic materials have a permeability of 1 at all flux densities and that annealed silicon steel has a permeability of about 4000 at 10,000 gausses. Thus, nickel and cobalt, at best, are about 200 times as permeable as non-magnetic materials, and only about 1/20 as permeable as commercial silicon steel.

A magnetic material called " Permalloy " has been developed by the Bell Telephone laboratories, consisting of about 78.5 per cent nickel and 21.5 per cent iron. This alloy has remarkably high permeability for low magnetizing force. A magnetizing force of 0.4 gilbert per centimeter produces a flux density of 3540 gausses. This results in a permeability of $\frac{3540}{0.4}$ or 8850, which is about 200 times as great as that of the best iron for this low magnetizing force.

Another alloy called " Hipernik," consisting of 78 per cent nickel and 22 per cent iron, also shows remarkable properties. A magnetizing force of 0.055 gilbert per centimeter produces 3300 lines per square centimeter; or a permeability of $\frac{3300}{0.055} = 60{,}000$.

In contrast with the above alloys, Ferranti Ltd. of England has developed a cast iron, called " Nomag." This has a permeability of 1.04 which is practically that of air, or of non-magnetic materials.

## SUMMARY OF CHAPTER VIII

### THE MAGNETIC CIRCUIT.

**1. The MAGNETIC FLUX follows a complete magnetic circuit just as the electric current follows a complete electric circuit.**

**2.** The magnetic flux is measured in LINES OF FORCE ($\Phi$), while the electric current is measured in AMPERES (I).

**3.** The magnetic flux is set up by the magnetic pressure, called the magnetomotive force, which is measured in gilberts. The electric current is caused to flow by the electric pressure, called the electromotive force, which is measured in volts.

**4.** The magnetic flux is limited by the reluctance of the magnetic circuit. The flow of the electric current is limited by the resistance of the electric circuit, which is measured in ohms.

FLUX DENSITY is the total flux in a circuit divided by the area of the path at right angles to the lines of force. Flux density, B, is measured in lines per square inch, or in lines per square centimeter. 1 line of force per square centimeter = 1 gauss.

$$B = \frac{\Phi}{A} \quad \text{or} \quad \Phi = B \times A.$$

Where A = area of the path.

The RELUCTANCE of non-magnetic materials is computed by an equation analogous to the equation for the electrical resistance.

| Reluctance | Resistance |
|---|---|
| $\mathcal{R} = \frac{K \times l}{A}$ | $R = \frac{K \times l}{d^2}$ |
| K for non-magnetic materials = 1. | K for copper = 10.4. |
| $l$ = length in centimeters. | $l$ = length in feet. |
| A = area in square centimeters. | $d^2$ = area in circular mils. |

Reluctances in series are added just as resistances in series are added.

Reluctances in parallel are treated just as resistances in parallel are treated.

The reluctance of a centimeter cube of air = 1 unit.

The reluctance of an inch cube of air = 0.394 unit.

The product of the amperes flowing in a coil times the turns in the coil is called the AMPERE-TURNS (NI) of the coil. MAGNETIC PRESSURE in gilberts, $\mathfrak{F}$ = 1.26 NI.

**5.** The law of the magnetic circuit is analogous to Ohm's law for the electric circuit.

OHM'S LAW.

| Magnetic Current | Electric Current |
|---|---|
| Flux equals pressure divided by reluctance. | Current equals pressure divided by resistance. |
| Lines of force equal gilberts divided by reluctance. | Amperes equal volts divided by ohms. |
| $\Phi = \frac{\mathfrak{F}}{\mathcal{R}}$ | $I = \frac{E}{R}$ |

Flux at the center of a Long Coil (air core) $= \frac{1.26 \text{ NI A}}{l}$ where A = area in square centimeters and $l$ = length in centimeters.

MAGNETIZING FORCE (H) in oersteds is the magnetic pressure drop per centimeter length of circuit, or,

$$H = \frac{1.26 \text{ NI}}{l}$$

PERMEABILITY is the ratio of the flux density in lines per square centimeter to the magnetizing force. It may be thought of as the number of lines per square centimeter which one gilbert per centimeter will set up in the material, at that degree of magnetization. The permeability of air or non-magnetic material is ONE. It is not constant for any given piece of iron or steel but depends upon the degree of magnetization.

The RELUCTANCE FOR A GIVEN PIECE OF IRON OR STEEL is not constant but depends upon the degree of magnetization.

## B-H CURVES.

In order to facilitate the computation of magnetic circuits containing iron and steel, curves are drawn between a series of values of the magnetizing force (H) in gilberts per centimeter and the corresponding values of the flux density (B) in lines per square centimeter. These are called magnetization curves.

For convenience, manufacturing companies in this country plot these curves between B in kilolines per square inch and H in ampere-turns per inch.

## TWO STAGES OF MAGNETIZATION. SATURATION POINT.

The magnetic curves have the same general shape for all magnetic materials. The magnetization may be divided into two stages. The first stage consists of that part of the curve where the flux density reaches only moderate values and the permeability is comparatively high, relatively only few ampere-turns being required. This continues until the saturation point is reached, beyond which it is increasingly difficult to add to the flux density. Each small increase of flux density requires a larger number of ampere-turns. The magnetization curve, therefore, makes a more or less sharp bend or " knee " at the saturation point.

It is not generally economical to operate machines with a flux density much beyond the saturation point.

## SERIES AND PARALLEL MAGNETIC CIRCUITS.

The rules for applying Ohm's law to series and parallel magnetic circuits are the same as for applying Ohm's law to series and parallel electric circuits.

## COMPUTATION OF FLUX FOR COMPOSITE CIRCUITS.

The flux which a given number of ampere-turns will set up in a composite circuit of magnetic material cannot easily be computed

directly. A cut and try method, however, may be used satisfactorily and an approximation made to any desired degree of precision. This may require several trial computations.

## HYSTERESIS.

When we remove the magnetizing force from a magnetic material, all the flux does not drop out of the material because there is a certain "lag" in the magnetic flux with regard to the magnetic force. This lagging is called hysteresis.

Since the flux in armature cores and alternating-current machines must be reversed many times a second, a certain amount of power must always be expended to overcome the effect of this lagging. Soft iron and annealed steel exhibit this property of hysteresis to a very limited degree. It is, therefore, customary to use these materials whenever frequent and large changes are required in magnetic flux. The hysteresis loss in watts in one cubic centimeter of iron or steel, in which the flux changes at the rate of one cycle per second, can be computed from Steinmetz's formula when the maximum flux does not greatly exceed 10,000 lines per square centimeter.

Steinmetz's formula for hysteresis loss:

$$P = \frac{KB_{max}^{1.6}}{10^7}$$

in which P = loss in watts per cubic centimeter of magnetic materials for one cycle per second,

$B_{max}$ = greatest flux density in lines per square centimeter during cycle,

K = Steinmetz's hysteresis constant depending upon material used.

## TRACTIVE FORCE OF ELECTRIC MAGNETS.

The general equation for the tractive force of magnets may be expressed as:

$$F = \frac{\Phi B_a}{72{,}000{,}000},$$

where F = tractive force in pounds, $\Phi$ = total lines of force in steel core, $B_a$ = lines per square inch in air at end of core.

## MAGNETIC QUALITY OF NICKEL AND COBALT.

Nickel and cobalt are magnetic to a slight extent, having a maximum permeability about two hundred times that of air and about 1/200 that of annealed steel.

## PROBLEMS ON CHAPTER VIII

**Prob. 44–8.** A wrought-iron ring 75 inches long is to carry a flux of 800,000 lines. Flux density is to equal 50,000 lines per square inch. (*a*) What is the cross-section area? (*b*) What is the permeability at this density? (*c*) How many ampere-turns are required? (*d*) What is the reluctance of circuit?

**Prob. 45–8.** In order to magnetize an iron rod, a magnetomotive force of 400 ampere-turns is necessary. How many volts must be applied to a coil of 200 turns and 68 ohms resistance?

**Prob. 46–8.** If wire in Prob. 45 is No. 24, B. & S. copper, how many feet of it are in the coil?

**Prob. 47–8.** It is desired to magnetize a cast-steel ring 15 inches in diameter to a density of 90,000 lines per square inch. 800 turns of No. 23 B. & S. copper wire are used. Average length of each turn is 14 inches. How many volts must be applied to coil in order to set up desired flux density?

**Prob. 48–8.** How many ampere-turns are necessary to magnetize a 10-inch cast-steel bar of 1.5 square inches cross-section so that it has a flux of 120,000 lines? Bar is bent into a ring.

**Prob. 49–8.** A cast-iron ring has a mean diameter of 20 inches and a cross-section area of 6.5 square inches. How many ampere-turns are required to set up a flux of 300,000 lines?

**Prob. 50–8.** What is the reluctance and permeability of ring in Prob. 49?

**Prob. 51–8.** What would be the reluctance and permeability of ring in Prob. 49–8 if 230,000 lines were set up?

**Prob. 52–8.** A magnetic circuit is made up of 120 inches of wrought iron, 8 square inches in cross-section; 0.45 inch of air, 9 square inches in cross-section. Find number of ampere-turns necessary to set up 600,000 lines of force in above circuit.

**Prob. 53–8.** A magnetic circuit is made up of:

75 inches of cast steel, 8 square inches cross-section,
65 inches of sheet steel, 7 square inches cross-section,
0.4 inches of air, 8 square inches cross-section.

Find current that must be sent through a coil of 20,000 turns in order to set up 720,000 lines in above circuit.

**Prob. 54–8.** If cross-section of each part of circuit in Prob. 53 were doubled, what current would be necessary?

**Prob. 55–8.** A magnetic circuit is made up of:

| | |
|---|---|
| 60 inches of wrought iron, | $B$ 80,000 lines per square inch, |
| 70 inches of sheet steel, | $B$ 105,000 lines per square inch, |
| 20 inches of cast iron, | $B$ 50,000 lines per square inch, |
| 0.3 inch of air, | $B$ 75,000 lines per square inch. |

Find ampere-turns necessary to maintain the circuit at these flux densities.

**Prob. 56–8.** If the total flux in circuit of Prob. 55 is 1,000,000, what must be the area of each part of the circuit?

**Prob. 57–8.** What is the reluctance of circuit in Prob. 55?

**Prob. 58–8.** Find the number of ampere-turns in the field coil of a generator, as in Fig. 16–8, the magnetic circuit of which is made up of the following parts: cast-iron ring, length 21 inches, flux density 40,000 lines per square inch; two wrought-iron cores each 4 inches long, flux density 80,000; two air gaps each 0.02 inch long, flux density 55,000; path through armature of standard annealed sheet steel 6 inches long, flux density 85,000.

**Prob. 59–8.** In Fig. 40–11, the field cores are of wrought iron, each 5 inches long and an average cross-section area of 20 square inches. Yoke is of cast steel 10 inches long, 15 square inches cross section; armature core is of sheet steel, magnetic path 5.5 inches long, with an average cross section of 11.5 square inches. Air gaps, each 0.25 inch long, of 38 square inches cross-section area. If 1,600,000 lines are desired in air gaps, of how many ampere-turns must the field coils consist?

**Prob. 60–8.** In order to raise a certain weight, the density in a horseshoe-shaped wrought-iron hoisting magnet must be 105,000 lines per square inch; length of iron circuit 20 inches. If there are 1428 turns carrying two amperes, how near to the weight must the magnet be brought to lift it? Neglect the reluctance of the weight.

From Caldwell's "Engineering Problems."

**Prob. 61–8.** It is desired to design a cast-steel horseshoe magnet to raise 1000 pounds. The length of magnetic path in the steel is 20 inches and there would be 2 air gaps of 0.004 inch each. Density in the

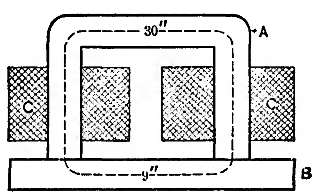



FIG. 30–8. A lifting magnet.

steel is to be 103,000 lines per square inch; in the air gap 97,000 lines. On account of its great cross section, neglect the reluctance of the weight. Find the diameter of the steel core of the magnet and the number of ampere-turns needed.

From Caldwell's "Engineering Problems."

**Prob. 62–8.** A U-shaped cast-steel hoisting magnet similar to Fig. 30–8 has the following dimensions:

| | |
|---|---|
| Length of path through U-shaped part | = 16 inches. |
| Length of path through flat piece | = 6 inches. |
| Length of air gaps | = 0.04 inch (each). |
| Cross section of U-shaped part | = 12 square inches. |
| Cross section of flat piece (wrought iron) | = 20 square inches. |
| Number of turns in coils (each) | = 1500 turns. |
| Current in coils | = 1.5 amperes. |

Find total flux.

**Prob. 63–8.** What is the tractive force between U-shaped part and flat piece in Prob. 62–8?

**Prob. 64–8.** What current must be used in the magnet coils of Prob. 62–8, if the tractive force is to be 2 tons?

**Prob. 65–8.** Fig. 30–8 represents a magnet used to hoist steel rails. Magnet core *A* is of cast steel, 10 square inches cross section. Rail *B* has a cross section of 15 square inches. There is a space of 0.05 inch between ends of core *A* and rail *B*, due to rust. The flux required is 1,100,000 lines. How many ampere-turns are required? Assume steel rail has same *B–H* curve as cast steel.

**Prob. 66–8.** What is the tractive force of magnet in Prob. 65–8?

**Prob. 67–8.** The lifting magnet Fig. 31–8 has the following dimensions: Core *A*, mean length of 11 inches, area of section 1 square inch. Core *B*, mean length 6.5 inches, area of section 0.9 square inch.

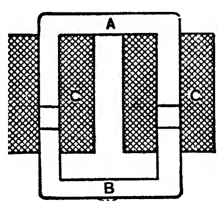



FIG. 31–8. A sucking magnet.

Material of both is sheet steel. Air space between ends of *A* and *B* is $\frac{7}{8}$ inch. If 100,000 lines are required in circuit to lift *B*, how many ampere-turns must be wound on *C*?

**Prob. 68–8.** The annealed sheet-steel core of a transformer weighs 280 pounds. If it is used on an alternating-current circuit of 133 cycles per second, what will be the hysteresis loss? $B_{max} = 65{,}000$ lines per square inch.

**Prob. 69–8.** What would the hysteresis loss be in Prob. 68–8, if the transformer were used on a 60-cycle circuit?

**Prob. 70–8.** Armature of generator in Prob. 59–8 weighs 50 lb. and makes 1200 rpm. What is the hysteresis loss?

**Prob. 71–8.** Assume 1 square inch of curved surface of coil is needed to radiate 1 watt, to prevent a coil from becoming overheated. What must be outside diameter of coil in Prob. 47–8 if it is to be used continuously? Length is 10 inches.

**Prob. 72–8.** On basis stated in Prob. 71–8, what must diameter of coil in Prob. 53–8 be, if length is 3 inches? Resistance = 40 ohms.

# CHAPTER IX

## INDUCTANCE

Thus far, we have considered the relations existing in electric circuits having resistance only, and carrying a steady current under the application of a steady or constant impressed voltage.

Let us now consider the electric and magnetic effects which take place when the current in a circuit changes, or is changed in value, due either to a change in the voltage applied to the circuit, or to a change in the magnetic field enclosing the circuit.

When the current in a circuit sets up a strong magnetic field, as it may when the circuit is in the form of a coil, any change in this current induces an electromotive force in the circuit itself. This electromotive force is in addition to the voltage impressed. Such a circuit is said to be **inductive,** or to have **self inductance**; and the voltage so induced is called the electromotive force of self induction.

When a change of current in one circuit magnetically effects another circuit and induces in it a voltage, the two circuits are said to be **mutually inductive.**

**1. Interlinkages.** It has been shown that wherever there is an electric current, there is always present a magnetic field which is everywhere at right angles to the current; and that it exists in closed lines or circuits about the conductor carrying the current. The electrical conductor is also part of a closed circuit if it carries a current. Therefore, the flux entirely encircles the conductor, or the current, and the current in the conductor entirely encircles the flux. They are thus said to "**interlink**" each other. Figure 9–7 shows the flux or magnetic lines interlinking a one-turn coil, while Fig. 1–9 illustrates a coil of 7 turns interlinking 5 magnetic lines. This gives $7 \times 5$ or 35 **interlinkages** or flux turns. Of course an increase or decrease of the current in the coil changes the strength of the field and the number of the interlinkages.

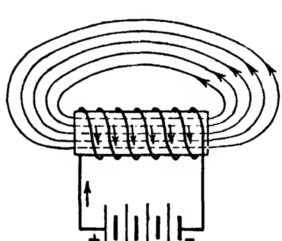



FIG. 1–9. Interlinkages of magnetic flux lines and the turns of a coil.

**2. Induced Electromotive Force. Electromagnetic Induction. Case I.** Consider Fig. 2*a*–9, which represents an insulated coil with its terminals connected to a galvanometer, or low-reading voltmeter. If a magnetic field is set up within the coil by any

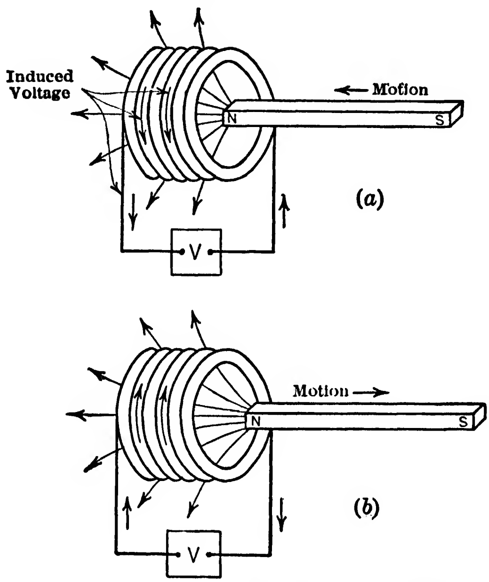



FIG. 2–9. Induced emf due to change in interlinkages. (a) Magnet moved into the coil. (b) Magnet withdrawn from the coil.

means (creating interlinkages of flux and turns), a voltage is **induced** in the coil **during the time the number of interlinkages is changing in value.** This is shown by a swing of the galvanometer needle.

If the magnet of Fig. 2*a*–9 is pushed into the coil, changing the interlinkages, it is found by experiment that an electromotive force will be induced in the coil in a direction indicated by the small arrows. When the motion of the magnet stops, that is, when the flux interlinking the coil no longer is changing in value, no voltage is induced.

As the magnet is withdrawn from the coil, again changing the interlinkages, an electromotive force will again be induced in the coil. However, the voltage is now in the **opposite** direction as indicated in Fig. 2*b*–9.

Note particularly that it is the **change,** or **rate of change,** in the number of interlinkages which sets up this electromotive force.

The average value of this electromotive force can be expressed by the equation

$$e = \frac{\Phi N}{10^8\, t} \tag{1}$$

where $e$ = average induced emf in volts;
$N$ = number of turns in the coil;
$\Phi$ = the magnetic lines (maxwells) linking the coil;
$t$ = time required for the change in flux;

and $\dfrac{\Phi N}{t}$ = average rate of change of the flux turns or interlinkages.

**When there is a change of one interlinkage per second in a circuit, one fundamental unit of pressure is set up.** This unit of pressure (called the abvolt) is 1/100,000,000 or $1/10^8$ of the practical unit of pressure (the volt); and hence, it is necessary to divide by $10^8$ all expressions involving induced pressure in volts. Stated in another way, when there is a change of $10^8$ interlinkages per second in a circuit, one volt is set up.

**Example 1.** What average voltage is induced in a circuit consisting of a coil of 800 turns, if the flux interlinking the coil is increased by 500,000 lines in 0.05 second?

**Solution.**

$$e_{av} = \frac{\Phi N}{10^8\, t}$$

$$= \frac{500{,}000 \times 800}{10^8 \times 0.05} = 80 \text{ volts.}$$

**Prob. 1–9.** How many volts, average, would be induced in the coil of Example 1 if it had 600 turns?

**Prob. 2–9.** If the flux in Example 1 were changed by twice the amount in the same time, how much voltage would be induced?

**Prob. 3–9.** How many turns are there in a coil, if the average induced voltage is 250 volts, when the flux is changed by 1,200,000 lines in 0.02 second?

**Prob. 4–9.** What time is required to change the flux by 2,400,000 lines in a coil of 1000 turns, if the average voltage induced is 3500 volts?

**Direction of induced voltage.** The induced voltages in the coils of Figs. 2*a*–9 and 2*b*–9 set up momentary currents through the coils and galvanometer, which are in the **same direction** as these voltages. If, in turn, the flux set up by these currents is investigated,

it is noted that in both cases it is in a direction to **oppose the motion of the magnet.** That is, the flux so set up **opposes** or **repels** the magnet as it is **moved into** the coil, and **attracts** the magnet as it is **moved out of** the coil. Note Figs. 3*a*–9 and 3*b*–9.

This effect was first stated by Lenz and is now called Lenz's law, part of which is:

**"An induced voltage sets up a current in a closed circuit, and a magnetic field, which is always in a direction to oppose, or stop, the action which produced it."**

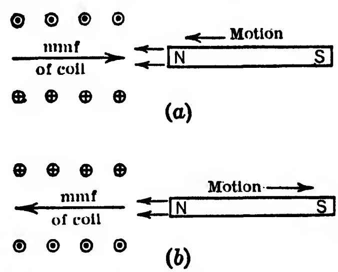



FIG. 3–9. The field set up by the induced current in the coil of Fig. 2–9 opposes the motion of the magnet: (a) As it is inserted in the coil; (b) As it is withdrawn from the coil.

This is part of the great physical law of the universe — the law of the "conservation of energy" as applied to the electric circuit. That is, induced electric currents, which represent energy, are produced by the expenditure of mechanical energy — in the illustrations above, by the movement of the bar magnet. The above fact should be clearly understood, for it applies to the action of all types of electrical machinery.

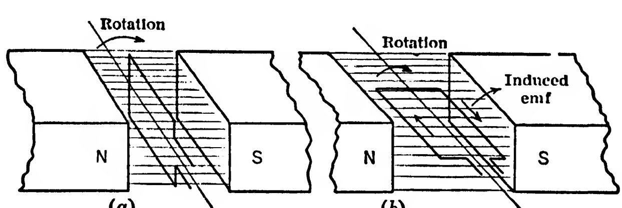



FIG. 4–9. Induced emf in a coil rotating in a stationary magnetic field. (a) Maximum number of magnetic lines linking the coil; no rate of change of linkages. (b) No lines linking the coil; maximum rate of change of linkages.

**3. Induced Electromotive Force. Case II.** In the previous article, the effect was considered when a changing magnetic field interlinked a stationary coil. Now consider a coil moving in a stationary magnetic field. In this case, the flux linking the coil is varied by the movement of the coil itself.

In Fig. 4–9 the one-turn coil revolves in a uniform magnetic field.

In (*a*) the plane of the coil is perpendicular to the magnetic field and interlinks the maximum possible flux. If the coil is rotated on an axis a quarter of a revolution, in a clockwise direction, it will be in position (*b*); and since the plane of the coil is parallel to the flux, there will be no interlinkages. If the coil be revolved clockwise another quarter of a revolution, there will be maximum flux interlinking the coil again. During the progress of the half revolution, then, the coil interlinks the maximum possible flux, first in one direction and then in the other. During the half revolution, a voltage will be induced in the coil in the direction shown by the arrows in (*b*). The average value of this voltage can also be expressed by an equation similar to equation (1), except that, since there has been a double interlinkage of flux, it would be written:

$$e_{av} = \frac{2\,\Phi N}{10^8\,t} \qquad (2)$$

where $t$ = time in seconds for the half revolution of the coil.

It is probably simpler in this case to visualize the sides of the coil as moving through or cutting across the flux. (The sides of the coil are here called " inductors " or " conductors.") Note that in the side of the coil moving **up** through the flux, Fig. 4*b*–9, the induced voltage is **away** from the reader. and in the side moving **down,** it is **toward** the reader.

From this fact, Fleming stated the definite relation between the direction of the flux, the direction of movement of the " conductor " and the direction of the induced electromotive force, known as Fleming's Right-Hand Rule. " If the thumb and first two fingers of the **right** hand are extended at right angles to one another, the thumb pointing in the direction of motion of the conductor and the first finger in the direction of the magnetic lines, the second finger will point in the direction of the induced electromotive force." This rule is further discussed in Art. 3, Chapter X. Note Figs. 9*a*–10 and 9*b*–10.

The voltage induced in the coil of Fig. 4–9 will be in the same direction if the field is rotated in a **counter-clockwise** direction while the coil is held stationary. This is equivalent to moving the coil clockwise through the stationary field, as already discussed. It is the **relative** motion of the two which determines the direction of the induced voltage.

If the coil of Fig. 4–9 is closed on itself, a current will flow in the direction of the induced voltage, and this current will set up a

magnetic field which opposes the motion of the coil. Figure 5–9 shows a cross-section view of the coil, and shows how the field set up by the current opposes the mechanical motion of the coil. Here again is shown the effect described in the previous article as Lenz's law.

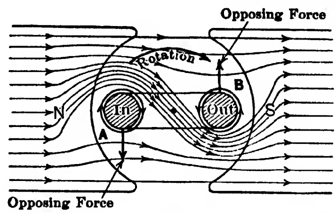



FIG. 5–9. The magnetic field set up by the induced current in the rotating coil opposes the motion of the coil.

**4. Electromotive Force of Self Induction.** Consider a wire 2000 feet long having 2 ohms resistance. Suppose it is stretched out "line and return" — 1000 feet each way — and 20 volts is impressed on it from a battery, as indicated in Fig. 6–9. Practically as soon as the switch is closed, the current will rise to 10 amperes in obedience to Ohm's law.

Now if the wire is wound into a coil, it still has 2 ohms resistance. When a pressure of 20 volts is now impressed on the coil, the ammeter will again register 10 amperes in obedience to Ohm's law, but some time will elapse before the current reaches this value. If the succeeding values of current are plotted against time, a curve

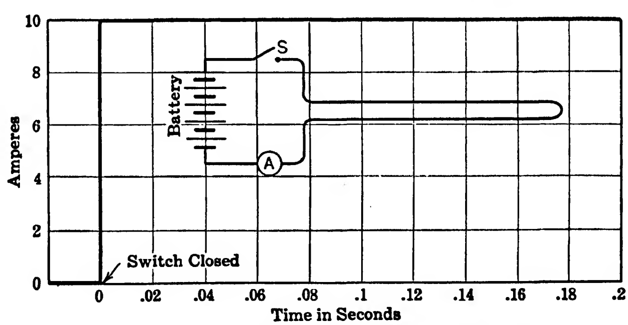



FIG. 6–9. Instantaneous rise of current in a straight wire, "line and return" — a non-inductive circuit.

would be obtained similar to that shown in Fig. 7–9. Note that it takes about 0.2 second after closing the switch for the current to rise to its full value of 10 amperes.

As the current increases in the coil, the flux increases, thereby increasing the interlinkages or flux turns. This, as has been shown, induces an electromotive force, commonly called an emf, in the coil. The value of this emf is determined by the rate at which

the flux increases and by the number of turns in the coil — or by the rate of change of linkages. This voltage, according to Lenz's law, and from the relations already shown, **opposes** the growth of the current and retards it so it cannot reach its maximum value at once.

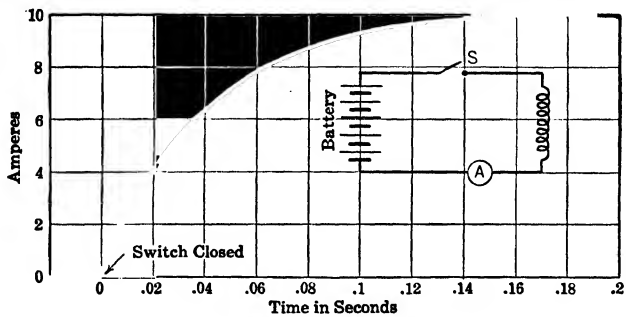



FIG. 7–9. Delayed rise of current in a coil of wire with an air core — an inductive circuit.

Now let the circuit be connected as in Fig. 8–9 with the switch, $S$, to short-circuit the coil. The fuse is inserted to protect the battery. With a steady current of 10 amperes flowing, the switch is closed and the coil short-circuited. The current in the coil will not immediately drop to zero, but will follow the curve of

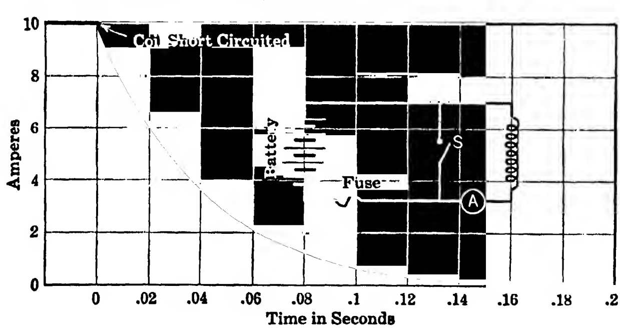



FIG. 8–9. Delayed decrease, or decay, of current in a short-circuited air-cored coil — an inductive circuit.

Fig. 8–9; for a decrease of current decreases the flux and therefore the linkages. This again sets up an induced voltage and a magnetic field which **opposes** the decrease in linkages, or in other words, tends to keep the current flowing. Here, also, is seen the operation of Lenz's law.

Again, if there is a steady current flowing in the coil in Fig. 7–9,

and the switch, $S$, is suddenly opened, it is found that a considerable voltage is induced in the coil. This is due to the sudden and rapid change in linkages which tends to keep the current flowing, as is evidenced by a spark at the breaking points of the switch.

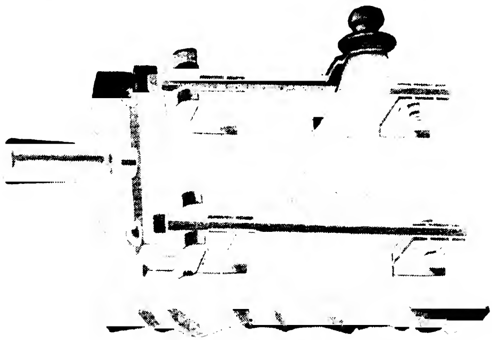


FIG. 9a–9. A "field discharge" switch. The extra clip on the upper blade serves to connect a discharge resistance across the field winding. *General Electric Co.*

This property in a single circuit, which opposes the building up and dying out of a current, is called the **Self Inductance** of the circuit. And the momentary electromotive force which is set up is called the **Induced Electromotive Force of Self Induction.**

Circuits consisting of coils in which strong magnetic fields are set up possess high self inductance; and the induced electromotive force is great, if one tries suddenly to build up, or destroy or change the current in such circuits.

In large generators, a sudden opening of the "field" switch in the circuit of the windings on the field poles may induce such a high voltage in these windings that it will break down the insulation. Because of this, these machines are equipped with a "field discharge switch" which, as it is opened, short-circuits these windings through a resistance, as shown in Figs. 9*a*–9 and 9*b*–9.

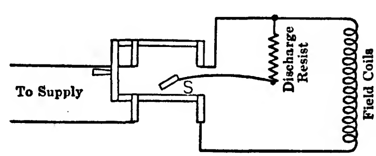



FIG. 9b–9. The circuits of a field winding and discharge resistance connected to a field discharge switch. As the switch is opened, the clip, *s*, connects the resistance across the field terminals before the connection to the power supply is broken. This serves to retard the decrease of current in the field circuit and prevents a high induced emf in the winding.

**Caution.** Voltages dangerous to life may be set up by the breaking of the field circuit of even small machines. Therefore, care must be used not to come into contact with the two terminals of a field winding at the time the circuit is broken.

**5. Growth and Decrease of Current in an Inductive Circuit.** It has been shown in the previous article that the voltage induced by an increasing current in a coil delays, or retards, the rise of current to the value determined by Ohm's law. By referring to Fig. 7–9, the current does not reach this Ohm's law value until about 0.2 second after the switch is closed. During the period between the closing of the switch and the 0.2 point of the curve, the impressed voltage, $E$, fulfills two functions. It overcomes, or balances, both the $IR$ drop due to the resistance of the coil, and also the emf of self induction due to the changing of the current. This relation can be expressed by the equation

$$E = iR + e \tag{3}$$

where $e$ = the voltage at any instant required to overcome the induced emf of self induction, and to which it is equal;

$i$ = the momentary current in the coil at the same instant;

$R$ = ohmic resistance of the coil;

$E$ = voltage impressed on the coil.

At the instant the switch is closed, the $IR$ drop is zero and the entire impressed voltage, $E$, is used to increase the current and overcome the induced emf, $e$. At this instant, the current is changing at the greatest rate and the induced emf is greatest. At the end of about 0.2 second, the current has reached its greatest value and is steady; and the entire impressed voltage, $E$, is now used to overcome the resistance or $IR$ drop of the circuit, while the induced emf, $e$, has dropped to zero.

During the rise of current, the induced voltage, $e$, opposes the impressed voltage, $E$, and therefore, the difference of the two voltages at any instant determines the value of the current at that instant. Thus in Fig. 7–9 at the end of 0.02 second, the current is 4 amperes, and since the impressed voltage is 20 volts and the resistance of the circuit is 2 ohms

$$20 = 4 \times 2 + e$$

or

$$e = 20 - 4 \times 2 = 12 \text{ volts.}$$

At the end of 0.04 second, the current is 6.4 amperes and changing at a much less rapid rate: thus the induced voltage is much less, or

$$e = 20 - 6.4 \times 2 = 7.2 \text{ volts.}$$

At the end of 0.2 second, the current is practically 10 amperes and has become steady and the induced voltage equals

$$e = 20 - 10 \times 2 \text{ or } 0 \text{ volts}$$

and has died out entirely.

In the case of a decreasing or dying current, at any instant it is directly due to the induced emf at that instant and inversely proportional to the resistance of the circuit. Thus in Fig. 8–9, at the end of 0.02 second after the coil is short-circuited, the current is 6 amperes and since $i = e/R$, the induced emf is

$$e = 6 \times 2 = 12 \text{ volts.}$$

At the end of 0.04 second, the current is 3.6 amperes and the induced emf is

$$e = 3.6 \times 2 = 7.2 \text{ volts.}$$

And when the current drops to zero the induced emf has died out entirely.

The growth of current in an inductive circuit is similar to the acceleration of a moving mass, such as a fly wheel on an engine, or an electric train. The train is propelled at constant speed against track and train resistance. At starting, the energy used to overcome train resistance is low and practically all the energy expended by the motors is used in acceleration. As the train nears its normal constant speed, the energy required for acceleration is low and most of the output of the motors is used to overcome train resistance. And when the train reaches normal speed, since there is no energy required for acceleration, **all** the energy of the motors is used in overcoming train resistance.

Similarly, at the instant of closing the switch on an inductive circuit, the $IR$ drop (equation 3) is zero and **all** the impressed voltage, $E$, is used in overcoming the induced voltage, $e$. As the current approaches a steady value, the induced emf, $e$, is low, and **most** of the impressed voltage is used in overcoming the resistance or $IR$ drop in the circuit. And when the current reaches **a** constant value, **all** the impressed voltage is used in overcoming the $IR$ drop, since the induced emf, $e$, is zero.

Again, the dying out of a current in an inductive circuit is similar to the deceleration of the train. When the power is turned

off the motors, the train does not stop immediately, but coasts along until the train resistance finally brings it to a stop. In fact, even by the application of the brakes, the train, due to its inertia or mass, cannot be brought to an immediate stop.

Similarly, the current in an inductive circuit, due to the induced emf of self induction, does not die out immediately upon short-circuit; but persists until the resistance of the circuit finally reduces it to zero.

Applying the brakes to the train in the illustration corresponds to connecting additional resistance in series with the field windings of a generator at the time of short-circuit, as is done by the "field discharge switch" previously described.

And finally, the sudden breaking of a highly inductive circuit, by opening the switch, piles up, or raises, the induced pressure to high values, corresponding to the forces exerted when the train is suddenly stopped by impact, as by a great boulder on the track, or by collision with another train.

Thus, induced electromotive forces of self induction act like the forces set up by the mass, or weight, of a body which oppose any change in its motion. Thus **inductance** in an electric circuit can be likened to **inertia** in mechanical devices.

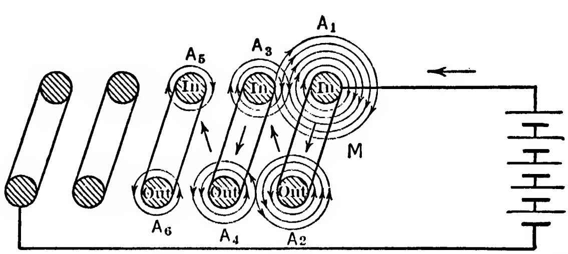



FIG. 10–9. The changing field about one wire in a coil cuts the other wires, and induces in them an opposing voltage.

**6. Cause of Self Inductance, Cutting of Magnetic Lines. Make and Break Spark Coils.** In some cases, as in Art. 3, it is perhaps easier to visualize the inductive effect as a cutting of magnetic lines, rather than as a change in interlinkages.

Consider Fig. 10–9 which represents a vertical cross section along the axis of a coil. A current is made to enter the coil at $A_1$. The magnetic field in growing around $A_1$ spreads out in ever widening rings as the current is increased, and cuts across $A_3$. This is shown in Fig. 11–9. This is equivalent to the wire $A_3$ moving to

the right, as indicated in Fig. 12–9. By applying Fleming's right-hand rule, it is seen that there would be a voltage induced in $A_3$ tending to send a current **out**. But a current **in** is being sent through $A_3$. Thus, this growing current **in** in $A_3$ is opposed by an electromotive force **out** induced in the growing field around $A_1$. Similarly, the current in $A_5$ is opposed by an emf induced by the growing field around $A_3$, and the current in $A_4$ by the growing field around $A_2$, etc.

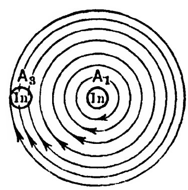



FIG. 11–9. The growing magnetic field about $A_1$ cuts $A_3$ and sets up in it a voltage " out."

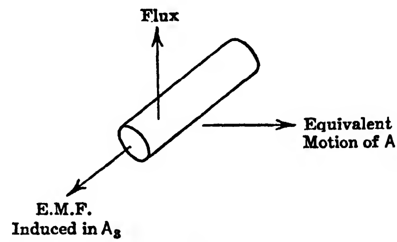



FIG. 12–9. Fleming's right-hand rule applied to the induced voltage in $A_3$ of Fig. 11–9.

Also, if the entire field within the coil is considered, it is seen that an electromotive force is induced in the coil, which tends to set up a current opposing the building up of a field within it. Thus a growing current in such a coil is said to be " choked " back, and takes some measurable length of time to come up to the full value as expressed by Ohm's law.

In a like manner, when the circuit is broken and the field around the wires and within the coil starts to die out, the wires are again cut by the field, though this time in a direction which sets up an emf tending to maintain the field and current as it is.

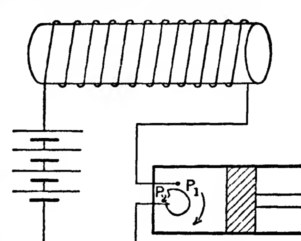



FIG. 13–9. A kick-spark ignition system.

If the magnetic field is sufficiently strong and the current is broken suddenly, the voltage induced by the dying field cutting the wires of the coil may be many times the original impressed voltage. Advantage is taken of this fact in the **Make and Break Spark Coil** used with marine gas engines.

A single coil of many turns, as in Fig. 13–9, is wound on a core of soft iron wires. A moving point, $P_2$, makes contact with a station-

ary point, $P_1$. Both are inside the cylinder of a gas engine. This causes a current to flow through the coil and sets up a strong magnetic field. When the point $P_2$ leaves $P_1$ there is a sudden breaking down of this magnetic field, which sets up enough induced emf to cause a spark to jump across the gap now existing between $P_1$ and $P_2$.

**7. Unit of Self Inductance. The Henry.** The practical unit of self inductance is called the **Henry,** after the American scientist, Joseph Henry. It is represented by the letter $L$.

A circuit is said to have **one henry of self inductance** when a change of current of one ampere sets up $10^8$ interlinkages, or flux cuttings, in the circuit.

This can be represented by the equation

$$L = \frac{\Phi N}{10^8 I} \tag{4}$$

where $L$ = self inductance in henries;
$\Phi$ = flux lines (maxwells).
$N$ = number of turns in the circuit;
$I$ = current in amperes.

Of course, if $10^8$ linkages are set up in one second the induced voltage will be one volt. In fact, a **second definition of a henry** is the inductance of a circuit in which a **change of one ampere per second induces one volt.** From this second definition, it is readily seen that the average induced voltage is equal to the inductance of the coil in henries times the average rate of change of current in the coil. This relation can be expressed by the equation

$$e_{\text{av}} = L\frac{i_1 - i_2}{t_1 - t_2} \tag{5}$$

where $e$ = average induced emf in volts;
$i_1$ = current in amperes at time $t_1$;
$i_2$ = current in amperes at time $t_2$;
$t_1 - t_2$ = time in seconds for the current to change from $i_1$ to $i_2$;
$L$ = self inductance in henries.

From Art. 5, and Fig. 7–9, at the end of 0.02 second, the induced voltage was found to be 12 volts; and at the end of 0.04 second, 7.2 volts. The average induced voltage during this time $\frac{12 + 7.2}{2}$

or 9.6 volts. And during the same period, the current rose from 4 amperes at 0.02 second to 6.4 amperes at 0.04 second. Substituting these values in equation (5) above

$$9.6 = L\frac{4 - 6.4}{0.02 - 0.04} \quad \text{or} \quad L = 0.08 \text{ henries.*}$$

When the inductance of the coil is known, equation (5) can be used to find the average voltage induced during a change of current.

The relation in equation (5) can also be expressed by the equation

$$e_{\text{av}} = L\frac{I}{t} \tag{6}$$

where $I$ = total change in current in time $t$, or $\frac{I}{t}$ = average rate of change of current.

Note that both equations (1) and (6) are expressions for average induced emf. If equation (1) be substituted in (6) above, it is written

$$\frac{\Phi N}{10^8\, t} = L\frac{I}{t}; \quad \text{and} \quad L = \frac{\Phi N}{10^8\, I}$$

which is the original expression for inductance according to definition, equation (4).

**It is important to note that the inductance of a circuit is fundamentally equal to the flux turns, or interlinkages per ampere.**

**Example 2.** A circuit has a self inductance of 2.4 henries. What voltage will be induced in the circuit if the current changes from 12 amperes to 4 amperes in $\frac{1}{10}$ second?

* If the time interval $t_1 - t_2$ used in finding the rate of change of current in the coil is large the value of inductance found will be somewhat inaccurate. The smaller the time interval is made, the more accurate the computed value of inductance becomes. The inductance of a coil can be found exactly from the equation

$$L = \frac{0.4343\, Rt}{\log_{10}\left(\frac{E}{e}\right)}$$

where $L$ = inductance of coil in henries;
$R$ = resistance in ohms;
$E$ = voltage applied to terminals of coil;
$t$ = any length of time after the voltage $E$ is applied, in seconds;
$e$ = induced voltage in coil at time $t$.

**Solution.**

$$e_{av} = 2.4\left(\frac{12 - 4}{0.10}\right) = 192 \text{ volts.} \quad \text{Ans.}$$

**Example 3.** How many flux turns or interlinkages did the change of current produce in the circuit of Example 2?

**Solution.**

$$L = \frac{\Phi N}{10^8\, I}$$

Change of current = 12 − 4 or 8 amperes.

$$2.4 = \frac{\Phi N}{10^8 \times 8}$$

$$\Phi N = 2.4 \times 10^8 \times 8 = 19.2 \times 10^8 \text{ interlinkages.} \quad \text{Ans.}$$

**Prob. 5-9.** The self inductance of the field windings of a generator is 35 henries and the current is 2.5 amperes. If the field switch is suddenly opened so that the current is reduced to zero in $\frac{1}{50}$ of a second, what average voltage is induced in the winding?

**Prob. 6-9.** If the current in Prob. 5 is allowed to die out gradually through resistance inserted across the terminals, what average voltage will be induced in the winding? Time of decay of current is 0.6 of a second.

**Prob. 7-9.** What is the inductance of a coil of 500 turns, if it sets up a flux of 2,000,000 lines when carrying a current of 8 amperes?

**Prob. 8-9.** How many turns are there in the circuit of Example 3, if the flux set up is 1,500,000 lines?

**Prob. 9-9.** In a coil, a current of 50 amperes is reduced to 20 amperes in $\frac{1}{20}$ second and 25 volts are induced. What is the self inductance of the coil?

**Prob. 10-9.** If it is desired to shut off the current in the coils of Prob. 5 and not induce more than an average of 440 volts, what time must the current take to die out?

**8. Computation of Self Inductance.** The flux at the center of a long coil was given in equation (9), Chapter VIII, as

$$\Phi = \frac{1.26\, NIA}{l}.$$

If the coil has an iron core of permeability, $\mu$, the above equation may be written

$$\Phi = \frac{1.26\, N\mu IA}{l} \tag{7}$$

where $\Phi$ = flux linking all the turns, since in such a coil the leakage is negligible;
$N$ = number of turns in the coil;
$I$ = current in the coil in amperes;
$A$ = area of core of coil in square centimeters;
$l$ = length of the core in centimeters.

If now the permeability is assumed constant, the value of the flux, $\Phi$, in equation (7), may be substituted in equation (1) and the average value of the induced electromotive force of self induction may again be written

$$e_{av} = \frac{\Phi N}{10^8 t} = \frac{1.26\ N^2 \mu A}{10^8\ l} \times \frac{I}{t} \tag{8}$$

where $\frac{I}{t}$ = the average rate of change of current. Since the induced electromotive force of self induction equals the self inductance multiplied by the rate of change of current, the inductance may be written

$$L = \frac{1.26\ N^2 \mu A}{10^8\ l} \tag{9}$$

It is to be noted that the emf of self induction, and therefore the inductance, is proportional to **square of the number of turns.** This is so, because the voltage induced per turn is due to the change of flux set up by a given change of current in **all** the turns. Since all the turns are in series the total voltage is proportional to the turns squared.

It is also to be noted that the **inductance depends upon the physical dimensions of the coil** and upon the permeability of the core.*

When the coil has an **iron core,** the flux per ampere, and therefore **the inductance, varies with the permeability.** When **no iron is present,** the flux set up per ampere is constant and **the inductance is also constant** and independent of the value of the current. The inductance of the windings in electrical machinery is also generally **considered constant.** Although these windings have magnetic paths of iron or steel, the maximum flux density at which these paths in any particular machine are worked is quite constant, and therefore the average permeability is constant.

* See Bulletin of U. S. Bureau of Standards, Vol. 8, No. 1.

**Example 4.** What is the inductance of a coil of 400 turns wound on an iron ring 12.75 inches mean diameter and 60 square inches cross-section? Assume average permeability of the core as 3000.

**Solution.**

$$L = \frac{1.26\ N^2\mu A}{10^8\ l}$$

$$\text{Length } l = 12.75 \times 3.1416 \times 2.54 = 102 \text{ centimeters.}$$

$$\text{Area of Core} = 60 \times 2.54^2 = 387 \text{ square centimeters.}$$

$$L = \frac{1.26 \times 400^2 \times 3000 \times 387}{10^8 \times 102} = 23.1 \text{ henries.} \quad \text{Ans.}$$

**Example 5.** Assume the current in Example 4 to change from 1.5 amperes to 0.1 ampere in 0.02 second. What average voltage would be induced?

**Solution.**

$$e_{av} = L\frac{i_1 - i_2}{t}$$

$$= 23.1\,\frac{1.5 - 0.1}{0.02} = 1620 \text{ volts.} \quad \text{Ans.}$$

**Example 6.** What change in flux would take place in the core of Example 4, under the conditions of Example 5?

**Solution.**

$$L = \frac{\Phi N}{10^8\ I}$$

$$23.1 = \frac{\Phi \times 400}{10^8 \times 1.4}$$

$$\Phi = \frac{23.1 \times 10^8 \times 1.4}{400} = 8.1 \times 10^6 \text{ lines.} \quad \text{Ans.}$$

**Prob. 11–9.** What is the inductance of a coil of a transformer having 300 turns, if the iron core is 30 inches long and 50 square inches in cross-section? Assume the average permeability of the core to be 4000.

**Prob. 12–9.** If the core in Prob. 11–9 had been made of wood, what would the inductance of the coil have been?

**Prob. 13–9.** If the change in Example 5 had taken place in $\frac{1}{120}$ of a second, what average voltage would have been induced?

**Prob. 14–9.** A coil of 600 turns is wound on a wrought-iron ring, the mean diameter of which is 15 inches and cross-section area 20 square inches. What is the inductance of the coil, if the average permeability is 1500?

**Prob. 15–9.** A current of 2.5 amperes is flowing in the coil of Prob. 14. If the current is reduced to 0.15 amperes, the decrease taking place in $\frac{1}{75}$ second, what average voltage will be induced?

**Prob. 16–9.** What is the flux set up in the core of Prob. 14–9 by the maximum current in Prob. 15–9? By the minimum current in Prob. 15–9? What is the change in flux set up by the change in current in Prob. 15–9? Permeability is assumed constant.

**Prob. 17–9.** A " choke " coil is formed by winding 200 turns on a ring 25 inches mean diameter, consisting of annealed sheet steel. Permeability is 4000 and cross-section area of the core is 200 square inches. What is the inductance of the coil?

**9. Energy Stored in the Magnetic Field.** When current is established in a coil of wire, as in Fig. 7–9, thereby setting up a magnetic field, an expenditure of energy is required.

In Art. 5, it was shown that the impressed voltage, $E$, fulfilled two functions; one, that of overcoming the resistance or $IR$ drop in the coil, and the other, that of overcoming the induced electromotive force, $e$, as indicated in equation (3).

At an instant $t_1$, when the current has risen to the momentary value of $i_1$ amperes, the rate of expenditure of energy in the circuit can be written

$$P = Ei_1 = i_1^2R + ei_1; \tag{10}$$

and it is seen that part of the power expended is used in $i_1^2R$, or heat losses in the coil; and part is used in forcing the current through the circuit against the electromotive force of self induction.

If the time $t_1$ be considered as very small, say $\frac{1}{1000}$ of a second, the current $i_1$ is exceedingly small and is changing at a rapid and almost constant rate. Therefore, the $i_1^2R$ loss in equation (10) can be neglected, and the induced electromotive force, $e$, is practically equal to the impressed voltage, $E$. and the equation may be written

$$P = Ei_1 = ei_1.$$

Hence the **energy** expended during the time $t_1$ may be written

$$W = Pt_1 = Ei_1t_1 = ei_1t_1. \tag{11}$$

But since the current rises from 0 to $i_1$ in time $t_1$, the average current during this time $t_1$ equals $i_1/2$; and the induced voltage, $e = Li_1/t_1$, from equation (6).

Substituting these values above in equation (11), the energy stored in the magnetic field for a current $i_1$ may be written

$$W = e\frac{i_1}{2}t_1 = L\frac{i_1}{t_1} \times \frac{i_1}{2}t_1;$$

or
$$W = L\frac{i_1^2}{2} \tag{12}$$

Thus the amount of energy stored in the magnetic field for any value of current $I$, may be written

$$W = L\frac{I^2}{2} \tag{13}$$

where $W$ = energy in watt-seconds or joules;
$L$ = inductance in henries;
$I$ = current flowing in amperes.

When the current reaches its steady or final value no energy is being expended in the field, and all the energy being expended may be accounted for as $I^2R$ loss in the electric circuit.

Again, when the coil is short-circuited, as in Fig. 8–9, the dying current flows as a result of the induced electromotive force, and the same energy is restored to the electric circuit by the decreasing magnetic field.

The relations discussed above have their counterpart in mechanics. Potential energy is stored in a magnetic field and is similar to the potential energy stored in a raised weight. If a weight $W$ is raised $n$ feet by means of a rope and pulley, $Wn$ foot-pounds of energy is stored in the weight. No energy is required to hold it in this position. But the energy so stored can be applied for use in various ways. Similarly, the energy stored in the magnetic field of an induction coil can be used, for instance, to give a spark to ignite the charge in a gas engine.

**10. Time Constant.** It is difficult to determine the exact time required for the current in an inductive circuit to reach a steady value; theoretically, this does not occur in a finite period of time.

The ratio of the inductance in henries to the resistance in ohms, or $L/R$, is called the time constant of the circuit. This happens to be the exact time in seconds required for the current to rise to 63 per cent of its steady or constant value. No particular importance is attached to this percentage, but the ratio of $L/R$ can be said to be a measure of the rapidity with which a current rises to its final value.

For the same impressed voltage on circuits of the same resistance, the larger the inductance, the longer it will take for the current to rise to its final value.

Thus in Fig. 7–9, the curve rises to 63 per cent of its final value,

or 6.3 amperes, in 0.039 second. The inductance of the coil then is

$$0.039 = \frac{L}{2} \quad \text{or} \quad L = 0.078 \text{ henries.}$$

In Fig. 6–9 the current rises instantly to 63 per cent of its final value and the inductance is zero.

**11. Mutual Inductance.** Consider two coils of wire, shown for simplicity of one turn each, as in Fig. 14–9. If the switch $S$ is closed, an increasing current is made to flow in coil $A$, and some of the flux so set up will thread coil $B$ and establish flux linkages with it. As long as these linkages are changing, a voltage will be set up, not only in coil $A$, in opposition to the growing current in that coil, as already discussed, but also in coil $B$. The induced voltage in coil $B$ will be in the same direction as that in coil $A$, as indicated by the arrow on coil $B$, since the change in linkages is in the same direction. Hence the current in coil $B$, if it be closed on itself, will be in the opposite direction to that in coil $A$, and will set up a flux or mmf in opposition to that of coil $A$. And again it is noted that an induced voltage sets up a magnetic field which opposes, or tends to destroy, the action which produced it.

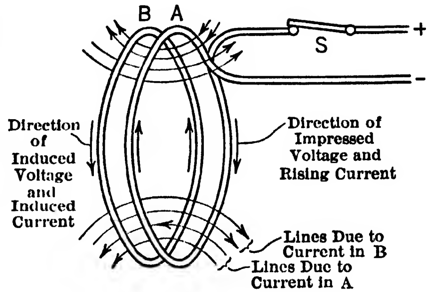



FIG. 14–9. When the switch, S, is closed, a growing current in coil A sets up a changing flux which links with the circuit of coil B, and induces voltages in both circuits in the direction of the arrows (shown on B). If circuit B is closed, a current flows in this circuit in the same direction as the induced voltage.

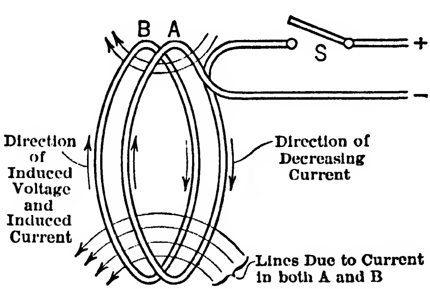



FIG. 15–9. When the switch, S, is opened, the decreasing current in circuit A sets up a voltage in both circuits which is opposite to that in Fig. 14–9, as shown by the arrows. A current will also flow in circuit B, if it is closed, in the direction of the induced voltage so long as the linkages are changing.

If the switch in Fig. 15–9 is opened and the current in coil $A$ is

allowed to die down, the decreasing linkages will set up a voltage in both coils in the opposite direction, as shown by the arrows; and the induced current in coil $B$ will now tend to set up a field in the **same** direction as that due to the decreasing current in coil $A$. Again it is noted that this induced field opposes the action which is taking place.

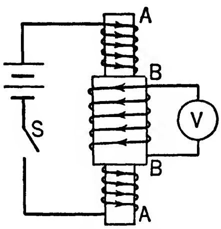



FIG. 16–9. A change of current in coil A induces a voltage in coil B.

If coil $B$ is open-circuited, no current can flow through it, and it therefore will set up no mmf, although a voltage will always be induced in the coil whenever the current in coil $A$ is changing. The amount of the voltage induced in the secondary coil, coil $B$, will depend upon how much of the flux from the primary coil, coil $A$, links the secondary and the relative number of turns in the two coils. It also depends upon the position and shapes of the two coils. The further coil $B$ is set from coil $A$ the less will be the mutual flux and the voltage induced in coil $B$. If the axes of the two coils are set at right angles, no flux from one will thread the other and no voltage will be induced in coil $B$.

Therefore, if two coils, having no electrical connection with each other, are so placed that the magnetic field set up by the current in one establishes a flux linking the other, any change in the value of current in the one will induce a voltage in the other. This effect is known as **Mutual Induction,** and the induced voltage in the second coil is called the **electromotive force of mutual induction.** The average value of this emf can be expressed by the equation

$$e_{2(\text{av})} = \frac{\Phi_m N_2}{10^8\, t} \tag{14}$$

where $e_{2(\text{av})}$ = average induced voltage in the secondary coil in volts;
$\Phi_m$ = mutual flux set up, linking both coils;
$N_2$ = number of turns in the secondary coil;
$t$ = time in seconds required to set up the flux $\Phi_m$.

**12. Mutual Inductance. Cutting of Magnetic Lines. Induction Coils.** Fig. 16–9 shows two coils $A$ and $B$ of insulated wire. Coil $B$ is wound over coil $A$ on the same core, but has no electrical connection with it. Fig. 17–9 represents a vertical cross-section

along the axis of the two coils. Assume the current to enter coil $A$ as indicated, and to be in a direction **in** at the top of the loops $A$, and **out** at the bottom of the loops $A_1$. This causes a clockwise field to grow around the top wires of coil $A$, and a counter-clockwise field around the bottom wires. This field spreads out in ever

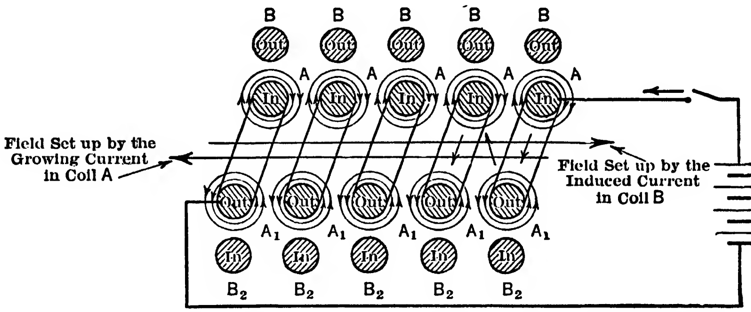



FIG. 17–9. A growing magnetic field around the wires of coil A cuts the wires of coil B.

widening rings and cuts across the wires of coil $B$, as indicated in Fig. 18–9, which shows only the top wires of both coils in Fig. 17–9. This is equivalent to the wires of $B$ (Fig. 18–9) moving downward and cutting the lines of coil $A$. This, as already described in Art. 6, induces a voltage in wires $B$, **out**. This is in the **opposite direction to the inducing current in** $A$. By applying the same reasoning to the bottom wires of the two coils, it is seen that a growing current **out** in the wires $A$, Fig. 17–9, induces a voltage **in** in wires $B$. Here again it is seen that the induced voltage in all the turns in coil $B$ is **opposite** in direction to the increasing current in coil $A$.

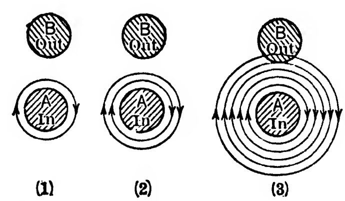



FIG. 18–9. How the growing magnetic field about A cuts the wires of B.

By employing the right-hand rule for determining the relative directions of the current in a coil and the magnetic field inside the coil, to Fig. 17–9, it is seen that the current in coil $A$ sets up a mmf or a field in direction from **right to left**, and in coil $B$ from **left to right.** That is, the magnetic field in the core due to the current in coil $B$, is opposed to the building up of a field due to the current in coil $A$.

As soon as the current in $A$ reaches its normal value, the lines of force around the wires no longer will be spreading out. There will be no lines cutting the turns of $B$ and the induced voltage and current in $B$ will die out. The induced current lasts **only as long as the primary current is growing.**

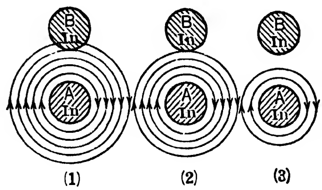


FIG. 19–9. How a dying magnetic field about A cuts the wires of B.

Thus in the case of the two coils there is opposition to the **building up** of a field by the current in coil $A$, due to its **self inductance,** as described in previous paragraphs, and also the opposition offered by the field set up by the current in coil $B$, due to **mutual inductance.**

In the case of a **decrease** of current in coil $A$, Fig. 17–9, the magnetic lines around the wires of coil $A$ contract and cut the wires of coil $B$ in the **opposite** direction; and set up a voltage and a current in $B$ in the **opposite** direction, as shown in Fig. 19–9. This, then, sets up a field in coil $B$ in the **same** direction as that in $A$, which opposes the decreasing of the magnetic field due to the dying current in coil $A$.

Thus the induced voltages in coil $B$ tend to set up currents in this coil, which **oppose** any **increase** or **decrease** in the existing magnetic field set up by a changing current in coil $A$.

**13. Induction Coils: Jump Spark.** One type of induction coil, often used for igniting the charge in a gas engine, depends upon this induced current in the secondary coil.

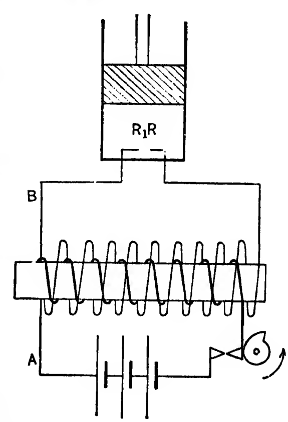


FIG. 20–9. A simple diagram of an ignition system.

The primary has comparatively few turns of heavy wire; the secondary has many turns of fine wire. A core of soft iron is put within the coils to strengthen the magnetic field. The secondary and the primary windings are so close together that practically all the flux set up by the primary cuts the secondary. The secondary circuit $B$, Fig. 20–9, has an air gap in it. This gap is between two points ($R$ and $R_1$) which are inside the cylinder of the gas engine. The primary circuit $A$ is connected to a set of battery cells. The cam first closes the

primary circuit and causes a field to be set up in the coil. Then, as the cam opens the primary circuit, the field is suddenly diminished as the current in the primary dies out. This causes a high induced emf to be set up in the secondary, because the field on dying out cuts the large number of turns in the secondary at a rapid rate. This induced emf is high enough to cause a spark to jump between the points $R$ and $R_1$, and ignite the charge of gas in the cylinder. There is a small spark when the circuit in the primary is made, because the **growing** field cuts the wires of the secondary coil. But the field **grows** more slowly than it **dies out** and thus the induced emf is not as great and the spark is much more feeble.

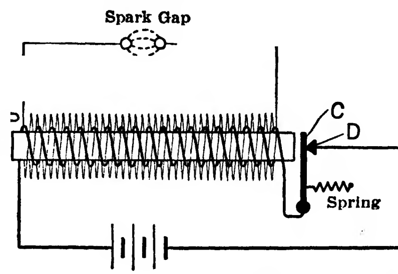


FIG. 21–9. A Ruhmkorff coil. The vibrating arm, C, sets up a high-voltage, high-frequency arc across the spark gap.

**14. Ruhmkorff Coils.** A Ruhmkorff coil is built on the same plan as the jump spark coil. The number of turns in the secondary is many times the number in the primary, and the voltage across the secondary is many times that across the primary. The voltage of the secondary will be in about the same ratio to the voltage of the primary as the number of turns in the secondary is to the number of turns in the primary. Of course what is gained in voltage is lost in amperage.

Fig. 21–9 shows a Ruhmkorff coil arranged with a magnetic interrupter. Current is supplied to the primary coil by the battery, through the movable arm $C$ in contact with the point $D$. The magnetic field set up pulls the arm against the core, breaking the circuit. This weakens the field and the arm is pulled back by the spring into contact with $D$, and the primary circuit is again established. By this means, a series of rapid interruptions is made in the primary circuit which causes a continuous discharge of high-frequency, high-voltage sparks across the gap in the secondary.

**15. Transformers** are mutual induction coils used on **alternating current** lines for raising or lowering the voltage. Each transformer consists of a primary coil and a secondary coil, both wound on a core of soft iron or annealed steel. See Fig. 22–9.

The action of a transformer follows the same principle as the

action of a jump spark coil, except that the current in the primary coil, instead of being periodically **broken,** is periodically **reversed** by being connected to a source of alternating current.

The current in the secondary coil thus opposes not only the dying out of a magnetic field in the core, but also the actual setting up of a field in the opposite direction. Since the current in the primary coil alternates in direction, the field in the core continually changes from a maximum in one direction to a maximum in the other; and there is thus induced in the secondary coil an alternating emf which tends to oppose the **continuous change,** according to Lenz's law.

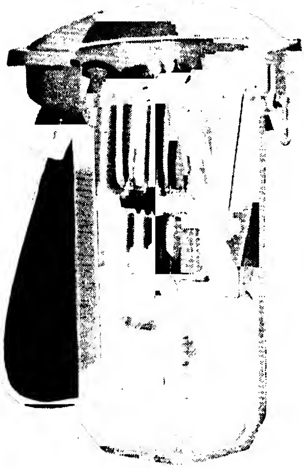


FIG. 22-9. A small distribution transformer.

**16. Computation of Mutual Inductance.** As has been stated, when two circuits are so situated that a change of current in one circuit causes a flux to thread through, or link, or cut the other circuit, the two circuits are said to possess **mutual inductance.**

The unit of mutual inductance is also the henry, and is indicated by the letter $M$.

When a change of one ampere in one circuit causes $10^8$ interlinkages or cuttings of flux in the other circuit, the mutual inductance of the two circuits is said to be **one henry.** This may also be expressed:

"When a change of **one ampere per second** in one circuit induces **one volt** in the other, the two circuits have a mutual inductance of **one henry.**" Both expressions have the same meaning.

The flux set up in the primary circuit can be written (from equation 7):

$$\Phi_1 = \frac{1.26\, N_1 I_1 \mu A}{l} \tag{15}$$

Now if it be assumed that two coils are wound interlaced on the same iron core so that all the flux set up in the primary circuit interlinks the other circuit, the value of $\Phi_1$ in equation (15) can be substituted for $\Phi_m$ in equation (14); and the average value of the

induced electromotive force of mutual induction can be written

$$e_{2(\mathrm{av})} = \frac{1.26\, N_1 N_2 \mu A}{10^8\, l} \times \frac{I_1}{t} \tag{16}$$

From the definitions above, it is seen that the first term in equation (16) is the coefficient of mutual inductance, or

$$M = \frac{1.26\, N_1 N_2 \mu A}{10^8\, l} \tag{17}$$

where $M$ = mutual inductance of the coils in henries;
$N_1$ = number of turns in the first, or primary coil;
$N_2$ = number of turns in the secondary coil;
$\mu$ = permeability of the core;
$A$ = area of the core in square centimeters:
$l$ = length of the core in centimeters.

Note that the equation is the same as that for the **self inductance** of the two coils wound as one on a core, **except** that instead of squaring the total number of turns, the number of turns in one coil is multiplied by the number of turns in the other.

**Prob. 18-9.** What is the mutual inductance of two coils wound on a circular wooden ring 12 square inches in cross-section, and having an average length of 50 inches? One coil contains 300 turns, and the other 600 turns.

**Prob. 19-9.** If the core of Prob. 18-9 were of sheet steel ($\mu = 3600$), what would the mutual inductance be?

**Prob. 20-9.** (*a*) If the current in the coil containing 300 turns of Prob. 18-9 were changed at the rate of 100 amperes per second, what voltage would be induced in the other coil? (*b*) Answer the same question for the coils in Prob. 19-9.

**Prob. 21-9.** If the current in the coil containing 600 turns, Prob. 19-9, is changed at the rate of 100 amperes per second, what voltage will be induced in the other coil?

**17. Coupled Circuits.** Circuits which possess mutual inductance are sometimes said, especially in radio work, to be coupled. If all the flux set up by one circuit interlinks all the turns of the second circuit, as assumed in the preceding article, the circuits are said to have **unity coefficient of coupling.**

In this case the self inductance of the first coil is

$$L_1 = \frac{1.26\, N_1^2 \mu A}{10^8\, l}.$$

The self inductance of the second circuit is

$$L_2 = \frac{1.26\, N_2^2 \mu A}{10^8\, l}$$

and the mutual inductance,

$$M = \frac{1.26\, N_1 N_2 \mu A}{10^8\, l}$$

Note that

$$M = \sqrt{L_1 \times L_2}$$

or

$$\frac{M}{\sqrt{L_1 \times L_2}} = 1. \tag{18}$$

The expression $\frac{M}{\sqrt{L_1 \times L_2}}$ is called the coefficient of coupling. However, it is **never quite equal to unity,** as indicated in equation (18), since actually no two coils can be so placed that all the flux set up by one coil links the other. In other words, the mutual flux, $\Phi_m$, in equation (14) is never quite equal to the flux of the primary coil $\Phi_1$ in equation (15).

Thus the expression for mutual inductance (equation 17) can be written more accurately as:

$$M = \frac{1.26\, N_1 N_2 K \mu A}{10^8\, l} \tag{19}$$

where $K$ is the coefficient of coupling.

In power transformers, where the very best possible coupling is often desirable, a coefficient of coupling of 0.98 or 0.99 can be obtained. And the coils are said to be closely coupled.

In radio transformers a coefficient of coupling of less than 0.50 is usually desirable for sharp tuning. Such a transformer is said to couple loosely the two circuits connected to its terminals. Fig. 23–9 shows a "vario-coupler" by which the coefficient of coupling can be varied by changing the position of the inner coil.

**Prob. 22–9.** Two coils have self inductances of 3.25 and 2.05 henries respectively. Their mutual inductance is 1.95 henries. What is their coefficient of coupling?

**Prob. 23–9.** Two coils have a coefficient of coupling of 0.82. The self inductance of the first is 1.82 henries, and of the second is 2.14. What is their mutual inductance?

**Prob. 24–9.** Two coils are wound on a paper tube 8 inches long and $\frac{3}{4}$ inch in diameter. The mutual inductance is 0.02 henry, and the coefficient of coupling is 0.92. If there are 2000 turns in the first coil, how many turns are there in the other?

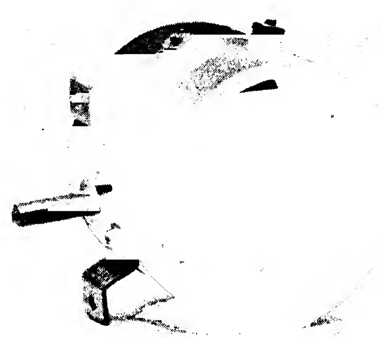


FIG. 23–9. A vario-coupler, or variometer. By varying the position of the inner coil the coefficient of coupling, or the mutual inductance, of the two coils can be changed. *General Radio Co.*

## SUMMARY OF CHAPTER IX

**LINKAGES.** In a coil carrying a current, the linkages are the product obtained by multiplying the flux in LINES threading through the coil by the number of TURNS in the coil. The linkages are also called the flux turns.

**INDUCED ELECTROMOTIVE FORCE.** If the linkages in an electric circuit, such as a coil, are changed, an emf will be induced in the coil DURING THE TIME the linkages are changing. This change may be made, either by moving an external magnetic field into a stationary coil, or by moving the coil through a stationary magnetic field. In the latter case it is convenient to visualize the sides of the coil as cutting the lines of the magnetic field.

When there is a change of $10^8$ linkages per second, or a cutting of $10^8$ magnetic lines per second in a circuit, one volt will be induced. The average value of this induced voltage may be expressed as

$$e_{av} = \frac{\Phi N}{10^8\, t}.$$

**DIRECTION OF INDUCED VOLTAGE: LENZ'S LAW.** A voltage induced by a changing magnetic field is in such a direction that it tends to set up a current and a resulting magnetic field, which is always in a direction to oppose the change in the existing field, i.e., to oppose the action which produces it.

SELF INDUCTANCE: EMF OF SELF INDUCTION. When a voltage is impressed on a coil of wire, the current does not rise immediately to the value determined by Ohm's law, because the changing magnetic field induces a voltage in the coil which opposes the rise of current. And when the circuit is broken, the dying current also induces a voltage which is in a direction to oppose the decrease in current. This property in a single circuit, which opposes any change in the value of the current and the magnetic field, is called the SELF INDUCTANCE of the circuit. The voltage induced by the changing magnetic field is called the ELECTROMOTIVE FORCE OF SELF INDUCTION.

INDUCTANCE in a circuit makes itself known only when the current is changing in value. It may be likened to inertia in mechanical devices, such as the moving mass of a train. The inertia reaction of this mass, or weight, opposes the acceleration of the train; and also opposes the deceleration or stopping of the train. It does not make itself felt while the train is traveling at a constant speed.

When a change of current in one circuit sets up linkages and an induced voltage in another entirely different circuit, the two circuits are said to possess MUTUAL INDUCTANCE.

HENRY: UNIT OF INDUCTANCE. When a change of current of one ampere per second causes a change of $10^8$ linkages, or flux cuttings, per second, i.e., sets up an induced emf of one volt in the same circuit, the circuit is said to possess a Self Inductance of one henry. When a change of current of one ampere per second in one circuit sets up one volt in another circuit, the two circuits are said to possess one henry of Mutual Inductance. The symbol for Coefficient of Self Inductance is L: for Mutual Inductance, M.

EQUATIONS FOR INDUCTANCE FOR COILS.

$$\text{Self Inductance, } L = \frac{1.26\, N^2 \mu A}{10^8\, l}$$

$$\text{Mutual Inductance, } M = \frac{1.26\, N_1 N_2 \mu A}{10^8\, l}$$

Inductance of coils depends upon their physical dimensions and the permeability of their magnetic circuits.

ENERGY IS STORED IN THE MAGNETIC FIELD as a current rises in a coil. This is potential energy and is comparable to that stored in a raised weight.

It is expressed by the equation

$$W = L\frac{I^2}{2}$$

The TIME CONSTANT of a circuit is the ratio of the Inductance in Henries to the Resistance in Ohms, or L/R. The ratio gives the time required for the current to rise to 63 per cent of its final value.

Circuits possessing MUTUAL INDUCTANCE are said to be COUPLED. They are CLOSELY COUPLED when a large fraction of

**the flux set up by one circuit threads through all the turns of the second circuit, and LOOSELY COUPLED when this fraction is small.**

**This fraction is called the COEFFICIENT OF COUPLING and equals** $\frac{M}{\sqrt{L_1 \times L_2}}$. **In radio, many coupled circuits are loosely coupled.**

**TRANSFORMERS consist of primary and secondary coils wound on a core of laminated annealed steel. An alternating current in one coil causes a continual change in the magnetic field. This induces an emf in the other coil. The ratio of the emfs across the two coils approximately equals the ratio of the number of turns in the two coils. Transformers can therefore be used to raise or lower the voltage in an alternating-current circuit. Efficiency, very high.**

## PROBLEMS ON CHAPTER IX

**Prob. 25–9.** A coil of 100 turns is linked with 100,000 magnetic lines. What average voltage is induced if the flux is reduced to zero in 0.015 second?

**Prob. 26–9.** When the flux in a certain coil is reduced from 60,000 lines to zero in 0.008 second, the average induced voltage is 15 volts. How many turns are there in the coil?

**Prob. 27–9.** If the current in the coil of Prob. 26–9 was 2.73 amperes, what is the inductance of the coil?

**Prob. 28–9.** The current in a circuit is changed from 0 to 80 amperes in 0.012 second. The average voltage induced is 160 volts. What is the inductance of the circuit?

**Prob. 29–9.** How many linkages are set up in the coil of Prob. 28–9?

**Prob. 30–9.** What average voltage is required to **reverse** the current of 80 amperes in 0.012 second in the circuit of Prob. 28–9?

**Prob. 31–9.** Find the inductance of a coil 14 inches long, 8 inches in diameter, having 200 turns of wire, (*a*) with air core; (*b*) with an iron core of 4000 permeability.

**Prob. 32–9.** A concentrated coil of 300 turns of small wire (to cut down the leakage) has an inductance of 0.0225 henry. The number of turns is increased to 600. What will be the approximate **change** in inductance?

**Prob. 33–9.** The inductance of the field winding of a small generator is 17 henries, and the resistance is 92 ohms. How long does it take for the current to reach 63 per cent of its final value?

**Prob. 34–9.** The inductance of the field winding of a generator is 42 henries. The current in the winding is changed from 2.1 amperes to 0.85 ampere in 0.025 second. What average voltage is induced?

**Prob. 35–9.** When the current in the field winding of Prob. 34–9 is 2.1 amperes, how much energy is stored in the magnetic field (*a*) in watt-seconds? (*b*) in foot-pounds?

**Prob. 36–9.** An iron ring ($\mu = 2000$) has a cross-section of 40 square inches, and an average length of 60 inches. How many turns of wire must be wound on it to produce an inductance of 12 henries?

**Prob. 37–9.** When 500 turns of wire are wound on an iron ring of 20 square inches cross-section and average length of 75 inches, the inductance is 1.2 henries. What value does this give for the permeability of the iron?

**Prob. 38–9.** If 1000 turns were wound on the ring of Prob. 37–9, what would the inductance be?

**Prob. 39–9.** On a steel ring of 4.00 inches mean diameter and 0.68 square inch cross-section, are wound two coils, one having 250 turns. The permeability of the steel is 3500. The coefficient of coupling is 0.85 and the mutual inductance is 0.50 henries. How many turns are there on the second coil?

**Prob. 40–9.** What is the self inductance of each of the two coils of Prob. 39–9?

**Prob. 41–9.** Find the self inductance of the primary coil of a transformer having 1000 turns. Length of magnetic circuit is 50 inches, area 300 square inches. Permeability of the iron is 2500.

**Prob. 42–9.** If the secondary coil of the transformer of Prob. 41–9 has 100 turns, what is the self inductance of its circuit?

**Prob. 43–9.** If the coefficient of coupling of the coils in Probs. 41–9 and 42–9 is 0.96, what is the mutual inductance?

**Prob. 44–9.** A transformer core of annealed sheet steel is 40 inches long and 120 square inches cross-section area. The primary coil has 2200 turns of wire and the secondary, 220 turns. Permeability of the core is 2100. What is the mutual inductance if the coefficient of coupling is 0.98?

# CHAPTER X

## GENERATORS

**1. The Dynamo** is a machine which may be used either to convert mechanical energy into electrical energy, or to convert electrical energy into mechanical energy. When the dynamo **is driven** by a source of mechanical power, such as a steam engine, turbine

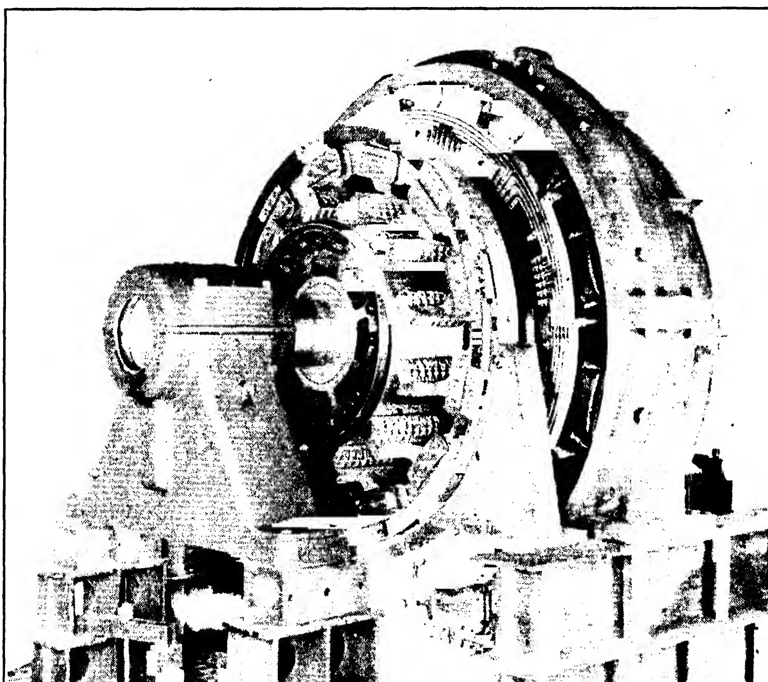


FIG. 1–10. A direct-current generator. *Crocker-Wheeler Electric Mfg. Co.*

or water-wheel, to supply an electric current, it is called a generator. When it is used as a source of mechanical power **to drive** machinery, pumps, blowers, cars etc., it is called a motor. The same machine may be used interchangeably, either as a generator, or as a motor. However, in practice there are many differences in the detail of design, and practically all commercial machines are

built and "rated" specifically for one class of service. Direct-current generators are rated as to their voltage, kilowatts they can deliver without overheating and the speed at which they are to run. Motors are rated as to their voltage, the horsepower they can deliver without overheating, and their speed. In this chapter, we will consider the dynamo used as a generator. Figure 1–10 illustrates a direct-current dynamo built for use as a generator.

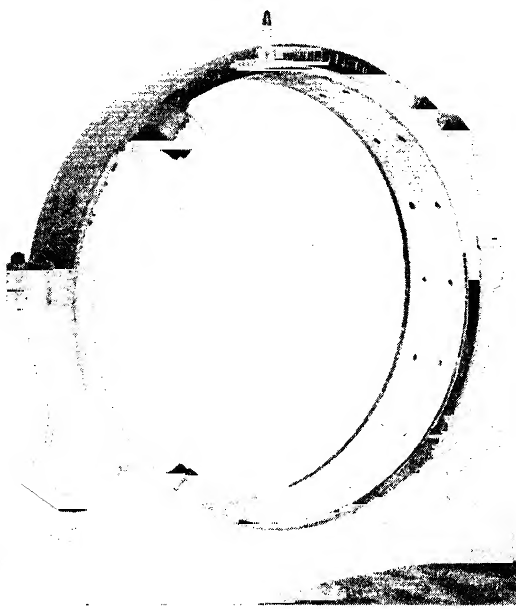


FIG. 2–10. The field ring of a direct-current generator. *The Crocker-Wheeler Electric Mfg. Co.*

The essential parts of a direct-current dynamo are discussed briefly below.

**The Field Frame** consists of the poles and yoke and carries the magnetic flux. The yoke, or field ring, shown in Fig. 2–10, is generally of soft cast steel, while the poles are generally of "laminated" sheet steel bolted to the field ring.

**The Field Coils** furnish the magnetomotive-force which sets up the flux in the magnetic circuit. They are mounted on the poles. Figure 3–10 shows a field frame with the field coils in place, Fig. 4–10, a field coil, and Fig. 5–10, a laminated pole piece.

**The Armature Core** is part of the magnetic path through the machine. It is generally constructed of laminated annealed sheet steel punchings. Silicon sheet steel is also often used.

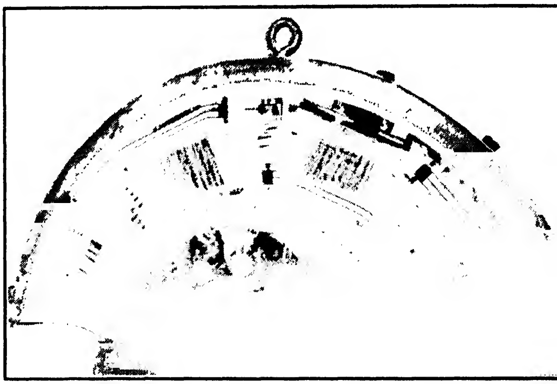


FIG. 3–10. A close-up of part of the same frame showing details of the field windings in place. *Crocker-Wheeler Electric Mfg. Co.*

**The Armature Winding** consists of insulated wire mounted on the armature core, and comprises the " conductors " in which the emf is generated (induced) in a generator, and in which the torque is developed in a motor. The conductors are usually placed in longitudinal slots in the surface of the armature core, and the winding is called a " drum " winding. Note Fig. 6–10.

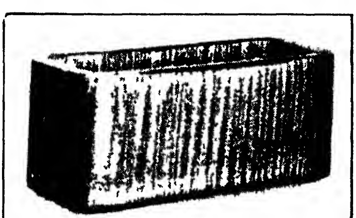


FIG. 4–10. A field coil of the machine of Fig. 3–10. *Crocker-Wheeler Electric Mfg. Co.*

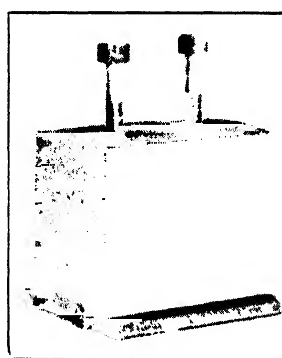


FIG. 5–10. A laminated main pole of the above machine. *Crocker-Wheeler Electric Mfg. Co.*

**The Commutator** rectifies the alternating emf generated in the armature " conductors " and connects them with the brushes.

The armature core and winding, together with the commutator,

are mounted on the shaft and comprise the rotating part of the machine.

**The Brushes** make a rubbing contact on the commutator and

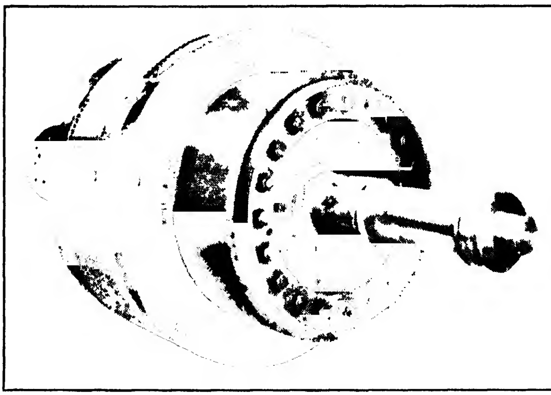


Fig. 6–10. A direct-current armature *Westinghouse Electric and Mfg. Co.*

conduct the current to, and from, the outside circuit. Each brush is supported and held against the commutator by a **brush holder,** which, in turn, is clamped to a **brush stud** insulated from, and supported by, the frame. The brushes, holders and studs are

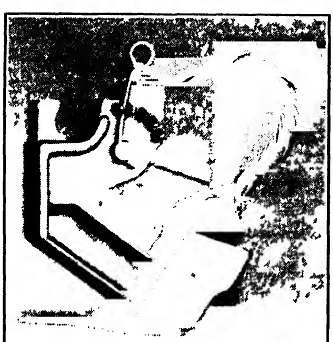


Fig. 7–10. Brush and brush holder for a direct-current dynamo. *Westinghouse Electric and Mfg. Co.*

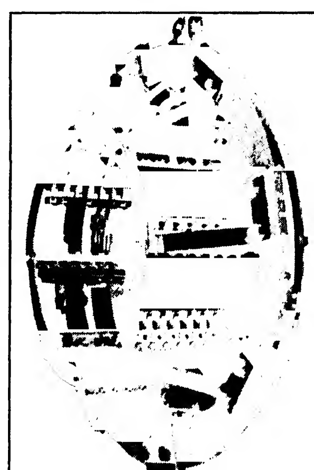


Fig. 8–10 Brush rigging for a direct-current dynamo. *Westinghouse Electric and Mfg. Co.*

stationary and are called the **brush rigging.** Note Figs. 7–10 and 8–10.

**2. Electromagnetic Induction.** In Chapter IX, Arts. 1 and 2, it was shown that if the flux linkages in a coil are changing, or if the sides of the coil, called " conductors," are moved across the flux of a magnetic field, a voltage will be " induced " in these conductors. This is in accordance with the " **Law of Electromagnetic Induction.**"

In fact, if a single wire is moved across the lines of a magnetic field, a voltage will be " induced " in the wire. For instance, if a wire is merely swung in the earth's magnetic field in such a way as to cut across the lines, we find that an emf is " induced " in the wire. This is shown by placing a galvanometer across the ends of the wire. The needle deflects every time the wire is swung. As long as the wire remains stationary, no voltage is set up. However, even if the wire moves, but in a direction parallel to the lines of force so that no lines are cut, no voltage is set up. This shows that if the conductor **moves and cuts flux,** a voltage will be induced in it.

**3. Direction of Induced EMF.** It was also shown in Arts. 2 and 3, Chapter IX, that an induced voltage sets up a current in a closed circuit, which in turn sets up a magnetic field which is always in a direction to oppose the action that produced it. Therefore, in a single wire or " conductor " moving in a magnetic field, there must be a definite relation between the direction of the inducing flux, the direction of movement of the conductor and the direction of the induced voltage.

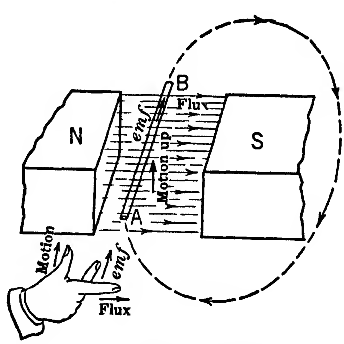



FIG. 9–10. Relation between direction of flux, direction of motion of a conductor and direction of induced voltage.

Consider Fig. 9–10. If the wire $AB$ is moved **up** across the lines of force as marked, a voltage will be set up in the wire, in the direction from $A$ to $B$. If the wire is moved **down** across the lines, the voltage will be set up in the opposite direction, from $B$ to $A$. If the direction of the flux, or magnetic field, were reversed, the voltage would then be set up in the reversed direction for the same motion of the conductor. These relations can be determined experimentally.

From the above effect, Fleming stated the relation between direction of flux, direction of motion and direction of induced voltage, known as Fleming's Right-Hand Rule. This rule has already been stated in Chapter IX, but is restated below:

**Extend the THUMB, FOREFINGER and MIDDLE FINGER of the RIGHT hand at right angles to one another. Let the THUMB point in the direction of the motion, the FOREFINGER in the direction of the magnetic flux, then the MIDDLE FINGER will be pointing in the direction of the induced electromotive force. As an aid to the memory, note that the initial letters of forefinger and flux are the same, and those of Middle Finger and (electro) Motive Force are alike.**

The hand in Fig. 9–10 shows the application of this rule to the case of the wire being moved UP.

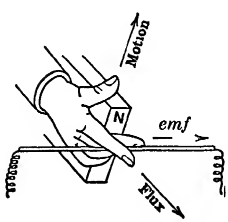


FIG. 10–10. An illustration of the "Right Hand Rule" for induced voltage.

Figure 10–10 also brings out this relation clearly.

**4. Amount of Induced EMF.** The amount of the induced electromotive force depends upon the number of magnetic lines cut per second; that is, upon the **rate of cutting flux.** In calculating the voltage generated, we start from the fundamental rule that if a wire (conductor) cuts across a magnetic field at the rate of 100,000,000 ($10^8$) lines per second, one volt will be generated. If two conductors are properly connected so that their electromotive forces add, two volts will be generated etc. This relation can be expressed by equation (1), Chapter IX.

$$E = \frac{\Phi N}{10^8 t}$$

where $E$ = the average electromotive force set up,
$\Phi$ = flux cut,
$N$ = number of conductors in series,
and $t$ = time of cutting in seconds.

**Example 1.** A wire passes at a constant velocity across the face of a field magnet, the flux density of which is 15,000 lines per square inch, in $\frac{1}{40}$ second. The dimensions of the face are 30 × 20 inches. What electromotive force is induced in the wire?

**Solution.**

$$\text{Flux } \Phi = 30 \times 20 \times 15{,}000 = 9{,}000{,}000 = 9 \times 10^6$$

$$\text{Lines cut per second} = 9 \times 10^6 \times 40 = 3.6 \times 10^8$$

$$\text{Generated emf} = \frac{3.6 \times 10^8}{10^8} = 3.6 \text{ volts.}$$

**Prob. 1–10.** How many volts (average) are induced in a wire that cuts $3 \times 10^7$ lines in 0.15 second?

**Prob. 2–10.** How many volts (average) are induced by 3 conductors in series, if they cut $4 \times 10^6$ lines in 0.10 second?

**Prob. 3–10.** A wire cuts through a field of 3,000,000 lines at an average rate of 3600 times per minute. How many volts (average) are set up in the wire?

**Prob. 4–10.** A wire 10 inches long moves through a magnetic field having a flux density of 80,000 lines per square inch. If the velocity at right angles to the field is 950 inches per second, what voltage is induced?

**Prob. 5–10.** If a wire, which is cutting through a magnetic field at the rate of 100 feet per second, induces 1.4 volts across its terminals, what must be its average velocity to set up 6 volts?

**5. Voltage Generated in a Single-Coil Armature.** Let us take the single-turn coil in Fig. 11*a*–10, which is similar to Fig. 4–9, and connect each end to one of two "slip-rings," as shown in Fig. 11*b*–10. Each slip-ring is continuous and insulated from the other, and from the shaft on which it is mounted. By means of a stationary brush bearing on each slip-ring, we will connect the coil to an outside circuit. Now let us revolve the coil in a clockwise direction. In Fig. 11*b*–10, the side of the coil, or conductor $AB$, is moving **down** across the flux, and a voltage is induced in the direction from $A$ to $B$ (as indicated by the arrow); while conductor $CD$ is moving **up** across the flux, and the voltage induced is in a direction from $C$ to $D$. This is in accordance with Fleming's Right-Hand Rule. Since the conductors in Fig. 11*b*–10 are moving at right angles to the flux, the voltage induced in them at this instant is at its greatest value. Note that the voltage induced in both conductors is additive, and in a direction to send current out through brush $F$, through the external circuit, and back through brush $E$, in the direction of the arrows.

Now consider Fig. 11*c*–10. The coil has turned through 90° and the conductors $AB$ and $CD$ are now in a position moving parallel to the flux, and no voltage is generated. The coil is said to be in a neutral, or zero position. Since no voltage is being generated, no current flows in the external circuit.

In Fig. 11*d*–10, the coil has turned through another 90° and the conductors $AB$ and $CD$ are again moving across the flux at the greatest rate, and the maximum voltage is again induced. However, the sides of the coil have interchanged position from that in

Fig. 11*b*–10, and conductor *AB* is moving **up** across the flux and the direction of the induced voltage is from *B* to *A*, while conductor *CD* is moving **down** across the flux and the direction of the induced voltage is from *D* to *C*. The induced voltage in both conductors is now reversed, and in a direction to send current out through brush *E*, through the external circuit, and back through brush *F* — in the opposite direction from that in Fig. 11*b*–10, as shown by the arrows.

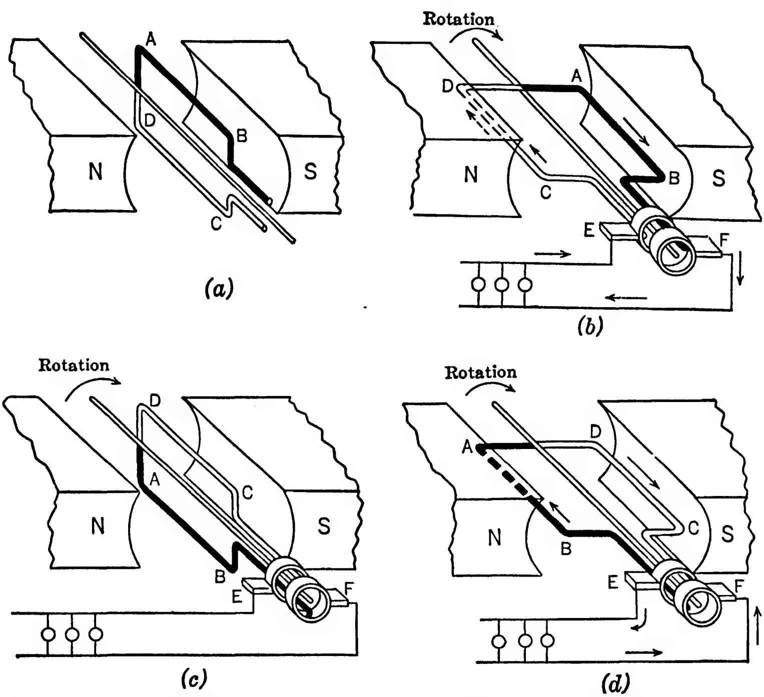



FIG. 11–10. Voltage induced in a coil of a single turn, connected to "slip rings."

After the coil has turned through another 90°, it will be in a position opposite to that in Fig. 11*c*–10 or in position (*a*), and the conductors will be moving parallel to the flux. The coil is again in a neutral, or zero position; no voltage is induced, and no current flows in the external circuit.

Thus, as the coil revolves, a voltage is induced in one direction during half the revolution, and in the reversed direction during the other half revolution. This forces a current

through the external circuit, first in one direction and then in the other. In other words, we have a very simple alternating-current generator forcing an alternating current through the external circuit.

Note here that the revolving coil does not generate a current. It generates, or induces, a voltage, which in turn forces a current to flow, if the coil is connected to a closed circuit.

**6. Curve of Electromotive Force.** In Fig. 11*a*–10, the coil is at the "zero position." When the coil is in the horizontal position of Fig. 11*b*–10, it would be 90° from the zero position. When the coil reaches the vertical position again, as in Fig. 11*c*–10, it would

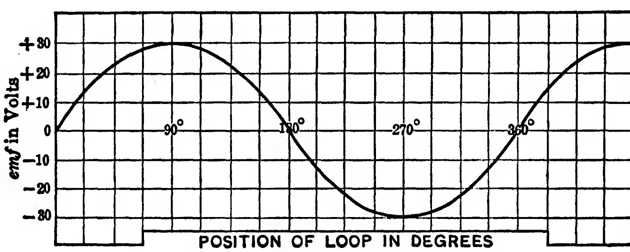



FIG. 12–10. The curve shows the voltage induced in the coil at the various instants during a revolution.

once more be in a zero position 180° from the first zero position. When the coil is again horizontal, as in Fig. 11*d*–10, it would be 270° from the first zero position; and after turning through another 90°, or 360° in all, it would be back in the original zero position.

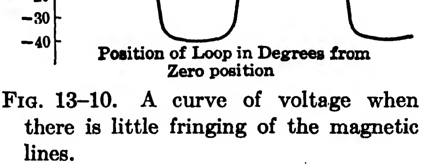



FIG. 13–10. A curve of voltage when there is little fringing of the magnetic lines.

We may plot a **voltage** curve using the **position** (in degrees from the zero position) as abscissae and the induced emf as the ordinates.

When the emf is in one direction, we call it positive (+), and in the other direction, negative (−).

If the flux between the poles is uniform, we shall obtain a curve which may have a shape something like that of Fig. 12–10;

or the curve may have the shape of Fig. 13–10, if the coil has an iron core and the pole faces are so shaped that the magnetic lines " fringe out " very little.

These curves show that the voltage (induced emf) rises rapidly to a maximum at the 90° position (horizontal) where the lines are being cut at the greatest rate. It falls again to zero at the 180° position (vertical), when the sides of the coil are moving parallel to the magnetic lines and are thus not cutting them. Then the sides begin cutting flux in the opposite direction and a voltage is induced in the opposite direction (which we have agreed to call " negative "). The voltage in this direction increases rapidly to a maximum at the 270° position (horizontal), where the lines are again being cut at the greatest rate (but in the opposite direction). Again it decreases, as it begins to approach the zero position (vertical), where the sides again move parallel to the flux lines; and again at this point, no voltage is induced. This " cycle " of events, as it is called, takes place every complete revolution in a two-pole machine; the maximum voltage induced depends upon the number of lines cut per second, at the instant when the sides of the coil are cutting at the greatest rate.

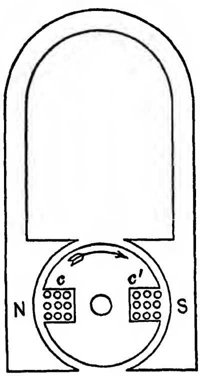


FIG. 14–10. A single coil magneto for generating alternating voltage for ignition purposes; $c$ and $c'$ indicate the conductors, or coil sides, in the armature winding.

In order to get a high maximum voltage in a generator, the coil must be mounted on an iron core to increase and concentrate the magnetic field. The coil also must be wound with a great many turns, so that the total voltage at the brushes is the sum of the emf's induced in the turns. A single-coil magneto is an example of a very simple alternating-current generator, and is shown in Fig. 14–10.

**7. Unidirectional or Direct Current.** If a direct current is to be obtained in the external circuit, that is, a current always flowing in the same direction, the " slip-rings " of Fig. 11–10 cannot be used. For we have just seen that an alternating emf is generated in the revolving coil, and, therefore, an alternating current will flow in the external circuit.

However, if the brushes can be connected, first to one end and then to the other end of the revolving coil, the change being made

just at the instant the voltage in the sides of the coil is reversing, the voltage across the brushes, and the current flowing through them, will always be in the same direction. That is, if the current to the brushes is **rectified**, a unidirectional, or direct current will flow in the external circuit.

This rectification can be accomplished by the use of one ring, split diametrically into two equal sections, or segments, insulated from the shaft, and from each other. This forms a very simple commutator. The ends of the coil are connected one to each segment of this commutator, as shown in Fig. 15–10. Note, in

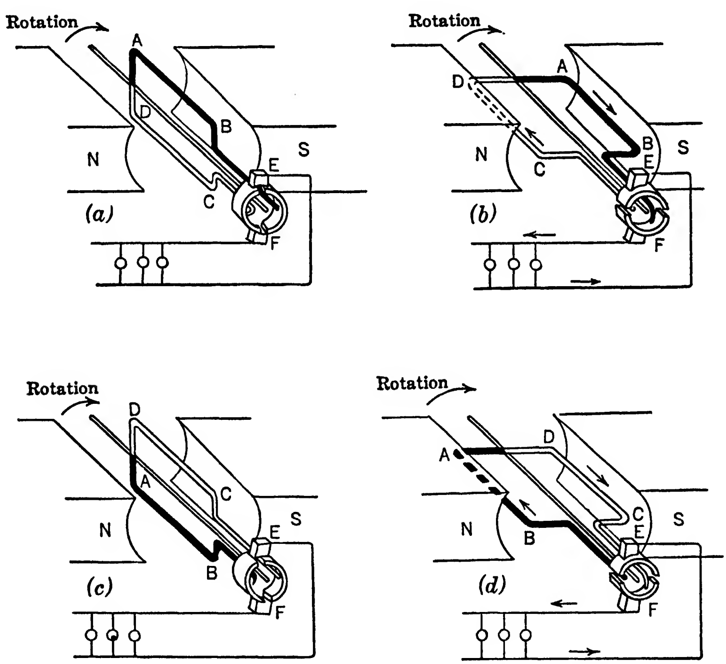



FIG. 15–10. An armature coil of a single turn fitted with a two-segment commutator.

Fig. 15*a*–10, that the brushes are so placed that, as the coil moves through the zero position, both commutator segments move out from under one brush and into contact with the other. In Fig. 15*b*–10, 90° from Fig. 15*a*–10, the direction of the induced voltage is such as to send current out through brush *F* and back through brush *E*, in the direction of the arrows. After another quarter

revolution, Fig. 15*c*–10, the voltage in the coil has dropped to zero and the commutator segments are again reversing the connection of the coil with the brushes. In Fig. 15*d*–10, after another quarter revolution, the coil sides and the commutator segments have interchanged position from that in Fig. 15*b*–10; and, although the direction of the emf is reversed in the conductors, the connections to the external circuit also are reversed. The induced voltage will again send current out through brush *F* in the **same** direction as before, as indicated by the arrows. That is, we have a direct current in the circuit. Note, however, that the voltage at the brushes, and hence the current, drops to zero twice in each cycle, and the voltage wave of Fig. 12–10 simply has its negative half reversed, as shown in Fig. 16–10.

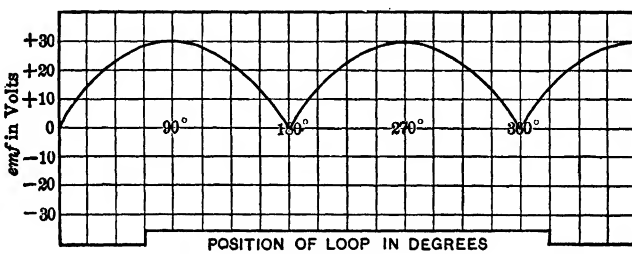



FIG. 16–10. The curve shows the voltage at the brushes at different instants during the revolution of a single coil.

Now a d-c voltage, varying rapidly from a given maximum to zero, cannot be used satisfactorily for direct-current service, for such a voltage would give a pulsating instead of a steady current.

We can improve our machine and obtain a steadier voltage at the brushes, if we use two coils and four commutator segments, as illustrated in Fig. 17–10. The voltage curve for a two-coil machine is illustrated in Fig. 18–10, in which the heavy line represents the voltage at the brushes.

We now have a very simple arrangement of commutator and winding to give us an elementary direct-current generator. The more turns we have in each coil, the higher will be the induced voltage; and the more coils and commutator segments there are, the steadier will be the voltage at the brushes. The arrangement in Fig. 17–10 is an open-circuit type of winding; that is, it is impossible to start at any one commutator segment and trace

through the entire winding to the starting point. One objection to the arrangement is that it is impossible to use that part of the emf of each coil shown by the dotted lines, *ab*, in Fig. 18–10; so it is not used in practice.

The commutator and other methods of arranging the coils of an armature will be further considered in the following paragraphs.

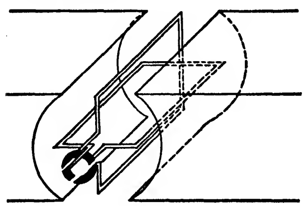


FIG. 17–10. A two-coil armature fitted with a four-segment commutator. Open-circuit winding.

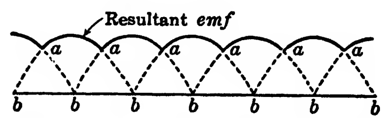



FIG. 18–10. The heavy line shows the voltage at the brushes of a two-coil armature. Open-circuit winding.

**8. The Commutator.** From the discussion above, it is seen that the function of the commutator is to continually reverse the connection of the armature coils to the outside circuit. In other words, it **rectifies** the alternating current which flows in the conductors or in the armature coils of the winding, because of the alternating emf induced in the armature conductors.

In the commercial machine, it consists of a set of copper bars or segments, so shaped that when assembled they make a hollow ring or cylinder. Each segment is insulated from all the others and from the metal frame that holds it. The bars are wedge-shaped, as shown in Fig. 19*a*–10, and the insulation between them consists of a mica strip of special grade. Figure 19*b*–10 shows two views of a commutator bar. The ends of the armature coils in the larger machines are soldered into the risers at one end of the bar. Note the bottom of the bar is shaped so that it can be clamped by a dove-tail arrangement. The method of connecting the arma-

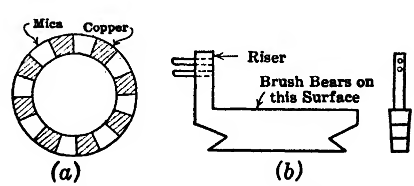



FIG. 19–10. (a) End view of the commutator for a small machine. (b) Two views of a commutator bar with riser for connecting to armature leads.

ture conductors to the commutator segments is shown in Fig. 20–10.

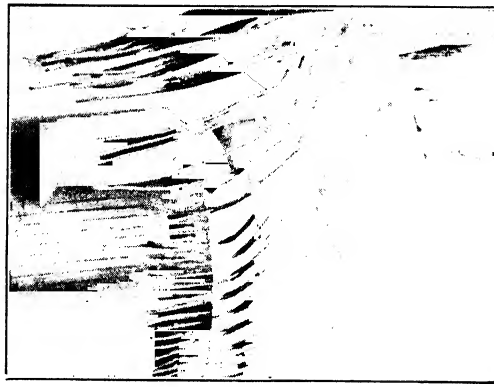


FIG. 20–10. How the armature conductors are soldered to the commutator segments. *General Electric Co.*

**9. Drum Armature Core and Winding.** By the term "armature winding" is meant, not only the group of conductors which cut the magnetic flux, but also all the wire in the coils which are made to revolve. This includes the parts of the coils which connect one conductor to another, but which do not cut the flux.

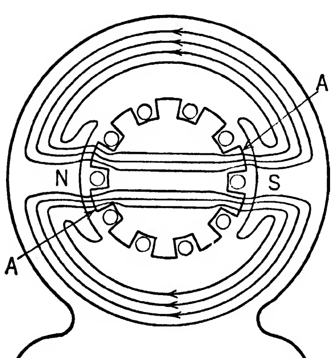



FIG. 21–10. The magnetic paths in a two-pole dynamo.

An armature winding, as previously stated, is always wound on an iron core. This core occupies nearly all the space between the pole faces, as indicated in Fig. 21–10, and leaves only the short air gaps, *AA*, in the magnetic path. This reduces the reluctance of the magnetic circuit of the machine and a high flux density is maintained in the air gaps.

In commercial machines, this iron core is in the shape of a cylinder or drum, and a great many coils are wound in slots on the surface of this drum. The winding takes its name from the shape of the core and is called a **drum**

winding. Drum armatures are used in all commercial machines today.

In the actual machine of moderate size, the core is made up of

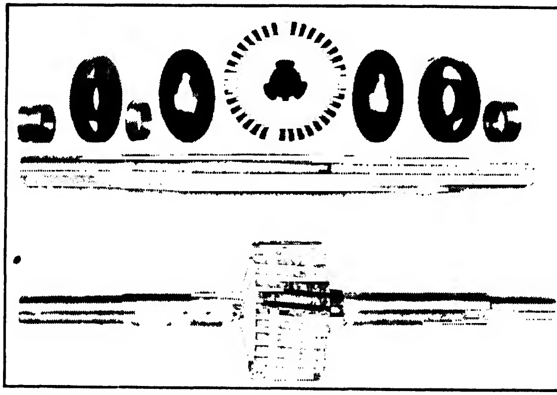


FIG. 22–10. A punching for the laminated core of an armature, and an assembled armature core mounted on the shaft. *General Electric Co.*

thin annealed steel punchings, or laminations, generally 0.014 inch thick. These punchings are built into a cylinder and keyed to the shaft. Slots are cut in the outside rim of these punchings, and, when they are assembled, form slots or grooves in which to place

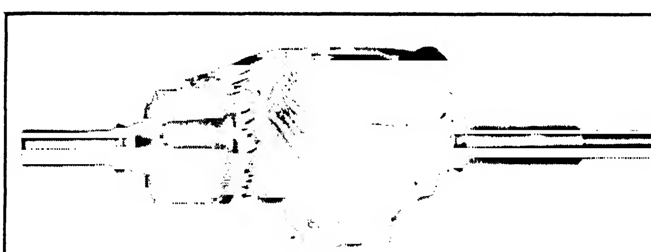


FIG. 23–10. Partially wound armature on a laminated core. *General Electric Co.*

the insulated conductors or coil sides of the armature winding. Figure 22–10 shows one of these annealed sheet-steel punchings and how they are assembled and attached to the shaft. Figure 23–10 shows a partially wound armature on such a core.

In large multipolar machines, the flux paths do not penetrate to

the center of the core, and the steel punchings are made in sections, forming a ring which is mounted on a **spider**, as illustrated in Fig. 24–10. This spider is then keyed to the shaft. The armature winding is wound in the slots on the surface of this structure.

The fundamental principles, underlying the generation of an emf and the action of the current in an armature, can best be studied by the aid of diagrams. There are almost innumerable ways of connecting the coils in drum windings; the circuits are somewhat complicated, and it is not easy to analyze just what takes

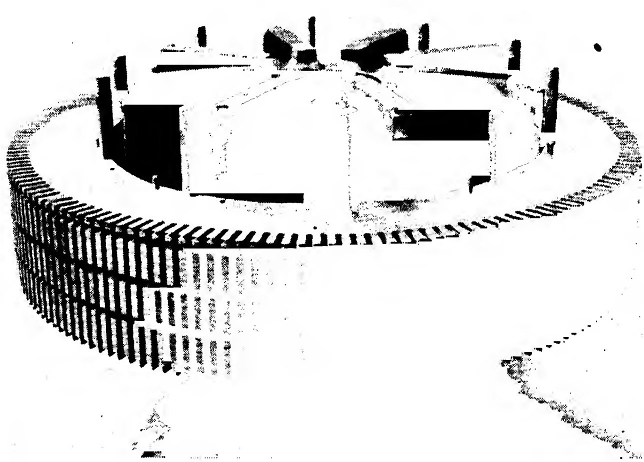


Fig. 24–10. Armature core and spider for a large machine. *Westinghouse Electric and Mfg. Co.*

place in this type of winding by aid of a diagram. We will, therefore, in our study make use of another type of armature called the **ring winding,** in which the circuits can be shown more clearly in the diagrams which follow. Drum armature windings will be further considered in Chapter XIII.

**10. Ring Armature Winding.** The core for a ring-wound armature consists of a ring, or hollow cylinder of steel, supported by a cast spider keyed to the shaft. The winding itself consists of insulated wire, wound spirally around the cylinder, Fig. 25–10. The winding may be one continuous coil with taps brought out at regular intervals to the commutator segments, or it may be made up of separate coils with the ends of each coil connected to adjacent

commutator segments. Note Fig. 26–10. In either case, the winding is continuous, or closes on itself; that is, it is possible to start from one commutator segment and trace through the winding to the starting point.

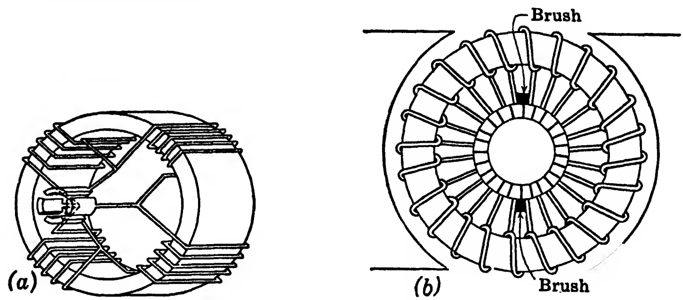



FIG. 25–10. Ring wound armatures.

In this type of armature, that part of the winding which lies **inside** the core does not cut the flux, and serves only as connectors for the conductors on the outside surface of the core. This results in a waste of wire and increased armature resistance. This feature, together with the fact that the drum armature wound of formed coils is cheaper to manufacture, has made the ring winding obsolete.

However, the action of the ring winding is fundamentally the same as that of the drum winding. And, since it is easier to represent a ring winding, this will be pictured throughout the chapter in our analysis of the action of a drum winding.

**11. Parallel Paths in a Two-Pole Armature.** Consider carefully the arrangement of the 6 armature coils in the simple two-pole generator, represented in Fig. 26–10. Each coil consists of 3 turns, or 3 conductors. Note that each coil is connected across the gap between adjacent commutator segments; and that there is a path in the armature from brush $E$, through the coils 6, 5 and 4 in series, to brush $F$. There is also a path in the armature from brush $E$, through coils 1, 2 and 3 in series, to brush $F$. If 20 amperes were flowing out from brush $F$, through the external circuit in the direction of the arrows, it would flow back through brush $E$. Here it would divide, and 10 amperes would flow down through coils 1, 2 and 3 to brush $F$; and 10 amperes also would flow down through coils 6, 5 and 4 to brush $F$, where the two currents would unite. The two brushes thus divide the armature into two parallel circuits.

**12. Voltage at the Brushes.** Consider the armature in Fig. 26–10 to be rotating in a clockwise direction. The voltage induced in each conductor (that part of the coil lying on the outside surface of the core) is in a direction determined by the Right-Hand Rule. In the conductors moving up on the left side of the armature, the induced voltage is **down,** or away from the reader, and is indicated by the small crosses (+) in the figure. In the conductors on the right side of the armature, the induced voltage is

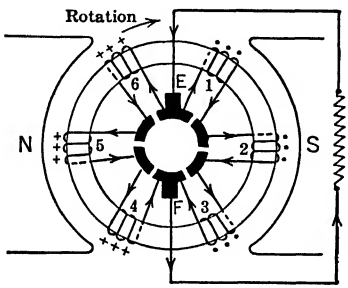



Fig. 26–10. The two brushes divide the armature circuit into two parallel paths. The voltage between brushes is the sum of the voltages induced in the conductors in one path.

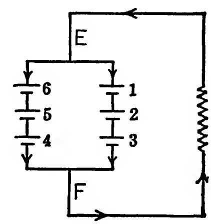



Fig. 27–10. Two sets of three battery cells in series, connected in parallel. The voltage across EF is that of three cells in series. The circuit is similar to that of Fig. 26–10.

**up,** or toward the reader, indicated by the dots (·). Note that there are nine conductors in each armature path and that the direction of induced voltage in each conductor is such that the voltages in each path are additive; that is, the voltage induced in each path is that of nine conductors connected in series. And the direction of the induced voltage through the armature circuit in either path is from brush *E* to brush *F*, as indicated by the arrows on the coil leads. Since these two paths are in parallel between the brushes, the induced voltage between brushes must be, according to the law of parallel circuits, the voltage of either path, not the sum of the voltages of the two paths.

The condition within the armature may be likened to that in Fig. 27–10, which represents a combination of battery cells, connected in two parallel circuits between the terminals *EF*. Cells 1, 2 and 3 are in series in one path, and the voltage across the terminals, *EF*, must be the sum of the voltages of the three cells in series. Similarly, the voltage across the terminals *EF* must be

the sum of the voltages of cells 4, 5 and 6. Since the two sets of cells are in parallel, the voltage across the terminals will be that of either path.

Obviously, the voltage of each path must be the same; or one set of cells will send a reverse current through the other set against its emf. This shows how essential it is in a generator to arrange the coils symmetrically around the armature, in order that the voltage of each path may be the same.

Note that Fig. 28–10 is the same as Fig. 26–10, except that the armature is shown in a position where each brush short-circuits a coil. These coils are short-circuited at the instant they are not cutting flux, and, therefore, have no voltage generated in them due to rotation. During rotation, as one coil moves out from under a brush, another moves forward to take its place. In Fig. 28–10, coils 3 and 6 are short-circuited and the voltage between brushes at this instant is that of two coils, or six conductors in series; while in Fig. 26–10, the instantaneous voltage is that of three coils or nine conductors in series. Since all coils cut the same flux during each revolution, the average value of the emf between brushes is essentially that of three coils or nine conductors in series.

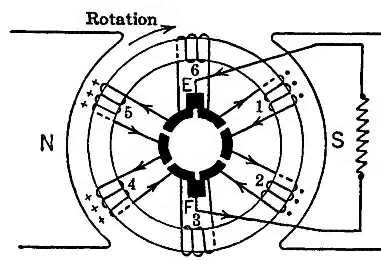


Fig. 28–10. As the armature rotates, two coils are short circuited. As one coil moves out from under a brush another one takes its place. The two brushes still divide the armature into two parallel paths.

Again, note that even with two coils short-circuited, as in Fig. 28–10, there are still two paths in parallel in the armature.

**Prob. 6–10.** If the generator in Fig. 28–10 were supplying 45 amperes to the external circuit, how much current would flow in each coil?

**Prob. 7–10.** If the average induced voltage in each conductor in Fig. 26–10 were 1.5 volts, what would be the average emf at the brushes?

**Prob. 8–10.** (*a*) In Fig. 29–10, the average voltage induced in each coil is 8 volts. What is the voltage at the brushes? (*b*) If this generator is delivering 15 amperes to the external circuit, how much current flows through the brushes? (*c*) How much flows through each path in the armature?

**13. Armature Resistance.** The resistance of an armature is merely the combined resistance of a series-parallel combination of resistances. Thus, the resistance of the armature in Fig. 28–10 would be the resistance of two parallel circuits, each of which consists of two coils in series. (Since coils 3 and 6 are short-circuited, they are cut out of the circuit.)

**Example 2.** Assuming the resistance of each coil in Fig. 28–10 to be 0.04 ohm, the resistance of one path made up of coils 1 and 2 in series $= 0.04 + 0.04 = 0.08$ ohm. The resistance of coils 4 and 5 in series $= 0.04 + 0.04 = 0.08$ ohm. Since the two paths are in parallel, the resistance of the armature $= \frac{0.08}{2} = 0.04$ ohm.

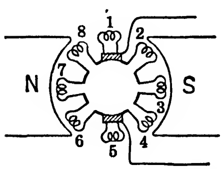



FIG. 29–10. An eight coil armature. The brushes short-circuit two coils.

**Prob. 9–10.** The resistance of each coil in Fig. 29–10 is 0.025 ohm. What is the resistance of the armature? Note that coils 1 and 5 are short-circuited.

**Prob. 10–10.** If the armature resistance of Fig. 29–10 were found to be 0.24 ohm, what is the resistance of each coil?

**Prob. 11–10.** The armature of a 2-pole, 2-brush machine is made up of 36 coils, each coil being composed of 3.5 ft of No. 12 copper wire. Compute the resistance of the armature. (Assume 2 coils are short-circuited.)

**14. Multipolar Generators, Armature Voltage and Current Relations.** A multipolar generator is one which has more than two poles. Generators of 5 kw capacity, and above, are usually of this type, because the magnetic circuits are shorter and the machine will weigh less for the same kilowatt output. Figure 30–10 represents a four-pole generator. In this machine, twice as many magnetic lines will be cut by the armature conductors during each revolution as in a two-pole machine, assuming the poles to be of equal strength. It can, therefore, be operated at a lower speed for the

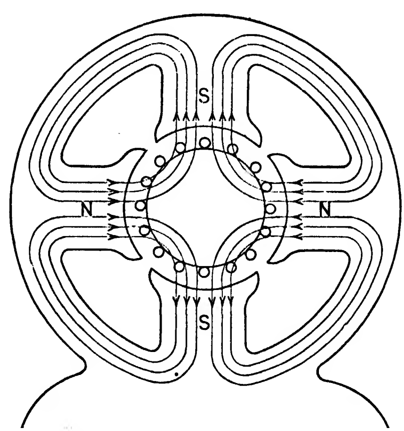



FIG. 30–10. The magnetic paths in a four-pole dynamo.

same generated voltage. In large slow-speed generators, there may be as many as 36 poles or more.

The number of brushes in large multipolar generators is generally the same as the number of poles. For small multipolar machines, armatures may be designed which have only two brushes, regardless of the number of poles. These will be discussed in Chapter XIII.

Carefully consider Fig. 31–10, which represents a four-pole generator with twelve armature coils and three conductors per coil. Note that the machine has four brushes, and that in the position shown, each brush short-circuits a coil. We thus have four coils short-circuited, leaving eight active coils at this instant.

Assume the armature rotates in a clockwise direction, as indicated in the figure, and note that there are two coils in series between any pair of brushes.

The voltage induced in each of the six conductors in coils 2 and 3, opposite a south pole, is additive and in a direction **up,** or toward the reader (Fleming's Right-Hand Rule), as indicated by the dots (·). The induced voltage in coils 2 and 3 is therefore in a direction in the armature circuit from brush $E$ to brush $F'$, making brush $F'$ positive.

The induced voltage in each of the six conductors in coils 5 and 6, opposite a north pole, is additive and in a direction **down,** or away from the reader, as indicated by the crosses (+); and in a direction in the armature circuit from brush $E'$ to brush $F'$, also making brush $F'$ positive.

Similarly, the direction of the induced voltage in coils 8 and 9, opposite a south pole, would be in a direction in the armature from brush $E'$ to brush $F$, making brush $F$ positive. Also the induced voltage in coils 11 and 12, opposite a north pole, is in a direction from brush $E$ to brush $F$, also making brush $F$ positive.

Note that there are two positive brushes, $F$ and $F'$, which are connected together at $P$, the positive terminal of the machine. Also, there are two negative brushes, $E$ and $E'$, which are connected together at point $N$, making this the negative terminal. Since there are four circuits in the armature, each consisting of two coils in series, and each circuit connected between the terminals $P$ and $N$, there are four paths in parallel. The voltage at the terminals of the machine (voltage across $PN$) is equal then to the voltage of two coils in series, or to the voltage of **one** path in the armature.

Again, it is seen that the armature winding must be symmetrical, with the same number of coils and conductors in each path, in order that the voltage induced in each path may be the same.

When a generator delivers current, this current must flow in the armature circuit in the direction of the induced voltage. The current in coils 2 and 3 flows in the direction from brush *E* to brush *F′*; the current in coils 5 and 6 flows from brush *E′* to brush *F′* where it unites with that from coils 2 and 3, and both currents flow out through brush *F′* to the terminal *P*. The current in coils 8 and 9 flows from brush *E′* to brush *F*; the current in coils 11 and 12 flows from brush *E* to brush *F*, where it unites with that from coils 8 and 9, and both currents flow out through

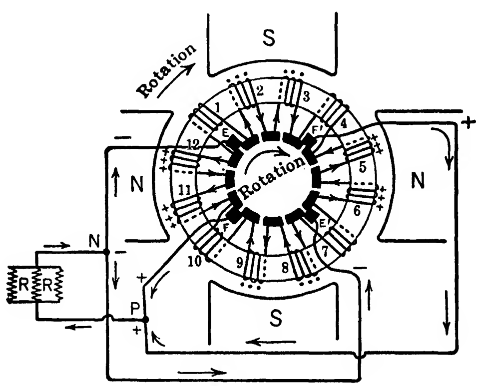



Fig. 31–10. A twelve-coil armature in a four-pole dynamo.

brush *F* to the terminal *P*. Here it unites with the current from the other half of the armature and flows into the external circuit.

If resistors, *R*, Fig. 31–10, were carrying 40 amperes, a current would flow to terminal, *N*, in the direction of the arrows, and divide, 20 amperes flowing to brush *E* and 20 amperes flowing to brush *E′*.

At brush *E*, this current of 20 amperes would again divide, 10 amperes flowing through coils 2 and 3 to brush *F′*, and 10 amperes flowing through coils 12 and 11 to brush *F*.

At brush *E′*, also, a current of 20 amperes would divide, 10 amperes flowing through coils 6 and 5 to brush *F′*, and 10 amperes flowing through coils 8 and 9 to brush *F*.

We thus have two currents of 10 amperes each, uniting at brush *F′*, and 20 amperes flowing out to terminal *P*; and also two

currents of 10 amperes each, uniting at brush $F$ and flowing out to terminal $P$. At $P$, the two currents, each of 20 amperes, unite and 40 amperes flow on through the resistors, $R$. Again it is seen that there are four paths in parallel in the four-pole, four-brush generator.

The circuits in Fig. 31–10 are similar to the parallel arrangement of battery cells in Figs. 32*a*–10 and 32*b*–10. The emf in each path is that of two cells in series. Each group of two series cells supplies one quarter of the current to resistors, $R$.

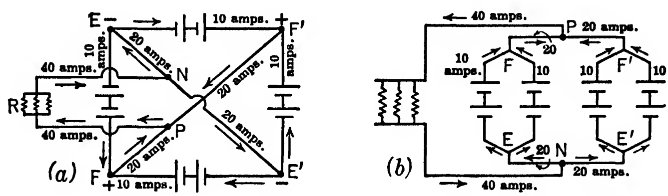



FIG. 32–10. (a) An arrangement of four sets of two battery cells in series, connected in parallel and feeding the resistor, R. A similar circuit to that in Fig. 31–10. (b) A simpler diagram for the circuit of Fig. 31–10.

It will be remembered that in a two-brush machine there are two parallel paths, the voltage of the machine being the voltage across either one of the paths. Similarly, in a six-pole, six-brush machine there would be six paths, and the voltage of the machine would be that on any one of the paths.

It can, therefore, be stated that there are generally* as many paths in an armature circuit as there are brushes; and that these paths are always in parallel, all the positive brushes being connected by jumpers, and all negative brushes connected in like manner. **The voltage of the machine is the voltage induced in any one path between any pair of positive and negative brushes.**

Note, also, that in the two-pole, two-path armature of Fig. 28–10, each path carries half the current. In the four-pole, four-path armature of Fig. 31–10, each path carries one-fourth of the current. Similarly, each path in a six-pole, six-path armature carries one-sixth of the current.

It may be stated, as a general rule, that **the current delivered by any multipolar generator divides in the armature into as many equal parts as there are paths in parallel.**

* The exception to this statement occurs in a wave-wound armature and is explained in Chapter XIII.

By comparing Figs. 28–10 and 31–10, the advantages of the four-pole machine are apparent. There are twice as many coils, and therefore, twice as many conductors on the armature in Fig. 31–10. However, the number of coils, or conductors, per path is the same as in Fig. 28–10; and if the poles are of the same strength, each conductor in Fig. 31–10 will cut twice as many magnetic lines per revolution. Therefore, the armature can be run at half the speed of the two-pole machine and yet generate the same voltage.

Also, since each path in the four-pole generator carries only one-quarter of the total current, it can deliver twice the current of the two-pole generator and have the same current per path.

Therefore, if the poles of the four-pole generator have the same strength as those of a two-pole generator, and the armature is wound with twice as many coils of the same size wire, the four-pole machine can be run at half the speed, and will generate the same voltage and supply twice the current. In other words, the kilowatt capacity will be doubled.

Additional comparisons can be made for machines of any number of poles.

**15. Armature Resistance of Multipolar Machines.** The armature resistance of the four-pole generator in Fig. 31–10 is the resistance of a parallel combination of four paths. Thus if each coil has a resistance of 0.03 ohm, the resistance of one path would be $2 \times 0.03$ or 0.06 ohm, the resistance of four paths in parallel, or the resistance of the armature equals

$$\frac{1}{\frac{1}{0.06}+\frac{1}{0.06}+\frac{1}{0.06}+\frac{1}{0.06}} = \frac{0.06}{4} = 0.015 \text{ ohm.}$$

It will be remembered that in a two-brush machine, the armature resistance equals the resistance of one path divided by 2. It may therefore be stated, as a general rule, that **the resistance of the armature of any machine equals the resistance of one path divided by the number of paths.**

In commercial machines, wound with a large number of coils, the resistance of one or two coils does not appreciably affect the resistance of the armature; so that the number of coils short-circuited may be neglected, and the resistance calculated from the size and total length of wire on the armature, as in the example below.

**Example 3.** What is the resistance of a 6-pole, 6-brush armature wound with 900 feet of No. 14 wire? Temp = 20° C.

**Solution.**

Resistance of 1000 feet of No. 14 wire (from wire table) = 2.525 ohms.

$$\text{Resistance of 900 feet} = 0.9 \times 2.525 = 2.273 \text{ ohms.}$$

$$\text{Resistance of one path} = \frac{2.273}{6} = 0.379 \text{ ohm.}$$

$$\text{Armature resistance} = \frac{0.379}{6} = 0.063 \text{ ohm.} \quad \textit{Ans.}$$

$$\text{Or, Armature resistance} = \frac{2.273}{6^2} = 0.063 \text{ ohm.} \quad \textit{Check.}$$

Thus, if the resistance of the short-circuited coils be neglected, it may also be stated, as a general rule, that **the resistance of an armature is equal to the resistance of the total length of wire in the armature divided by the square of the number of paths.**

**Prob. 12–10.** There are 480 feet of No. 16 copper wire in a 4-pole, 4-brush armature. The winding consists of 20 coils with 4 coils short-circuited. Temp = 20°. Calculate:

(*a*) Total resistance of wire in the armature;
(*b*) Resistance of one path in the armature;
(*c*) Armature resistance.

**Prob. 13–10.** After the generator of Prob. 12–10 has been run two hours, the temperature rises to 75° C. What is the armature resistance at this temperature?

**Prob. 14–10.** An 8-pole, 8-brush armature is wound with 1200 feet of No. 10 copper wire. What is the resistance of the armature at 20° C., if the short-circuited coils are neglected?

**Prob. 15–10.** The resistance of a 4-pole, 4-path armature was measured at 20° C. and found to be 0.04 ohm. There were 36 segments on the commutator. The wire used was No. 12 copper. Assume 4 coils short-circuited. How many feet of wire were used in winding the armature?

**16. Commutation.** We have seen in Figs. 28–10 and 31–10 that the brushes are so placed that they short-circuit a coil of the armature winding when the coil is not cutting flux; that is, the coil is in a neutral position. If the brushes were not set at this point, we would have the condition illustrated in Fig. 33–10. With the brush in the position shown, the emf induced in the coil, due to its movement across the magnetic flux, forces a heavy local current through the coil and the brush, as indicated by the arrows.

When commutator segment 2 moves out from under the brush, this current is interrupted and an arc is formed. Therefore, brushes **must** be set at the neutral point.

When a coil moves past the neutral position, the reversal of its connection to the brush by the commutator reverses the direction of the current in the coil. This process is called commutation.

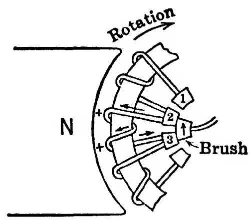



FIG. 33–10. Illustrates the circulating current flowing in an armature coil if short-circuited when cutting magnetic flux.

Figure 34–10 (*a* to *e*) represents part of the coils of an armature, shown in a horizontal plane for convenience, and a positive brush placed to short-circuit a coil in the neutral position. Let us assume that 30 amperes are flowing through the brush; consequently, 15 amperes must flow through each path or coil. In (*a*), the brush is connected to commutator segment *C*, and the direction of the currents in the coils is indicated by the arrows. In position (*b*), commutator segment *B* is just making contact with the brush and the commutation of coil 3 is beginning. At this point, the contact resistance between commutator segment *B* and the brush is higher than that through coil 3 to the brush; hence, the current divides and most of it, say 10 amperes, flows through

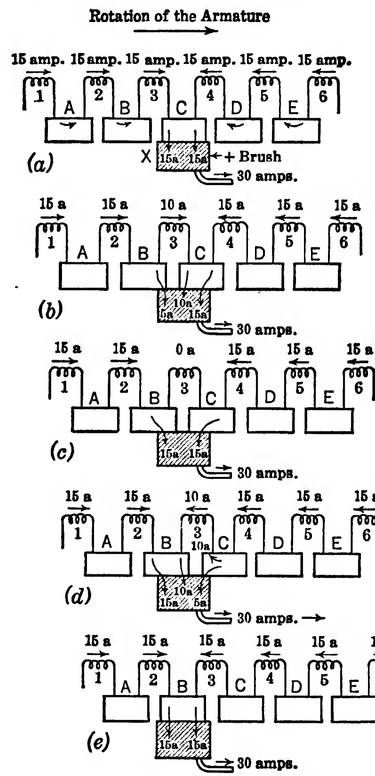



FIG. 34–10. Illustrates the process of commutation. Ideal condition. The connections of coil 3 to the brush X are reversed as the armature moves from position (a) to position (e).

coil 3 to the brush, while only 5 amperes flows directly from coil 2 to the brush. As the area of the brush-contact with commutator segment *B* increases, the resistance of this path decreases and increased current flows from segment *B* to the brush; while the current in coil 3 decreases until, at position (*c*), the current in coil 3 drops to zero. At this point, the contact resistances between the brush and commutator segments *B* and *C* are the same, and, moreover, coil 3 is at the neutral point.

As the commutator moves into position (*d*), contact resistance between commutator segment *C* and the brush increases and the current rises in the **opposite** direction in coil 3 to, say 10 amperes, while only 5 amperes flow directly from *C* to the brush. As the area of contact between *C* and the brush decreases, this current decreases; and consequently, the current in coil 3 rises until segment *C* breaks contact with the brush, as in (*e*). Fifteen amperes are now flowing through coil 3 in the opposite direction, and the commutation of the coil is completed.

This ideal condition, however, does not occur in the actual machine, because of the **inductance** of the armature coil. We have seen in Chapter IX that the current in an inductive circuit takes some time to die out. The events described above occur in a **very small** period of time, and the current cannot reverse as quickly as described. There is, therefore, a local current flowing in coil 3 and the brush, when in position (*c*), due to the emf of self induction as shown in Fig. 35–10. Furthermore, the current of 10 amperes in coil 3, Fig. 34*d*–10, cannot flow against this emf of self induction. Therefore, the total current of 15 amperes, in addition to the local current due to self induction, flows from commutator segment *C* to the brush. When contact with segment *C* is broken, then, both the load current in the coils and this local current are interrupted, and sparking occurs.

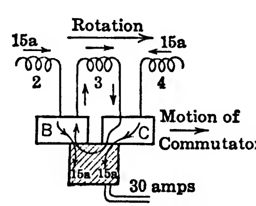



FIG. 35–10. When a coil, in a neutral position, is short-circuited by a brush the counter emf of self-induction causes a circulating current to flow.

To obtain sparkless commutation, therefore, the brushes of a generator must be shifted in the **direction of rotation,** so that the coils undergoing commutation are cutting enough flux in the **opposite direction** to generate sufficient voltage to counteract the emf of self induction in these coils, and allow

the current to be almost instantly reversed. This flux in the fringe of the next pole, which reverses the current in the short-circuited coil, is called the **commutating flux.** In the actual machine the position of this commutating flux and, therefore, the proper position for the brushes is further shifted. This is described in Art. 22.

Sparking, or arcing, at the brushes of a d-c generator or motor must be avoided. Arcing pits and wears away the edges of both the commutator bars and the brushes, and increases brush-contact resistance. For successful operation, the surface of the commutator must be smooth, and the brushes should make intimate contact with the commutator over their entire surface. One cause of sparking is due to "high mica." If the mica strips between commutator segments do not wear away as fast as the copper bars, the mica protrudes above the surface of the commutator, as illustrated in Fig. 36*a*–10. To avoid this possibility, the commutators in many machines are "undercut," as illustrated in Fig. 36*b*–10.

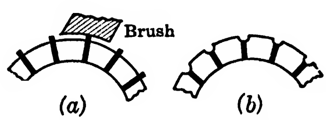



FIG. 36–10. (a) A commutator with "high mica." (b) An undercut commutator.

The wearing away of the commutator bars is due, not only to friction, but also, to an electrochemical action which takes place between the copper and carbon brushes when the machine supplies a current.

**17. Average EMF of a D-C Generator. Fundamental Equation.** It was shown in Art. 4, and also in Chapter IX, that a conductor must cut magnetic flux at the rate of 100,000,000 ($10^8$) lines per second to generate 1 volt. In Arts. 12 and 14, it was also shown that armature windings are divided by the brushes into parallel paths, and that the generated emf between brushes is that induced in the conductors in series in one path of the winding.

Not all the conductors on an armature, or in any one path, are active. That is, not all the conductors are cutting flux at any one instant. In the actual machine, generally from 60 to 80 per cent of the conductors are active. Neither do all the conductors generate the same emf at any one instant. Some conductors lie under a pole face where the flux density is greatest; others lie in the fringe of flux near the edges of the poles — called the pole fringe — where the flux density is not so great; while others lie in the space midway between the poles and do not generate any voltage. (A certain proportion of this latter group may be short-circuited by the brushes.)

The voltage of the generator may be obtained by computing the instantaneous emf generated in each of the conductors in series in a path, and then adding all these voltages.

However, the above method of calculating the emf of a generator is not so readily applied as that of calculating the **average** induced voltage.

To compute the average emf induced in an armature, it is necessary to know the **average** rate at which each conductor cuts the magnetic flux (which is the same for each conductor), and also the number of conductors in series in one path in the armature.

Let $E$ = average emf generated in the armature,
$\Phi$ = flux per pole in the air gap,
$P$ = number of poles in the machine,
$N$ = speed in revolutions per minute,
$Z$ = number of conductors on the armature,
$a$ = number of parallel paths in the armature winding.

Then $\Phi \times P$ = flux cut by each conductor in one revolution,

and $\frac{N}{60}$ = revolutions per second.

Then $\frac{\Phi \times P \times N}{60}$ = flux cut per second per conductor,

and $\frac{Z}{a}$ = number of conductors in series in each path.

The average emf induced in the armature can then be expressed as:

$$E = \frac{\Phi \times P \times N}{10^8 \times 60} \times \frac{Z}{a}$$

which is generally written

$$E = \frac{\Phi PZN}{10^8 \times 60\, a} \tag{1}$$

The relations expressed by this equation are so important that it is often called the fundamental equation of the generator.

**Example 4.** The generator in Fig. 31–10 has 36 conductors, 4 poles and 4 paths. Assume the flux per pole is 3,000,000 lines and that the armature is running at 1100 revolutions per minute (rpm). What will be the average value of the emf generated?

**Solution.**

$$E = \frac{\Phi PZN}{10^8 \times 60 \times a} = \frac{3{,}000{,}000 \times 4 \times 36 \times 1100}{10^8 \times 60 \times 4} = 19.8 \text{ volts.}$$

Note here that in any given generator, the number of poles, $P$, the number of armature conductors, $Z$, and the number of parallel paths, $a$, is fixed, so that only the terms $\Phi$ and $N$ in equation (1) can be changed. Thus the emf of the machine depends directly upon flux per pole and the speed at which the machine is driven. If the flux per pole is doubled, speed remaining constant, the emf will be doubled; or if the speed is doubled, flux remaining constant, the emf will be doubled.

**Prob. 16–10.** A 6-pole, 6-path d-c generator has 3,000,000 lines per pole. The armature has 216 conductors. Speed = 1200 rpm. What is the emf of the generator?

**Prob. 17–10.** What emf is induced in an 8-pole, 8-brush generator having 1000 conductors? Speed = 750 rpm. $\Phi$ = 4,500,000 lines per pole.

**Prob. 18–10.** What emf would be induced in the generator in Prob. 17, if the speed were increased to 1000 rpm? All other data the same.

**Prob. 19–10.** What emf would be induced in the generator in Prob. 17, if the flux per pole were reduced to 4,000,000 lines? All other data the same.

**Prob. 20–10.** A bipolar generator has a field strength of 60,000 lines per square inch of pole face. Each pole face has an area of 35 square inches. Speed = 1750 rpm. Number of conductors is 240. What is the emf of the machine?

**18. Saturation, or Magnetization, Curve.** It was shown in the previous article that the emf of a generator is directly proportional to the flux per pole, if the machine is driven at constant speed. Now the flux is produced by the ampere-turns of the field coils; and we have found that the flux varies with, but is not directly proportional to the ampere-turns, as witnessed by the shape of the *B–H* curves in Chapter VIII.

The relation existing between the field ampere-turns and the flux per pole in a generator driven at constant speed is shown in Fig. 37–10. This curve does not start at zero flux, but at point $R$, due to the residual magnetism in the poles. At low values of flux, most of the field ampere-turns are used in overcoming the reluctance of the air gap, and the curve is straight. But as the field poles approach saturation, the curve begins to bend at the point $S$, as shown. This is known as the saturation or magnetization curve of the machine.

Since the emf, or volts generated, is directly proportional to the

flux per pole, and since the field ampere-turns are directly proportional to the field current (as the turns in the field winding are fixed), the saturation curve can be plotted between field current and volts generated. This is shown as curve $M$ in Fig. 38–10.

If the saturation curve of a generator is obtained, say at the normal speed of the machine, the curve for any other speed may be calculated, since emf is directly proportional to speed. For instance, in Fig. 38–10, if curve $M$ is obtained at a speed of 1000

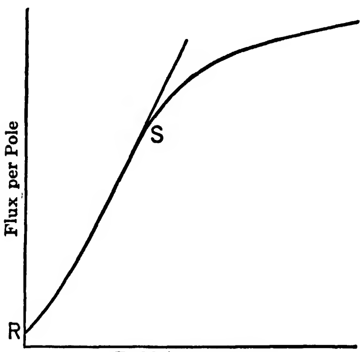



FIG. 37–10. Saturation, or magnetization, curve of a generator plotted between field ampere-turns and resulting flux per pole.

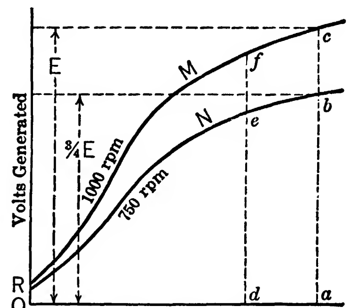



FIG. 38–10. Saturation curves plotted between field current and generated voltage.

rpm, the curve $N$ at $\frac{3}{4}$ speed, or 750 rpm, can be calculated. For a field current $oa$, the voltage generated at 1000 rpm is $ac$. At 750 rpm, for the same field current $oa$, the voltage generated will be $ab$, exactly $\frac{3}{4}$ of the voltage $ac$; or $\frac{ab}{ac} = \frac{750}{1000}$; also at a field current $od$, $\frac{de}{df} = \frac{750}{1000}$. That is, the ordinates of points on curve $N$ are just $\frac{3}{4}$ of those on curve $M$ for the corresponding field currents. Curves at any other speed may be similarly determined.

**19. Determination of the Saturation Curve of a Generator.** When the normal saturation curve of a generator is to be determined experimentally, a current is supplied to the field winding from a separate source of power, and controlled by a rheostat inserted directly in the field circuit. An ammeter, inserted in the field circuit, measures the field current; and a voltmeter, connected directly across the armature terminals, reads the emf (since the $IR$ drop in the armature is negligible). The machine, of course, is not

loaded; that is, it is not delivering current. It is driven at rated speed and successive values of emf are obtained from the voltmeter readings for various values of field current. Speed must be kept constant while the readings are being taken.

If the machine has a high resistance field winding such as in a "shunt" generator, it is convenient to use a "potentiometer" connected rheostat, which enables the curve to be started from the

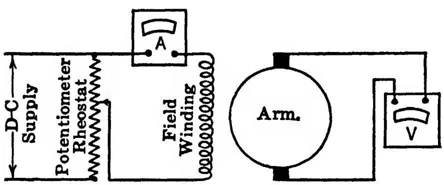



FIG. 39–10. Connections for obtaining a saturation curve.

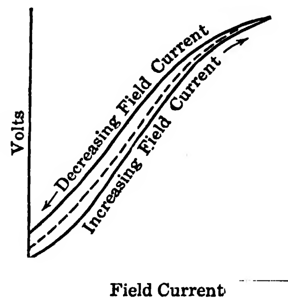



FIG. 40–10. A saturation curve plotted for both increasing and decreasing values of field current. The difference in the curves is due to hysteresis. Broken line is the average, or mean, of the two curves.

zero value of field current. The connections for obtaining a saturation curve are shown in Fig. 39–10. If the turns in the field winding are known, the curve can be plotted between volts and field ampere-turns.

The values of field current should be **increased** continuously in one direction from zero to a maximum for a curve of increasing field current; or decreased continuously from a maximum to zero for a curve of decreasing field current. These two curves are not the same, as shown in Fig. 40–10, due to the lagging hysteresis effect in the iron of the magnetic circuit. These two curves are equivalent to a portion of a hysteresis loop of a magnetic circuit, composed partly of iron and partly of air.

The performance of a generator under load can be determined from the saturation curve. When this is desired, it is convenient to plot the curve between **mean** values (the broken curve in Fig. 40–10) and **ampere-turns per pair of poles.** Curves so plotted will be used later in this chapter.

**20. Function of a Generator.** Sources of mechanical energy for driving generators are generally steam and water turbines or steam and internal combustion engines. In general, each of these machines operates at its best efficiency at some particular speed and so it is operated approximately at this speed. Speed may be

held constant by some governing device. Generators are, therefore, driven by these machines at constant, or approximately constant, speed.

We know that it is desirable to have a constant voltage on lamps and other appliances. For this reason, electrical energy should be distributed and used at constant pressure (except for series lighting circuits). Therefore, a **satisfactory generator, driven at a constant speed, should deliver electrical energy** from its terminals **at a constant, or approximately constant, voltage.**

From the equation $E = \frac{\Phi PZN}{10^8 \times 60\,a}$, we know that if the speed, $N$, and flux, $\Phi$, in a generator are constant, the emf is constant. However, it is not so easy to keep the terminal pressure constant; first, because there is a loss of pressure due to the resistance ($IR$) drop in armature; and second, it is difficult to keep the magnetic flux constant. As we know, the flux is obtained by a current flowing in the field winding placed on the poles of the machine. There are several possible ways of connecting the field windings, or exciting the fields, as it is called. These are discussed below.

**21. Field Excitation of D-C Generators.** Direct-current generators and motors may be classified according to the way the field windings are connected. Generators may be, (*a*) **separately excited**; or (*b*) **self-excited.**

(*a*) In the **separately excited** generator, current is supplied to the field coils from an outside source, as from a storage battery, or from another generator. See Fig. 41*a*–10. The field winding consists of a large number of turns of small wire carrying a low value of current; therefore its resistance is comparatively high. This winding, as are all field windings, is made up of as many identical coils as there are poles, and the coils are connected in series. The machine is operated with a rheostat inserted in series with the field circuit.

(*b*) In the **self-excited** generator, current to excite the field coils is drawn from the armature of the machine itself. **Self-excited** generators may be of three types, **shunt, series** and **compound.**

In the **shunt-generator,** the field circuit is connected directly across the armature terminals, generally in series with a field rheostat, as shown in Fig. 41*b*–10. This winding is really in **shunt** with the **external circuit,** and this method of connecting it gives it this name. Note that after the current leaves the positive brush, it divides; one part flowing through the load and the other part flowing through the field coils. This shunt winding must have a

comparatively high resistance so that it will not take too much current from the armature and, therefore, must consist of a relatively large number of turns to supply the necessary flux.

The **series generator** has a field winding of comparatively few turns, connected in **series** with the armature and the load, as in Fig. 41*c*–10. Since the winding must carry the entire armature-and-load current, it is wound of large wire and the resistance is relatively low.

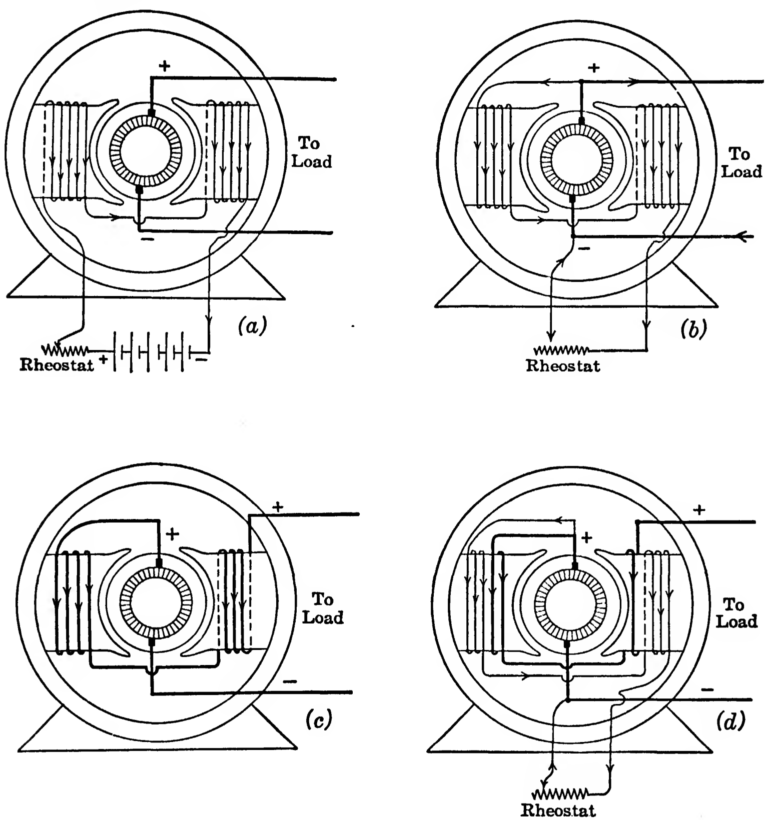



FIG. 41–10. (a) Arrangement and connection of the field coils in a separately excited generator. (b) Connections in a shunt generator. (c) Connections in a series generator. (d) Connections in a compound generator—short-shunt.

The **compound generator** is a combination of the shunt and series types, having both the high-resistance shunt winding and the low-resistance series winding. Each of the two windings is divided into as many coils as there are poles, and all the coils of each winding

are connected in series. Thus in the compound generator, each pole of the machine has wound upon it two coils; one a part of the series winding and the other a part of the shunt winding. See Fig. 41*d*–10. When the current in the series coil is in such a direction that it sets up a flux in the **same** direction as that set up by the shunt coils — that is, when the series coils aid the shunt-field flux — it is called a **cumulative** compound generator. When the series coils set up a flux, or mmf, in the opposite direction to that of the shunt-field winding, it is called a **differential** compound generator.

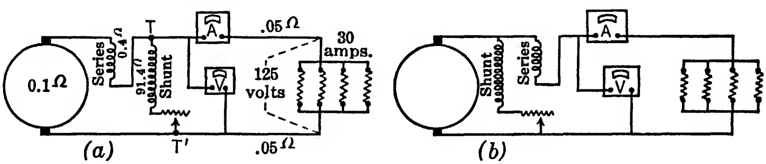



Fig. 42–10. Conventional diagram of a compound generator connected to a load. (a) Long-shunt connection. (b) Short-shunt connection.

When the shunt field with its rheostat is connected as in the conventional diagram of Fig. 42*a*–10, it is called a **long-shunt** connection; when connected as in Fig. 42*b*–10, it is called a **short-shunt** connection. Note that with the long-shunt connection, the series winding must carry the shunt-field current.

**Example 5.** The long-shunt compound generator of Fig. 42*a*–10 is supplying 30 amperes to a load at 125 volts. Resistance of line wires is 0.05 ohm each. Resistance of shunt-field circuit is 91.4 ohms. Resistance of armature is 0.1 ohm, and that of the series field is 0.40 ohm. (*a*) What is the terminal voltage of the generator? (*b*) The shunt-field current? (*c*) The current in the armature and series field? (*d*) The emf of the generator?

**Solution.**

(*a*) Terminal voltage $= 125 + (0.05 + 0.05)30 = 125 + 3$
$= 128$ volts across terminals $TT'$.

(*b*) Shunt-field current $= \dfrac{128}{91.4} = 1.4$ amperes.

(*c*) Armature and series field current $= 30 + 1.4 = 31.4$ amperes.

(*d*) $IR$ drop in armature and series field $= 31.4(0.1 + 0.4) =$ 15.7 volts.

Emf of generator $= 128 + 15.7 = 143.7$ volts.

**Prob. 21–10.** A shunt generator has a terminal voltage of 220 volts when delivering 65 amperes to a load. (*a*) If the resistance of the armature is 0.035 ohm, and that of the field circuit is 160 ohms, what is the generated emf? (*b*) If the resistance of the field winding is 108.5 ohms, how much resistance is there in the shunt-field rheostat?

**Prob. 22–10.** If the generator in Example 5 has 4 poles, 4 brushes, a flux per pole of 2,500,000 lines and 360 conductors on the armature, at what speed does it run?

**Prob. 23–10.** Answer the questions in Example 5, if the generator is connected short-shunt.

**Prob. 24–10.** Assume the generator in Fig. 42*a*–10 has the following data: armature resistance = 0.08 ohm; series-field resistance = 0.06 ohm; resistance of shunt-field winding = 80.5 ohms; resistance in the field rheostat = 45 ohms; speed = 900 rpm; flux per pole = 3,000,000 lines. There are 6 poles and 6 brushes. If the terminal voltage is 230 volts when the machine is delivering a load of 50 amperes, how many conductors are there on the armature?

**Prob. 25–10.** A shunt generator is supplying 60 amperes to a load at a terminal voltage of 230 volts. Resistance of the armature = 0.15 ohm; resistance of the field circuit = 59 ohms; number of poles = 4; rpm = 1140; conductors on the armature = 180; flux per pole = 3,500,000 lines. How many parallel paths are there in the armature?

**22. Armature Reaction.** If the armature of the two-pole generator of Fig. 43–10 (*a–k*) is rotated clockwise, the direction of the emf in the conductors and, consequently, the direction of the current which will flow, if the external circuit is closed, is indicated by the dots and crosses in Fig. 43*a*–10. This agrees with Fleming's Right-Hand Rule. For the sake of clearness, the conductors are not placed in slots, and the commutator is not shown. The brushes are here placed on the axis *xy*, called the geometrical or mechanical neutral. This is the position which we have previously considered to be the proper one to short-circuit the armature coils while they are not cutting the flux (except for the slight shift necessary, due to the self inductance of the coils undergoing commutation).

Figure (*b*) shows the flux, due to the mmf of the field windings, in the air gaps and armature core **when no current flows in the armature conductors.** This flux is parallel to the axis of the poles, 90° to the brush axis *xy*, and is the only flux in the air gaps.

Now consider Fig. (*c*) which shows **only** the flux set up by the armature conductors **when they are carrying current.** The conductors act like the turns of a coil and set up an mmf and a flux, shown by the arrows, downward in the diagram at 90° to the field mmf and flux. Note that this armature mmf, which is called the **armature reaction, does not occur** when the machine is running **without load.**

Thus it is seen that when the generator supplies a load, there are two separate mmf's at 90°, acting on the magnetic circuit at the

same time. Since most of each of these mmf's is used to set up the flux in the air gaps, fluxes may be added in the same manner we add mmf's. So these two fluxes, or fields, set up a resultant field, or flux, displaced from the axis of the field poles, as shown by the

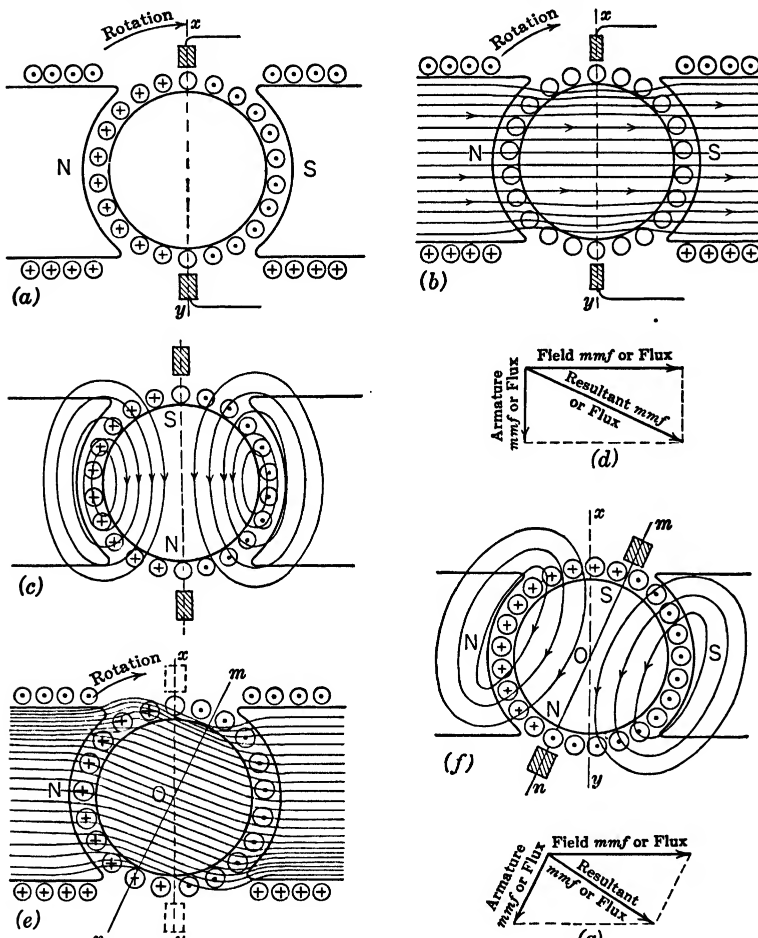



FIG. 43–10. Armature reaction in a two-pole generator. (a) Direction of induced voltage and current in armature conductors. (b) Direction of mmf, or field, set up by field ampere-turns only. (c) Direction of mmf, or field, set up by armature ampere-turns only. (d) Resultant of both armature and field mmfs. (e) Distortion of flux in the air gap due to current in both armature and field windings. (f) Direction of armature mmf after the brushes have been shifted. (g) Resultant of both armature and field mmfs after brushes have been shifted.

vectors in Fig. (*d*). Note in Fig. (*e*) that the resultant flux in the air gap has now shifted, becoming more dense in the trailing pole tips (the tips of the poles from which the armature conductors recede) and less dense in the leading pole tips. Also note that the neutral axis has shifted to the position *mn*, and the brushes (shown dotted) are no longer at the proper position for commutation. To prevent sparking, the brushes must be shifted to this new neutral axis, *mn*. There is now, due to the new position of the brushes, a change in the direction of the current in those conductors within the angles of brush shift, *xom* and *yon* in Fig. (*f*). The direction of the armature mmf, or flux, has now changed so that it is in a direction in the figure, downward and to the left, at an angle greater than 90° with the field mmf. This is shown by the vectors in Fig. (*g*), and it is to be noted that the resulting mmf is less than before.

This decrease in mmf can be determined if we consider Fig. (*h*). The conductors within the angle of brush shift, $S$, set up an mmf in opposition to that of the field poles; and since the conductors within an equal angle, $S'$, have an identical effect, we can divide the armature conductors into two groups: those within the double angles $S + S'$, and those outside these angles.

Figure (*i*) shows only those conductors within the double angles. These conductors act like a coil whose ampere-turns set up an mmf in direct opposition to that of the field ampere-turns. They decrease the effective flux in the air gaps and, of course, decrease the emf of the machine. The ampere-turns due to these conductors are called the **demagnetizing or back ampere-turns.** Note that the turns of this "coil" are equal to the number of conductors within **one double angle.**

The remaining conductors on the armature, shown in Fig. (*j*), have also the effect of a coil, the ampere-turns of which set up an mmf at 90° to the field mmf and simply distort the field flux. These are known as the **cross-magnetizing ampere-turns,** and affect the emf of the generator only slightly.

There are, therefore, in addition to the mmf of the field, two distinct mmf's set up in the armature of a generator when it is carrying a load — **the demagnetizing mmf** and the **cross-magnetizing mmf.** These two mmf's comprise what is called **armature reaction.** The relation between the mmf's set up by the field ampere-turns, the back ampere-turns and the cross ampere-turns is illustrated by the vectors in Fig. (*k*).

Armature reaction in multipolar machines is a little more difficult to visualize, but occurs in exactly the same way as in the two-pole generator.

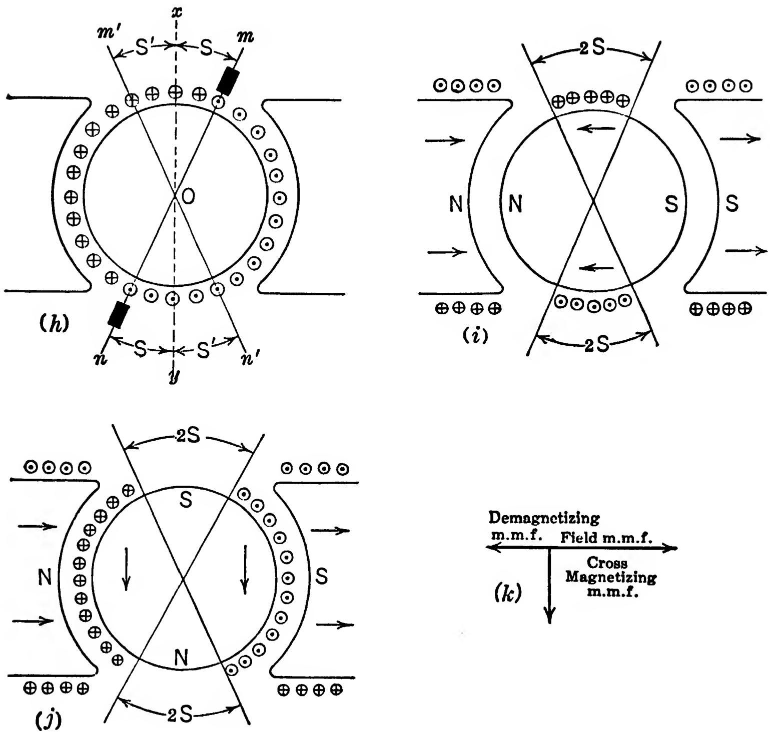



FIG. 43–10 (*Continued*). Armature reaction in a two-pole generator. (h) S plus S′ is the double angle of brush shift. (i) Demagnetizing armature ampere-turns. (j) Cross-magnetizing armature ampere-turns. (k) Three distinct mmfs due to field ampere-turns, armature demagnetizing ampere-turns and armature cross-magnetizing ampere-turns.

**23. Distribution of Flux in the Air Gaps.** The distribution of the flux in the air gaps of a generator may conveniently be shown by the curves of Fig. 44–10. The armature is shown as a flat surface with the poles in their relative position to the armature conductors. In (*a*), the curve shows the distribution of the field flux in the air gap when no current flows in the armature conductors. The height, *h*, of the curve is proportional to the flux density in the air gaps, and is uniform under the entire surface of the pole face. The flux is zero over a considerable space between the poles so

that the brush can easily be set in a neutral position. Compare Fig. 43*b*–10.

The curve in (*b*) represents only the air gap flux due to the

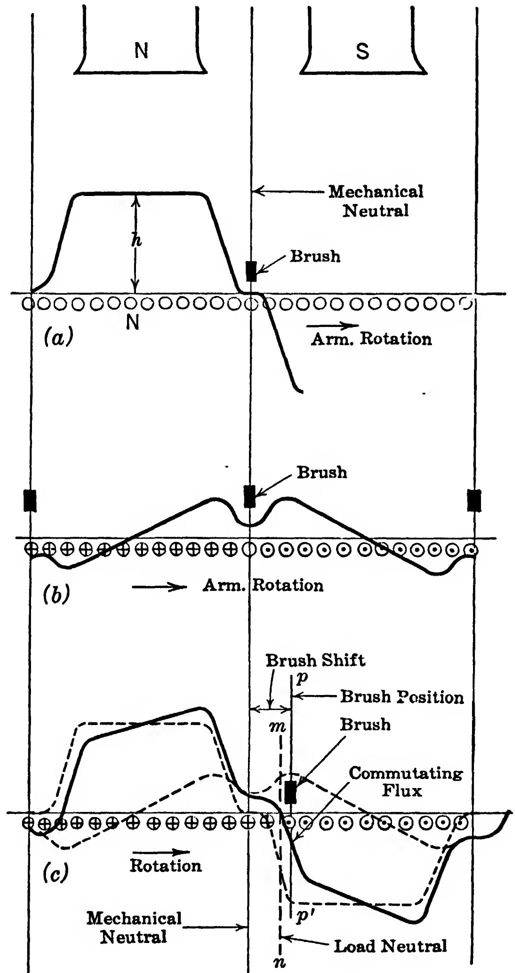



FIG. 44–10. Distribution of flux in the air gap. (a) Flux curve due to field mmf. (b) Flux curve due to armature mmf. (c) The heavy line is the combination of the two flux curves in (a) and (b).

current in the armature. Note that this curve is displaced 90° from that in (*a*). Compare Fig. 43*c*–10.

The heavy curve in (*c*) is the resultant of the field and armature

fluxes in (*a*) and (*b*). Note that this curve is distorted; that the flux density is greatest under the trailing pole tip; and that the position of zero flux, and therefore of the brush, is shifted in the direction of rotation, to the line *mn*. Compare Fig. 43*e*–10. In fact, the brush must be shifted to the position *pp'* in order that the coil undergoing commutation may cut some flux in the opposite direction (the commutating flux). This is due to the self inductance of the coil, as previously explained in Art. 16. Also note that this flux curve is very steep as it passes through the position *pp'*; therefore, the neutral zone is narrow and the brush position must be adjusted between very close limits.

The shape of the resultant flux curve in (*c*) changes somewhat with different values of armature current, so the proper position for the brushes also changes with armature current. The brushes are generally set in the position to give the best commutating performance at, or somewhere near, full-load armature current.

**24. Performance Curves.** The electromotive force of a generator has been defined as the voltage generated by the machine. The **terminal voltage** depends not only upon how much emf is induced, but also upon what current is being delivered by the machine. For constant-voltage service, the terminal voltage of an ideal generator must not change, regardless of the amount of current delivered. As more and more current is taken from the actual machine, the terminal voltage generally changes. This relation between terminal voltage and current output plotted as a curve is called a **load curve.** If this load curve is obtained under conditions such that the terminal voltage at full-load current output is the rated voltage of the generator, the curve is called the **external characteristic.** In the choice of a generator for a specified service, this characteristic curve gives us the data we must have concerning the performance of the machine.

**Note.** Most modern d-c generators are equipped with additional windings on auxiliary poles spaced midway between the main poles called "interpoles," or "commutating poles." In many cases, machines of both moderate and large size are also equipped with "compensating" windings. These windings are placed in slots in the faces of the main poles. By these means, the shape of the external characteristic may be somewhat altered, and better performance, or specified performance for special service, obtained.

However, in order to properly appreciate the function of these

additional windings, the performance of the standard types of d-c generators, not so equipped, is first discussed in the following paragraphs.

**25. Load Curve of the Separately Excited Generator. External Characteristic.** See Fig. 41a–10. The relation between terminal voltage and load current, or the load curve, of a separately excited generator can be obtained by test, or may be calculated.

To obtain a load curve by test, the machine is connected as in Fig. 45–10, and the load current, measured by ammeter $A_L$, is increased in increments from zero to at least the rated value; while readings of the voltmeter, which indicates terminal voltage, are taken for each value of load current. The field current must be held constant and, therefore, an ammeter in the field circuit is also necessary. The machine must be driven at constant speed.

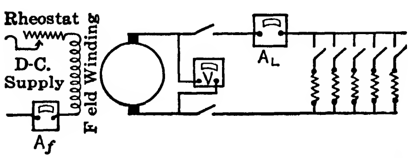


FIG. 45–10. Connections for obtaining the characteristic curve of a separately excited generator.

If the **external characteristic** is to be obtained by test, the proper value of field current is determined by first taking full-load current from the machine and adjusting the field to give the rated, or normal terminal voltage. The machine must be driven at normal speed.

The general shape of the load curve is shown in Fig. 47–10. Note that as the current output is increased, the terminal voltage decreases. A generator in which the terminal voltage drops, as the load current is increased, is said to have a **drooping** characteristic. This droop in the curve with increased load current is caused by two things:

**First,** the increase in $IR$ drop in the armature, brushes and brush contact;

**Second,** the demagnetizing action of the back ampere-turns due to the shifting of the brushes.

Let us now consider a low-voltage two-pole generator with a rated full-load current of 100 amperes. Assume the armature resistance (with brushes) to be 0.03 ohm, and the brush shift to be such that there are 6 conductors in the double angle. (Note Fig. 43i–10.) The magnetization curve for this machine is shown in Fig. 46–10.

At 25 volts, no load, the field current produces 1500 ampere-

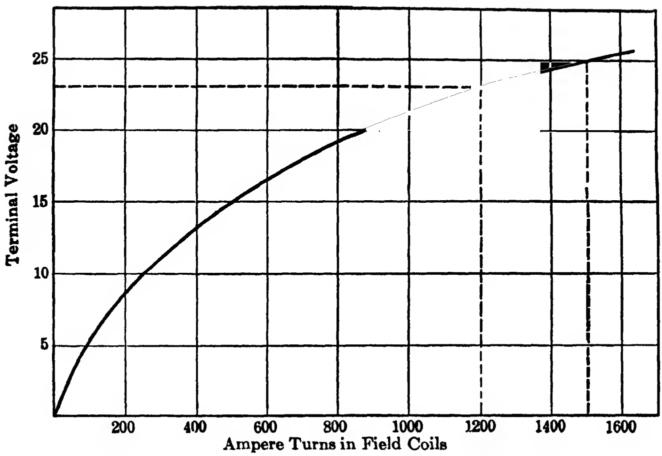



FIG. 46–10. A magnetization or saturation curve.

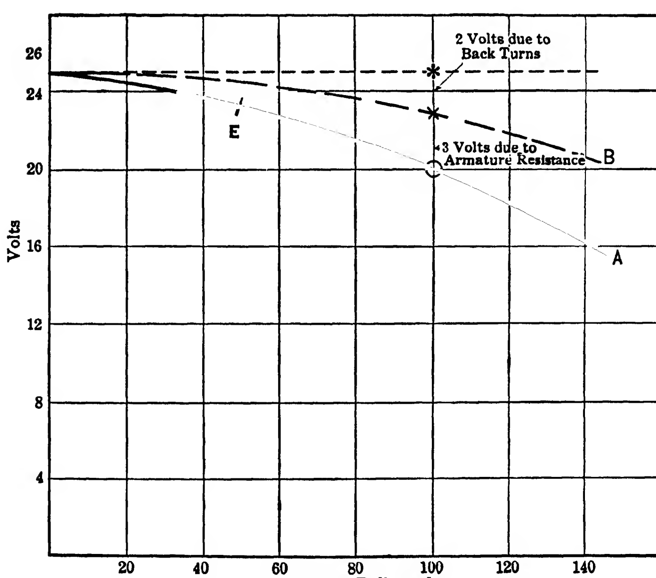



FIG. 47–10. Curve A is the external characteristic of a separately excited generator, having the magnetization curve of Fig. 46–10.

turns in the field winding, Fig. 46–10. When the machine is delivering 100 amperes, 50 amperes must flow through each conductor on the armature (as there are two paths in a two-path machine). Since there are 6 back turns on the armature, due to the position of the brushes, there must be 6 × 50 or 300 demagnetizing ampere-turns in the armature at full-load current. This would reduce the magnetizing force of the field from 1500 to 1500 − 300, or 1200 ampere-turns, which in turn would reduce the voltage to 23 volts, shown as point $x$ on curve $B$, Fig. 47–10. The generator, therefore, generates only 23 volts at full-load current, instead of 25 volts as at no load.

At half load of 50 amperes, the demagnetizing action of the "back" turns would be 25 × 6, or 150 ampere-turns, and the total magnetizing effect of the field equals 1500 − 150, or 1350 ampere-turns. This would produce 24.1 volts, according to Fig. 47–10. Thus we might draw a curve between the actual generated voltage and the load on the generator. This curve would be like curve $B$, in Fig. 47–10.

The curve $A$ of terminal voltage is now obtained by subtracting the armature $IR$ drop from this generated voltage curve. Thus the point $O$ of 20 volts is obtained by subtracting 100 × 0.03, or 3 volts, from the voltage of point $x$, or 23 volts.

We have seen that for half load, the generated voltage is 24.1 volts. The armature $IR$ drop is 50 × 0.03 = 1.50 volts. The terminal voltage for half load is 24.1 − 1.50 = 22.6 volts, which is point $E$ on curve $A$.

If 20 volts, the terminal voltage at full load of 100 amperes as found above, is the normal or rated voltage of the generator, curve $A$ is the external characteristic curve of the machine. This curve shows the "inherent" or automatic behavior of the generator as load is applied.

Thus the performance, or external characteristic, or load curve of a separately excited generator can be calculated as follows:

**First,** from the saturation curve and back ampere-turns for various loads, determine the curve between generated voltages and load currents;

**Second,** subtract the respective values of armature $IR$ drop from the generated voltages for the different load currents.

**Note.** In the application of the saturation curve in the above discussion, both the field ampere-turns and the back ampere-turns for two poles are used. Since the field mmf is due directly to a

pair of poles, and the back ampere-turns are due to the shifting of two brushes, we may compute these two effects in terms of pairs of poles. Thus in multipolar machines, we use this same relationship; and the saturation curve may be plotted, **using field ampere-turns per pair of poles, and not the total number of field ampere-turns.**

From the discussion above, it is obvious that the separately excited generator does not give a constant terminal voltage with change in load (current output), although the droop of the curve is not excessive. The main objection is that it requires a separate generator to excite the field, which adds to the complication of circuits and increases the cost. Separately excited generators are generally used only where the armature voltage is not suitable to excite the field. They are used for low-voltage machines for electroplating, and as high-voltage machines for supplying the plate voltage for radio transmitting tubes.

**Example 6.** The saturation curve of a 15-kw, 230-volt, 6-pole, 6-brush generator operated at 1200 rpm is shown in Fig. 48–10. The armature has 380 conductors and a resistance of 0.18 ohm.* The brushes have a 4° lead (forward shift). Full-load current is 65 amperes. The resistance of the field circuit with field rheostat is 125.5 ohms, and the field is separately excited from 110-volt mains. The field winding has 700 turns per pair of poles.

(*a*) Determine the field current and the ampere-turns in the field per pair of poles.

(*b*) What is the no-load voltage of the machine?

(*c*) What is the full-load voltage?

**Solution.**

(*a*) $$\text{Field Current} = \frac{110}{125.5} = 0.875 \text{ ampere.}$$

$$\text{Field ampere-turns} = 700 \times 0.875 = 613 \text{ ampere-turns per pair of poles.}$$

* In all problems in this text, unless otherwise stated, the term armature resistance includes the resistance of the armature windings and of brush contact. The resistance of the windings is constant at normal temperature, but the resistance of brush contact decreases in almost direct proportion to the increase in current. This causes the voltage drop between each brush and the commutator to remain practically constant from $\frac{1}{2}$ load up to heavy overloads. For carbon brushes of medium hardness, this drop is 1 volt per brush or 2 volts per machine.

The American Institute of Electrical Engineers advises that an allowance be made for 2 volts drop for the brush-contact resistance of d-c machines with ordinary carbon brushes, and that the resistance of the armature be considered that of the windings only. However, it usually causes very slight error to consider the resistance of windings and contact as constant, as will be done in this text.

(Note here that 700 ampere-turns per pair of poles requires $\frac{700}{700}$ or 1 ampere in the field; 350 ampere-turns requires $\frac{350}{700}$ or 0.5 ampere in the field; and a scale of field current can also be laid out for the saturation curve as shown in Fig. 48–10.)

(*b*) From the saturation curve, 613 ampere-turns will generate 245 volts — no-load voltage.

(*c*) Back turns in the armature per pair of poles

$$= \frac{4 \times 2}{360} \times 380 = 8.45 \text{ turns (not ampere-turns).}$$

Back ampere-turns per pair of poles at full-load current

$$= 8.45 \times \frac{65}{6} = 92 \text{ ampere-turns.}$$

Resulting ampere-turns per pair of poles at full-load current

$$= 613 - 92 = 521 \text{ ampere-turns.}$$

Volts induced by 521 ampere-turns (from saturation curve, Fig. 48–10) = 229 volts.

Voltage drop in the armature at full load = 65 × 0.18 = 11.7 volts.

Terminal voltage at full load = 229 − 11.7 = 217.3 volts.

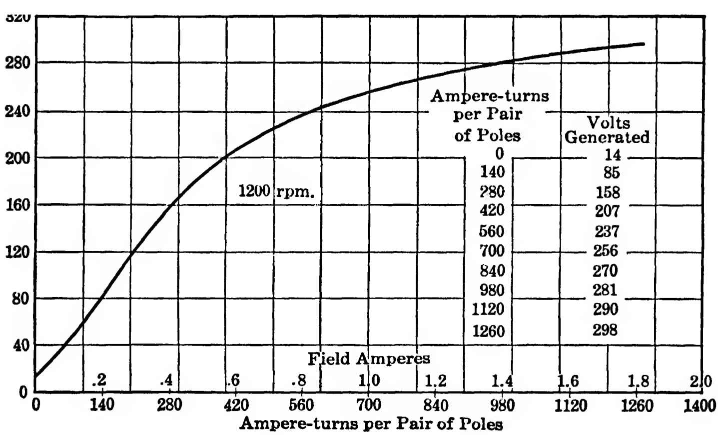



FIG. 48–10. Saturation curve for the generator of Example 6.

**Prob. 26–10.** What will be the terminal voltage of the generator in Example 6 when delivering 20 amperes? Same field excitation (current).

**Note.** In solving this, and all problems involving the use of the saturation curve, the student is advised to plot the curve on finely divided coordinate paper.

**Prob. 27–10.** Determine the terminal voltage of the generator in

Example 6 for load currents of 0, 10, 20, 30, 40, 50, 60, and 65 amperes, and plot a load curve for this machine.

While the terminal voltages, determined in Example 6 and the problems above, are points on a load curve, they **are not** points on the normal external characteristic curve.

By definition, the normal external characteristic or performance curve of a separately excited generator is obtained at that field excitation which will develop the rated terminal voltage when the machine is carrying full-load current. This may be determined as follows:

**Example 7.** (*a*) What field current in the generator of Example 6 will produce rated terminal voltage at full-load current? (*b*) What no-load voltage will this field current generate? Speed, 1200 rpm.

**Solution.**

(*a*) Rated terminal voltage = 230 volts.
Armature *IR* drop at full load = $65 \times 0.18 = 11.7$ volts.
Voltage generated at full load = $230 + 11.7 = 241.7$ volts.
From saturation curve (Fig. 48–10) effective ampere-turns = 595.
From Example 6, back ampere-turns = 92.
Field ampere-turns must be $595 + 92 = 687$
Field current = $\frac{687}{700} = 0.98$ ampere.

(*b*) From the saturation curve (Fig. 48–10), 687 ampere-turns generate 255 volts, which is the no-load voltage.

**Prob. 28–10.** Construct the external characteristic curve of the generator in Example 7. Plot values of terminal voltage for the same load currents as in Prob. 27–10.

**26. Regulation and Control.** Another way of stating the performance or characteristic of a generator is to give its "voltage regulation."

By **regulation** is always meant some change that automatically takes place when the load is changed. Unless it is otherwise stated, the change in the load is always assumed to be between full load and no load. The American Institute of Electrical Engineers gives a definition for regulation as the difference between no-load and full-load terminal voltage, expressed as a percentage of full-load voltage, the voltage at full load being the normal, or rated voltage of the machine. Thus,

$$\frac{E \text{ (no load)} - E \text{ (full load)}}{E \text{ (full load)}} \times 100 = \text{percentage regulation.}$$

The smaller the value of the regulation, the better is the performance of the machine for most purposes.

By **control** is meant some change that is **made to take place** in the load curve by the manipulation of auxiliary apparatus. For instance, as will be seen in the next article, an attendant may **change** the **field current** of a generator by adjusting the resistance in the field circuit, and therefore change the terminal voltage.

**Regulation** refers to some change in terminal voltage that is inherent in the design of the machine, and takes place when the load is changed. **Control** refers to some change in terminal voltage which can be produced by manipulation of auxiliary apparatus, whether the load is changed or not.

The regulation of a generator may be determined under test by putting full-load current on the machine, when it is driven at normal speed, with the field current adjusted to give the rated terminal voltage. The load is then thrown off, the speed and field current being held constant, and the no-load voltage measured.

Or the full-load and no-load voltages may be computed from the saturation curve, the back ampere-turns and armature resistance drop, as in Example 7. The regulation of the generator in that example is—

$$\frac{255 - 230}{230} \times 100 \quad \text{or} \quad 10.85 \text{ per cent.}$$

**Prob. 29–10.** (*a*) What will be the no-load voltage and the percentage regulation of the generator in Example 7, if operated as a 240-volt generator (240 volts terminal voltage at same full-load current)? (*b*) If operated as a 200-volt generator? (*c*) What conclusions can you draw as to why you obtain the different values you do for percentage regulation in (*a*) and (*b*)?

**Prob. 30–10.** (*a*) Construct the external characteristic curve of a 10-kw, 230-volt, 1100-rpm, separately excited generator having the following data:

VALUES FOR SATURATION CURVE AT 1100 RPM

| Ampere-turns per Pair of Poles | Generated Volts | Ampere-turns per Pair of Poles | Generated Volts |
|---|---|---|---|
| 0 | 10 | 600 | 254.5 |
| 150 | 96 | 750 | 271.0 |
| 225 | 140 | 900 | 282.0 |
| 300 | 177 | 1050 | 289.5 |
| 375 | 204 | 1200 | 295.0 |
| 450 | 226 | 1350 | 300.0 |

Number of poles = 4. Number of brushes = 4.
Brush lead = 4.5°. Number of armature conductors = 320.
Turns in field per pair of poles = 750. Full-load voltage = 230.
Armature resistance = 0.35 ohm. Resistance of field coils = 60 ohms.
Voltage across the field circuit = 110 volts. (*b*) Determine the percentage regulation. (*c*) What is the resistance of the field circuit?

**Prob. 31–10.** (*a*) Redraw the saturation curve of the generator of Prob. 30–10 at a speed of 1300 rpm, and construct the external characteristic of the generator at this speed. Full-load voltage is 230 volts. Field is excited from a 110-volt circuit. (*b*) Determine the percentage voltage regulation. (*c*) What is the resistance of the field circuit?

**Prob. 32–10.** Redraw the saturation curve for the generator in Prob. 30–10 at a speed of 950 rpm, and construct the external characteristic at this speed. Full-load voltage = 230 volts. Field is excited from a 110-volt circuit. What is the percentage regulation of the generator at this speed?

**Prob. 33–10.** From the results of Probs. 30–10 to 32–10, what conclusions can you draw as to the effect on regulation and droop of the characteristic of operating a separately excited generator either above or below rated speed? Explain why the results differ as they do.

**27. Voltage Control of the Separately Excited Generator.** It was shown in the previous articles that the characteristic curve shows the " inherent " or automatic performance of the machine when the load on it is changed. We have seen that the terminal voltage decreased, or dropped, with increase in load, which is undesirable for constant-voltage service.

However, the voltage of a separately excited generator can be **controlled** by the rheostat, shown in Fig. 45–10, in series with the field circuit. An attendant, by **increasing** this resistance and thus **decreasing** the field current, can **lower** the terminal voltage, or, by **decreasing** this resistance and **increasing** the field current, can **raise** the terminal voltage. The greater the droop of the characteristic with increase of load, the greater must be the amount of resistance " cut out " of the rheostat. To keep the terminal voltage constant, with a rapidly changing load, necessitates continual changing of the resistance in the rheostat, which obviously is a disadvantage. The rheostat can, however, be adjusted to give any desired terminal voltage for a given load, increasing it to the point where all the resistance in the rheostat is " cut out."

**Example 8.** How much resistance must be added, or " cut in," to the rheostat in the field circuit of Example 6 in order to adjust the no-load voltage to 217.3 volts? Voltage applied to the field circuit to remain at 110 volts.

**Solution.**

In Example 6, the resistance of the field circuit is 125.5 ohms and the resulting field current gives a full-load terminal voltage of 217.3 volts.

To generate 217.3 volts at no load (from Fig. 48–10) requires 460 ampere-turns, and $\frac{460}{700}$ or 0.657 ampere in the field circuit.

$$\frac{110}{0.657} = 167.5 \text{ ohms in the field.}$$

Resistance added to the field circuit = 167.5 − 125.5 = 42 ohms.

**Prob. 34–10.** What must be the change in resistance of the field rheostat of Example 6 to produce a terminal voltage of 220 volts at a load of 30 amperes?

**Prob. 35–10.** What is the resistance in the field rheostat in Prob. 30–10? In Prob. 31–10?

**Prob. 36–10.** How much resistance must be inserted in the field rheostat of the generator in Prob. 30–10 to produce a terminal voltage of 175 volts when the machine is delivering 25 amperes?

**Prob. 37–10.** What must be the current in the field winding of Prob. 30–10 to produce a terminal voltage of 245 volts at a load of 30 amperes and a speed of 1000 rpm?

**Prob. 38–10.** At what speed must the separately excited generator of Prob. 30–10 be operated in order to produce 220 volts at full load, if the resistance of the field circuit is increased to 200 ohms? Field is excited from 110-volt mains.

**28. Building Up and Characteristic Curve of the Series Generator.** The series generator is self excited and the armature, the load and the field windings are in series and carry the same current, as shown in Fig. 41*c*–10. The conventional diagram is shown in Fig. 49–10. When the generator is not delivering current, there is no current in the field windings and no mmf is set up. However, we know that if the field poles have at any time been magnetized, there is a small amount of flux in the air gap due to " residual " magnetism in the poles. So even at no load, a small voltage will be generated. If now the terminals of the generator are connected to an external circuit, some current will flow in the circuit, and hence, in the field windings. If this current in the windings is in the right direction to increase the field strength, the voltage across the brushes will rise. As the resis-

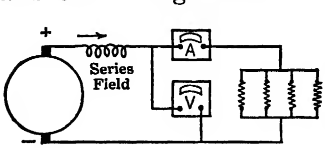


FIG. 49–10. Series generator connected to a load.

tance of the external circuit is decreased, more and more current will flow in the field windings and the voltage will continue to rise. This is called the " building up " of the generator.

The terminal voltage continues to rise until a certain point is reached, and if the current output is increased further, the terminal voltage drops as shown in Fig. 50–10. This point depends upon the shape of the magnetization curve, the armature reaction and the total resistance of the circuit.

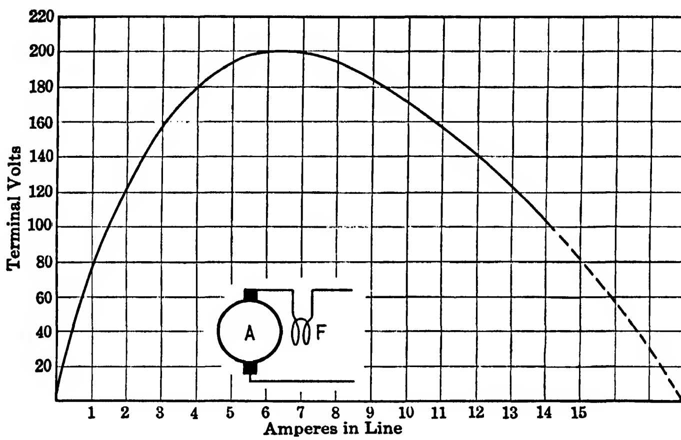



FIG. 50–10. The external characteristic of a series generator.

The curve, plotted between terminal voltage and current, is the characteristic curve of the series generator. The shape of this curve is very similar to the magnetization curve, but falls below it. There are two reasons for this drop in voltage:

**First,** the demagnetizing effect of the back ampere-turns in the armature tends to offset any increase in the magnetic field that the increased current in the field coils would tend to set up. This demagnetization by the back ampere-turns is exactly like that in the separately excited generator.

**Second,** the *IR* drop in the armature and series field coils continues to increase as the current output is increased. But the increase in *IR* drop finally becomes greater than the increase in generated voltage, which results in an actual **decrease** in the terminal voltage.

This two-fold loss in voltage can best be illustrated by considering the curves in Fig. 51–10. If we separately excite the field coils, the saturation curve may be obtained, as already described. It is

shown as curve $A$ in the figure, plotted between induced volts and field current, which in the series machine is also the load current. Curve $B$ shows the actual voltage generated when the machine is self excited and various currents are delivered to the external circuit. The difference between this curve and curve $A$ is due to the demagnetizing effect of the back ampere-turns in the armature. Curve $C$ is the actual external characteristic, or performance curve, of the generator and shows the terminal voltage at any load current. The difference between this curve and curve $B$ is due to the $IR$ drop in the armature and series field.

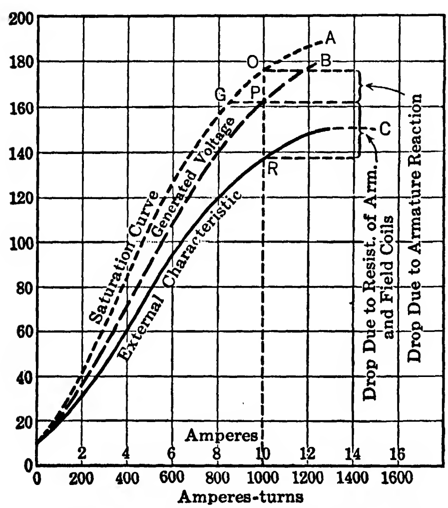



FIG. 51–10. External characteristic of the series generator of Example 9.

From the shape of the external characteristic, it is apparent that the terminal voltage of the series generator varies so widely with change in the load current that it is unsuitable as a source of supply for constant voltage service. Practically, its only use today is as a "series booster" for raising the voltage on long d-c feeder lines. It was formerly used to supply d-c series arc lights, but now alternating current is used almost exclusively as a supply for series street lighting.

The abscissae of the curves in Fig. 51–10 can represent not only amperes delivered by the generator, but also the ampere-turns per pair of poles, and points on curve $C$ can be calculated from the saturation curve, as in the example below.

**Example 9.** The saturation curve $A$ in Fig. 51–10 is that of a series generator with 2 poles, 2 brushes, and 100 field turns per pair of poles. Brushes are so set that there are 30 back turns on the armature. Resistance of the armature and series field = 2.5 ohms.

At a load of 10 amperes, field ampere-turns = $10 \times 100 = 1000$ ampere-turns; 1000 ampere-turns will generate 176 volts, which gives us point $O$ on curve $A$.

Back ampere-turns with 10 amperes load = $\frac{10}{2} \times 30$ = 150 ampere-turns.

Resulting ampere-turns = 1000 − 150 = 850 ampere-turns.

From the saturation curve *A*, we find 162 volts, point *G*, actually generated in the machine when it is delivering 10 amperes load. This gives us point *P* on curve *B*.

Volts drop due to the back ampere-turns = 176 − 162 = 14 volts.

*IR* drop in armature and series field with 10 amperes load = 10 × 2.5 = 25 volts.

Terminal voltage = 162 − 25 = 137 volts. This gives us point *R* on curve *C*.

Thus at 10 amperes load, the emf of 176 volts, set up by the ampere-turns of the field, is decreased 14 volts by the back ampere-turns of the armature to 162 volts. The *IR* drop in the armature and series field

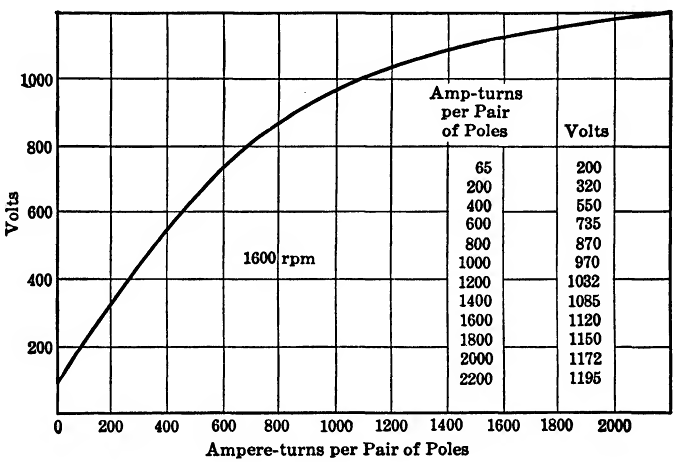



FIG. 52-10. Magnetization curve for the series generator of Prob. 39-10.

further decreases the voltage by an additional 25 volts, giving a terminal voltage of 137 volts at this load. Other points on curve *C* for different load currents can be found in the same manner.

**Prob. 39-10.** Figure 52-10 is the magnetization curve of a series generator. The other data are:

Turns in field per pair of poles = 100.
Number of armature conductors = 800.
No. of poles = 4. No. of brushes = 2.
Brush shift = 9°. Speed = 1600 rpm.
Field resistance = 2.5 ohms. Armature resistance = 3.5 ohms.

Construct the external characteristic, carrying the values for load current to 20 amperes.

**Prob. 40-10.** Construct the external characteristic for the generator

in Prob. 39–10, if it is run at 1200 rpm. What effect does decreasing the speed have on the characteristic of the series generator?

**29. Field-Resistance Line.** We know, from Ohm's law, that the current flowing through a circuit of constant resistance is directly proportional to the voltage impressed upon it.

For instance, if a field circuit has a resistance of 20 ohms, the current flowing in it will be 2 amperes when 40 volts is impressed; 3 amperes when 60 volts is impressed; 4 amperes when 80 volts is impressed, etc. If a curve be plotted between current as abscissae, and impressed voltage as ordinates, it will be a straight line starting at the origin, as curve $A$, Fig. 53–10. Curve $B$ is the resistance line for a field circuit of 40 ohms: 2 amperes flows when 80 volts is impressed; 3 amperes flows when 120 volts is impressed, etc. Curve $C$ is the resistance line for a circuit of 60 ohms resistance.

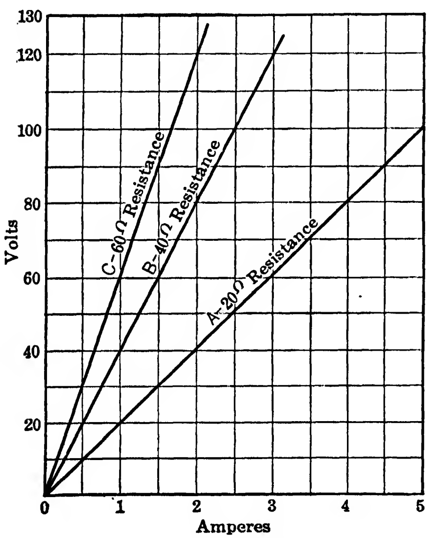



FIG. 53–10. Field resistance lines.

Note that the greater the resistance, the steeper is the slope of the line. If such a curve be drawn with volts and amperes plotted to the same scale, the resistance is equal to the tangent of the angle the line makes with the abscissa, for the ordinate is $E$, the abscissa is $I$, and $\frac{E}{I} = R$.

The use of the field-resistance line will be employed in the next article.

**30. Building Up of Voltage of the Shunt Generator.** The voltage of a shunt generator, as of all self-excited generators, builds up because of the residual magnetism in the magnetic path. Since the field circuit of the shunt generator is connected directly across the brushes, Fig. 41*b*–10, the voltage generated by the residual magnetism forces a small current through the field circuit. This increases the flux, if the current set up is in the right direction,

and a higher voltage is generated, which, in turn, forces more current through the field circuit. This again increases the flux and the induced voltage, and so on until the resistance of the field circuit and the bend in the magnetization curve prevent any further increase in voltage. Figure 54–10 shows the saturation curve of a shunt generator together with its field-resistance line,

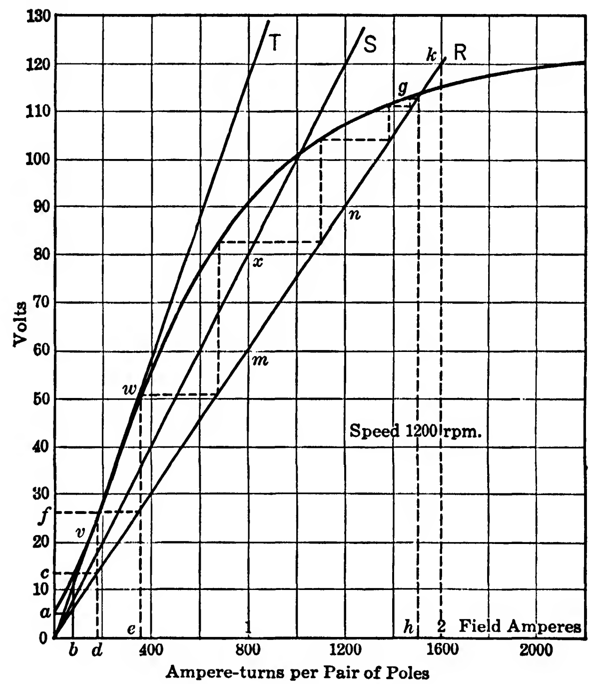



FIG. 54–10. The building up of a shunt generator. The no-load voltage is determined by the point where the field-resistance line crosses the saturation curve.

*R*, plotted to the same scale. Note, from the resistance line *R*, that with 60 volts across the field circuit (point *m*), 1 ampere flows in the field; with 90 volts across the field circuit (point *n*), 1.5 amperes flow. The resistance of the field circuit then must be $\frac{60}{1} = \frac{90}{1.5} = 60$ ohms.

As the generator comes up to speed, the voltage *oa*, or 5 volts, is generated in the armature by the residual magnetism. Since the field circuit is directly across the brushes, the current *ob*, or about

0.1 ampere, is forced through the field. The field current *ob* generates the voltage *oc*, or about 13 volts, which in turn forces the current *od*, or about 0.225 ampere, through the field. The current *od* will generate the voltage *of*, about 26 volts. This voltage forces more current through the field and the machine continues to build up. At point *g*, about 113 volts, the current *oh*, or about 1.925 amperes, is forced through the field, which in turn generates about 114 volts, or just enough pressure to force the current *oh* through the field and the building-up process stops. The machine builds up to the point where the field-resistance line crosses the saturation curve.

The voltage cannot build up beyond 114 volts. Consider the point *k* on the resistance line above the saturation curve. For this point, a field current of 2 amperes is required, but 2 amperes will generate only about 115 volts. Since 120 volts are required to force 2 amperes through the field, and this value of field current generates only 115 volts, it is apparent that the machine cannot build up to the point *k*.

Let us now consider the effect of increasing the resistance of the field circuit. If the resistance is increased to 80 ohms by means of a field rheostat, there must be 80 volts across the field to force 1 ampere through the field circuit, and the new field-resistance line, *S*, must pass through the point, *x*, in the figure. The machine would now build up to only about 101 volts. Thus by adjusting the resistance of the field circuit, we can control the value of the voltage to which the machine will build up.

If the resistance of the field circuit is increased to that of resistance line, *T*, about 117 ohms, the machine will build up to any point between *v* and *w*, and will be unstable. If the resistance is increased even slightly above this amount, the machine will not build up much above the voltage due to residual magnetism. This is called the **critical field resistance.**

**31. Generator Fails to Build Up.** If a self-excited generator fails to build up, it is usually because the field coils have been connected to the brushes in such a way that the current in them tends to set up a field in the opposite direction to the residual magnetism. Thus, when the voltage first induced in the armature by the residual magnetism sends a current through the field coils, this current neutralizes the residual magnetism, rather than adds to it, and the induced voltage drops, instead of building up. The remedy is to reverse the field coil connections to the brushes.

It may be necessary, however, to remagnetize the machine by lifting the brushes and sending a slight current, from some outside source, through the field. A few dry cells usually will supply all the current needed in this case.

**32. Load Curve of the Shunt Generator. External Characteristics.** A shunt generator, Fig. 41*b*–10, after building up to voltage, is loaded by being connected as shown in the conventional diagram of Fig. 55–10. The voltmeter connected across the terminals indicates terminal voltage and the ammeter measures the load current, as this current is increased. An ammeter should also be inserted in the field circuit to indicate the values of the field current. The generator must be driven at constant speed.

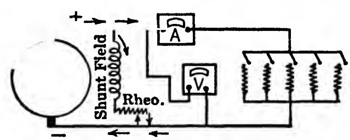


Fig. 55–10. A shunt generator connected to a load.

If the values of the voltmeter and the ammeter readings are plotted, a curve, similar to *C* in Fig. 56–10, is obtained. At no load, the terminal voltage is practically that determined by the field ampere-turns, just as in the separately excited generator. As more and more current is delivered, the voltage drops further below that at no load. This is similar to the performance of the separately excited generator, except that the voltage of the shunt generator falls off more rapidly with increase of load.

This is because there is another factor in the shunt generator which tends to lower the terminal voltage as the load is increased.

In both machines, the terminal voltage decreases due to the increase in back ampere-turns and armature resistance drop.

In the separately excited generator, the fields are excited from a separate source of power, and the field current is constant. In the shunt generator, the field windings are connected directly across the machine terminals, and any decrease in terminal voltage further decreases the field current, with a resulting additional decrease in flux and terminal voltage.

The terminal voltage of a shunt generator then decreases with increase of load because of three effects:

(*a*) The increase in the demagnetizing, or back ampere-turns;

(*b*) The *IR* drop in the armature;

(*c*) The decrease in shunt-field current due to the effect of (*a*) and (*b*) on the terminal voltage.

In Fig. 56–10, curve *A* shows what the performance of the machine would be if the only cause for drop in voltage were the

demagnetizing, or back ampere-turns. Curve *B* shows the loss in voltage due to both back ampere-turns and the armature resistance drop. And curve *C* shows the actual curve of the machine in which the effect of decreased field current is added to the other two effects.

It might appear that the decrease in field current and the resulting decrease in terminal voltage would continue indefinitely when

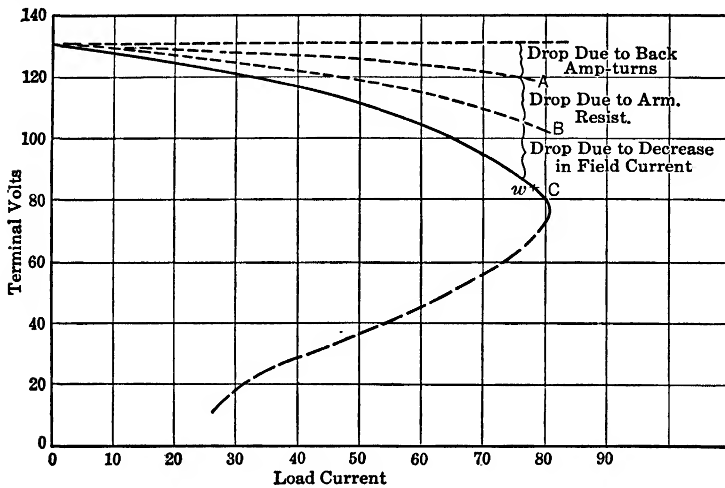



FIG. 56–10. The external characteristic of a shunt generator.

a load of any value is put on the machine, thus automatically decreasing the voltage to zero. This would be the case, if it were not for the fact that the iron circuit is to some degree saturated; and that, due to the shape of the magnetization curve, a given drop in terminal voltage, say 10 per cent, with consequently a 10 per cent decrease in field current, results in much less than a 10 per cent decrease in flux.

If sufficient additional load is supplied by the machine, the terminal voltage rapidly falls and the " break-down " point, *w*, on curve *C* is reached, and the curve becomes unstable. This is considerably beyond the full-load current in most machines. If the external resistance is further reduced, the load current reaches a maximum and then actually decreases; and the curve bends back on itself as the broken portion of curve *C* shows. The machine now may be short-circuited with still further decrease in current.

The reason why the current actually decreases is due to the fact that below the break-down point, the low voltage on the field winding has reduced the flux to the straight portion of the magnetization curve, where the slope is the steepest. At this point, the terminal voltage decreases faster than does the resistance of the external circuit. At short-circuit, the current is due to the emf induced by the residual magnetism alone, reduced somewhat by the back ampere-turns.

The load curve of the shunt generator, curve $C$ in Fig. 56–10, also can be approximately determined graphically from the saturation curve of the machine and the field-resistance line, as in the example below.

**Example 10.** Consider a 4-kw, 1200-rpm shunt generator with the following data: full-load current is 36 amperes; armature conductors are 200, and brush lead is 9 degrees; number of paths in the armature is 2; field turns per pair of poles are 2000; resistance of armature and brush contact is 0.20 ohm. The magnetization curve for the machine at 1200 rpm is plotted in Fig. 57–10, and both field current and ampere-turns per pair of poles are indicated on the plot. Resistance of shunt-field circuit is 178.6 ohms.

**Solution.**

Since the resistance of the field circuit is 178.6 ohms, the voltage drop in this circuit, for instance with 0.4 ampere, is $178.6 \times 0.4$ or 71.4 volts. This gives us point $n$ on the field-resistance line of Fig. 57–10. Other points can be obtained in similar manner. The field-resistance line, $R$, thus cuts the saturation curve at 127 volts. **This is the no-load voltage.**

The terminal voltage, corresponding to various armature currents, can be determined by means of the following construction.

**We will compute the terminal voltage for an armature current of 30 amperes.**

Back turns in the armature $= \dfrac{2 \times 9}{360} \times 200 = 10$ turns per pair of poles.

Demagnetizing or back ampere-turns at 30 amperes armature current $= \frac{30}{2} \times 10 = 150$ ampere-turns.

Armature $IR$ drop at this current $= 30 \times 0.2 = 6$ volts.

Note that the **ratio** of armature $IR$ drop to back ampere-turns equals $\dfrac{6}{150}$ for 30 amperes armature current. This **ratio** is constant at all loads.

For instance, at 60 amperes armature current:

Demagnetizing ampere-turns $= \dfrac{60}{2} \times 10 = 300$ ampere-turns.

Armature $IR$ drop $= 60 \times 0.2 = 12$ volts.

**Ratio** of armature $IR$ drop to back ampere-turns $= \frac{12}{300} = \frac{6}{150}$, as before.

Also at 250 amperes armature current (assumed for convenience) the ratio $= \frac{50}{1250} = \frac{12}{300} = \frac{6}{150}$, as before.

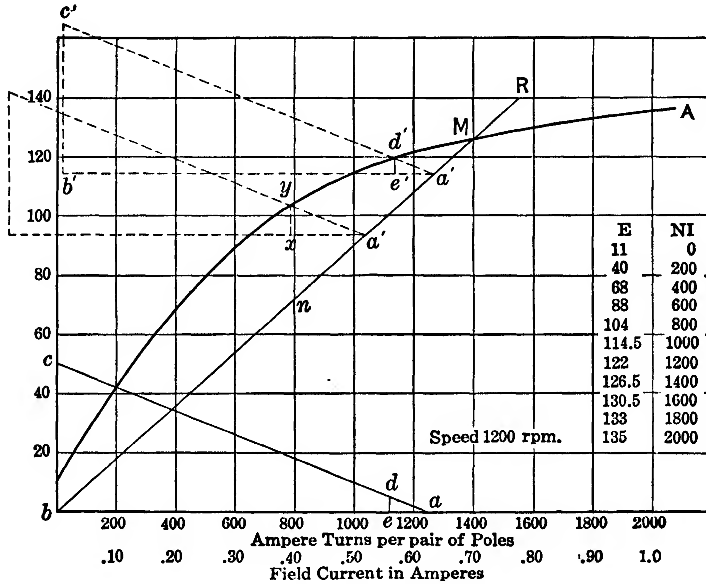



FIG. 57–10. A method of determining the load curve of a shunt generator.

On Fig. 57–10, lay off $bc$ equal to 50 volts and $ba$ equal to 1250 ampere-turns. Cut out the triangle, or preferably construct one to the same scale from another piece of the same graph paper. (It is well to use paper ruled into divisions of $\frac{1}{20}$ of an inch.) Lay off $de$ on the triangle equal to 6 volts (armature $IR$ drop with 30 amperes) and $ae$ will equal 150 ampere-turns. Now place the triangle on the graph with the point $a$ on the line $R$ and the line $ab$ extending to the left and parallel to the horizontal axis. Keeping $ab$ parallel to the horizontal axis, slide the point $a$ along the line $R$ until a position is found where the point $d$ falls on the line $A$. This position is indicated by the triangle $a'b'c'$. The ordinate of the point $a'$ will be the terminal voltage when the armature current is 30 amperes. In this case, it reads 114 volts.

The reasons why this procedure gives the terminal voltage are as follows: If the terminal voltage is 114 volts, then the field current is $\frac{114}{178.6}$ or 0.638 ampere, which would produce $2000 \times 0.638$ or 1275 ampere-turns in the field. But the 150 back ampere-turns of the armature

decrease this to 1270 − 150 or 1120 ampere-turns, which generates a voltage of 120 volts as shown by the point $d'$ on curve $A$. The armature $IR$ drop of 6 volts, however, must be taken from this voltage to find the terminal voltage. The line $d'e'$ subtracts 6 volts from $d'$ on curve $A$ and gives us 114 volts for the terminal voltage.

The terminal voltage can be found for any other armature current by means of the same triangle. For instance, for 50 amperes armature current, the armature drop is $50 \times 0.2 = 10$ volts, and the back ampere-turns are $\frac{50}{2} \times 10 = 250$. Lay off $yx$ equal to 10 volts in Fig. 57–10 and $a'x$ equal to 250 ampere-turns, and slide point $a'$ along the line $R$ with the base horizontal until point $y$ falls on the line $A$. The ordinate of $a'$ is then the terminal voltage at 50 amperes armature current. In this case, the terminal voltage is 93 volts. If the terminal voltage is determined for a sufficient number of values of armature current, a load curve can be plotted. The load curve is plotted between the terminal voltage and the **line current** (not the armature current). The line current in each case is found by subtracting the shunt-field current from the armature current, as shown below.

| Armature Current | Terminal Voltage | Shunt-field Current | Line Current |
|---|---|---|---|
| 30 amperes | 114 | $\frac{114}{178.6} = 0.638$ ampere | 29.4 amperes |
| 50 amperes | 93 | $\frac{93}{178.6} = 0.522$ ampere | 49.5 amperes |

**Prob. 41–10.** Plot the magnetization curve of Fig. 57–10 on fine coordinate paper; draw the field-resistance line; construct the triangle as described in Example 10 and determine the terminal voltage of the generator at 35 amperes armature current. (*a*) What is the field current at this load? (*b*) What is the line current?

**Prob. 42–10.** Determine the terminal voltage of the generator in Example 10 for various armature currents to short-circuit, and plot the resulting load curve.

If the **external characteristic** of a shunt generator is to be determined by test, the field current must be adjusted by field rheostat to that value which will give the normal or rated terminal voltage with full-load current output at the rated speed, just as in the case of the separately excited generator. The load is removed, and with speed held constant, the no-load voltage is obtained. Without changing the resistance of the field circuit, the load is increased in steps to at least full load, while values of load current and terminal voltage are obtained.

Or the external characteristic can be determined graphically, from the saturation curve, by first finding the position of the field-resistance line, and therefore, the field excitation which will give rated voltage at rated current output. This will be further discussed in the next article.

**33. Regulation of the Shunt Generator.** As in the case of the separately excited generator, the characteristic curve shows the inherent or automatic behavior of the shunt generator under load. This " inherent " performance can also be expressed as the regulation of the machine. The definition of regulation for a shunt generator is exactly the same as for the separately excited machine; namely, the difference between no-load and full-load terminal voltages, expressed as a percentage of the full-load voltage — full-load voltage being the normal voltage of the generator. Since the full-load terminal voltage of the characteristic curve must be the normal voltage of the generator, the full-load and no-load voltages used in calculating regulation must be the full-load and no-load points on the external characteristic curve.

In computing the regulation from the saturation curve and the field-resistance line, it must be remembered that the triangle, as used in Example 10, is plotted from armature resistance drop and armature back ampere-turns. The armature current in a shunt generator is the sum of the line current and the field current. But, in this case, the field current at normal full-load voltage is not known. It can, however, be approximated, and the position of the field-resistance line and the value of the no-load voltage determined, as in the example below.

**Example 11.** Let us connect the 230-volt, 15-kw separately excited generator of Example 6 as a shunt generator, and determine the position of the field-resistance line for normal terminal voltage at full load. This will also give us the no-load voltage, so that both the characteristic and regulation can be determined. The saturation curve for this machine, Fig. 48–10, is replotted in Fig. 58–10. From Example 6, back turns in the armature per pair of poles are 8.45, and resistance of armature and brush contact is 0.18 ohm.

At 200 amperes (to obtain triangle of convenient size):

$$\text{Armature } IR \text{ drop} = 200 \times 0.18 = 36 \text{ volts.}$$
$$\text{Back ampere-turns} = \tfrac{200}{6} \times 8.45 = 282.$$

Lay out the triangle in Fig. 58–10 with $ab$ equal to 282 ampere-turns and $bc$ equal to 36 volts.

First assume that line current equals armature current. Armature

*IR* drop at full load then equals 65 × 0.18 or 11.7 volts, and point *d* on the triangle can be located.

Move triangle up the saturation curve until the base *b′a′* is parallel to horizontal axis at the 230-volt ordinate and point *d′* is on the saturation curve. The abscissa of point *a′* is equal to *bx* or 670 ampere-turns in the field. $\frac{670}{700}$ = 0.96 ampere in the field. The corrected armature current at full load is now 65 + 0.96 or 65.96 amperes, and a new armature *IR* drop, 65.96 × 0.18 or 11.9 volts is laid off, as *fg* on triangle *abc*.

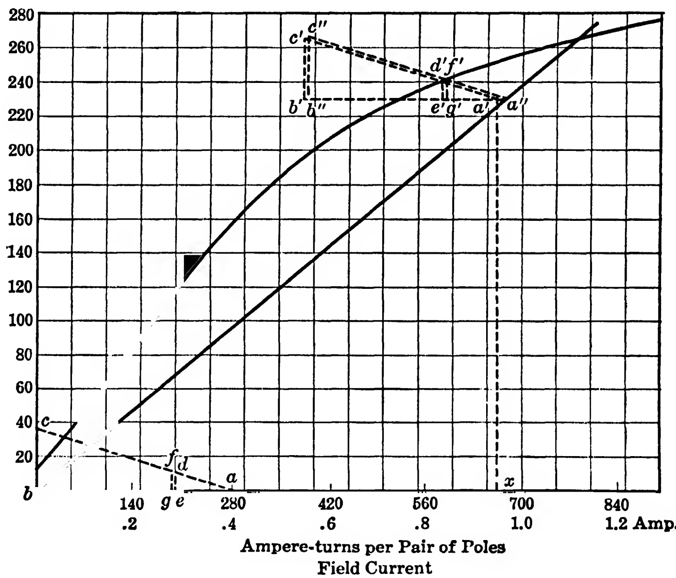



Fig. 58–10. A method of determining that position of the field-resistance line (or resistance of field circuit) which will give normal terminal voltage of a shunt generator at full load, Example 11.

Now apply triangle as before with point *f′* on the saturation curve and *a″* is a point on the more precise position of the field-resistance line. This line is now drawn through the point *a″* and cuts the saturation curve at 264 volts, the no-load voltage.

The regulation of this generator used as a shunt machine can now be calculated as

$$\frac{264 - 230}{230} \times 100 \quad \text{or} \quad 14.8 \text{ per cent,}$$

which is considerably higher than the percentage regulation of this same machine used as a separately excited generator. Points on the characteristic curve for various load currents can now be determined.

**Prob. 43–10.** The data for a certain 5-kw, 115-volt, 1400-rpm shunt generator, together with its magnetization curve, are as follows:

Full-load voltage = 115 volts. Shunt-field turns per pair of poles = 600.
Armature and brush contact resistance = 0.255 ohm.
Brush lead = 3.34°. Number of paths in armature = 2.
Armature conductors = 124. Full-load line current = 43.5 amperes.

(a) What is the resistance of the field circuit at rated voltage, full load?

(b) Determine and plot the external characteristic as far as the short-circuit value of line current.

(c) What is the percentage regulation?

MAGNETIZATION CURVE AT 1400 RPM

| Ampere-turns per pair of poles | Volts generated | Ampere-turns per pair of poles | Volts generated |
|---|---|---|---|
| 0 | 8 | 750 | 113 |
| 50 | 14 | 900 | 123 |
| 150 | 30 | 1050 | 130.5 |
| 300 | 56 | 1200 | 135.5 |
| 450 | 80.5 | 1400 | 140.0 |
| 600 | 99.5 | 1700 | 144.5 |

**Prob. 44–10.** Determine and plot the external characteristic of the shunt generator of Prob. 43–10 as far as the short-circuit value of line current, when it is run at 1500 rpm. What is the resistance of the field circuit and the percentage regulation? Full-load voltage, 115 volts.

**Prob. 45–10.** Repeat Prob. 43–10, when the machine is driven at 1300 rpm. What is the resistance of the field circuit and the percentage regulation? Full-load voltage, 115 volts.

**Prob. 46–10.** From the results of Probs. 43, 44, and 45 above, what is the effect on the droop of the characteristic curve, and on regulation, of operating the generator above rated speed? Below rated speed? Explain why these results differ as they do.

**34. Voltage Control of the Shunt Generator.** While, as we have seen, the characteristic curve of a shunt generator does not show an excessive drop in voltage until it is well loaded, still the drop is usually too much for the satisfactory operation of lamps or other constant-voltage appliances.

We have also seen in Art. 30, that the shunt generator builds up to a no-load voltage, determined by the resistance of the field circuit and the shape of the magnetization curve. The field resistance and, therefore, the no-load voltage can be controlled by a rheostat connected in series with the field coils.

In the same way, the terminal voltage can be controlled when the machine is carrying a load, and the shunt generator **made** to produce a constant terminal voltage. This means that, as the load changes, the setting of the field rheostat must be adjusted, usually by hand. Of course, the rheostat cannot raise the voltage beyond the point where all the rheostat resistance is cut out, but it can be used to lower the voltage at light loads. At full load, the resistance in the rheostat is cut out sufficiently to produce normal terminal voltage. When, however, the load on the machine decreases and the terminal voltage tends to rise, enough resistance is turned in to the rheostat to keep the voltage at its rated value. Thus, to keep the terminal voltage of the shunt generator constant, with changing load, involves a continual changing of the field rheostat setting, just as in the case of the separately excited generator. In fact, this disadvantage is even more marked in the shunt machine because, as we have seen, the characteristic of this machine falls off more than does that of the separately excited generator.

It is possible, also, to keep the terminal voltage of a shunt generator constant by means of an automatic device, such as the Tirrill Regulator, which cuts resistance in and out of the field rheostat, as the load changes. But such methods of voltage control are expensive, and hand control is inconvenient, so that shunt generators are not in general use where there are close voltage requirements. It is customary in such cases to use compound generators which can be built to furnish practically a constant terminal voltage, as will be explained later.

**Prob. 47–10.** What resistance must be cut in to the field circuit of the shunt generator of Prob. 45–10 so that it will have a no-load voltage of 115 volts at 1300 rpm?

**Prob. 48–10.** How much of the resistance must be cut out of the field rheostat of Prob. 43–10 to produce a terminal voltage of 125 volts when half load (approximately 23 amperes armature current) is being taken from the generator?

**Prob. 49–10.** How much resistance must be cut in to the field circuit of Prob. 43–10 to produce a terminal voltage of 100 volts at full-load line current?

**Prob. 50–10.** Plot a load curve and determine the percentage change in terminal voltage between no-load and full-load line currents of the shunt generator of Prob. 43–10, when 9 ohms have been added to the resistance of the field rheostat. Speed = 1400 rpm. What effect does added resistance in the field circuit have upon the droop of the curve

and percentage change in terminal voltage between no load and full load? Explain why this is so. Note that this change in voltage is not the regulation, since the full-load voltage in this and the following problem is not the rated voltage of the machine.

**Prob. 51–10.** Plot a load curve and determine change of terminal voltage between no-load and full-load line currents of the generator of Prob. 43–10, when 5 ohms have been cut out of the field rheostat. Speed = 1400 rpm. What effect does decreased resistance in the field circuit have upon the droop of the load curve, and upon percentage change in terminal voltage between no load and full load? Explain why these results differ, as they do, from those in Prob. 50–10.

**35. The Compound Generator. Characteristic Curve.** We have seen in the previous articles that the drop in voltage with increased load, which occurs in the shunt generator, makes it unsatisfactory as a source of constant-voltage supply. This is especially true for lighting circuits, where a small change in voltage makes a very considerable change in the light, or candlepower, of incandescent lamps.

It is the load current in a shunt generator which produces the armature *IR* drop, armature reaction and decrease in shunt-field current, all three of which in turn cause the terminal voltage to drop off. If this load current is carried through a set of series coils of large cross-section and low resistance, also wound on the poles of the machine, the ampere-turns of this additional winding can be made to aid the ampere-turns of the shunt winding and increase the flux, and thus compensate for armature drop and armature reaction. Such a machine can be made to automatically supply a practically constant terminal voltage, even with change in load, and is called a **compound generator.** It is essentially a shunt generator with a few series turns added to the field.

The connections and arrangement of the two windings are shown in Fig. 41*d*–10. The series winding may be connected, "long shunt" or "short shunt," as described in Art. 21, Figs. 42*a*–10 and 42*b*–10.

The compound generator "builds up" and operates at no load, exactly as a shunt machine. The no-load voltage is controlled by a rheostat in series with the shunt field.

If the number of the added series ampere-turns are just sufficient at full-load current to compensate for armature *IR* drop, armature reaction and *IR* drop in the series winding itself, the terminal voltage at this load will be equal to the no-load voltage. In this case, the machine is said to be "**flat-compounded**" and the

characteristic curve will be similar to curve *A* in Fig. 59–10. Note that the voltage tends to rise slightly at first, and then to drop to the value of the no-load voltage when full load is reached. The shape of the curve is due to the fact that as the iron approaches saturation, the series ampere-turns do not increase the flux as much as at light load. Flat-compounded generators are often built to produce rated terminal voltage at one-third load and at full load. They are often used in installations where the feeders are short and sufficiently large so that their resistance drop is negligible.

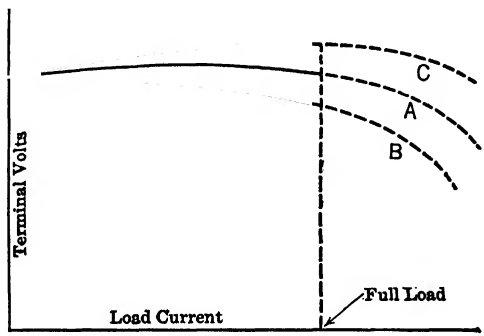



FIG. 59–10. Characteristic curves of the compound generator; A, flat-compounded; B, under-compounded; C, over-compounded.

If at full load, the number of series ampere-turns is insufficient to compensate for the armature and series-field drop and the armature reaction, the characteristic droops, as shown in curve *B*, Fig. 59–10. The full-load voltage is less than that at no load, and the generator is said to be **under-compounded.** Under-compounded generators are seldom used.

If the series ampere-turns at full load are **more** than sufficient to compensate for the armature and series-field drop and the armature reaction, the terminal voltage at this load will be higher than at no load and the characteristic rises, as shown in curve *C*, Fig. 59–10. The machine is now said to be **over-compounded.**

Over-compounded generators may be said to be the standard machine for d-c power stations, and are always used where the load is at considerable distance from the machine. As the load increases, the voltage at the load tends to fall, due to increase in *IR* drop in the feeders. However, if the generator terminal voltage rises just enough to counteract the feeder drop, the voltage at the load will remain constant. Note the example below.

**Example 12.** Consider a compound generator supplying a load of 200 amperes over a pair of long feeders of 0.075-ohm resistance. See Fig. 60–10. The no-load voltage of the generator is 240 volts, and it is desired to maintain a pressure of 240 volts at the load. At full load, the feeder drop is $200 \times 0.075 = 15$ volts.

If the machine were flat-compounded, at full-load current, the pressure at the load would be $240 - 15$ or 225 volts. However, if enough series turns were added so that the terminal voltage rose from 240 volts at no load to 255 volts at full load, the pressure on the load at 200 amperes would be $255 - 15$ or 240 volts, the same as at no load.

The ideal condition of 240 volts at all loads, however, cannot be exactly obtained, for feeder *IR* drop varies in a straight-line ratio with current, and the characteristic is a **curved line.** Therefore, if the compounding is correct for full-load current, the load voltage for intermediate currents will be slightly high, as shown by the difference in voltage between point *a* on the characteristic curve and point *b* on the feeder resistance, or *IR* drop line *R*, of Fig. 60–10.

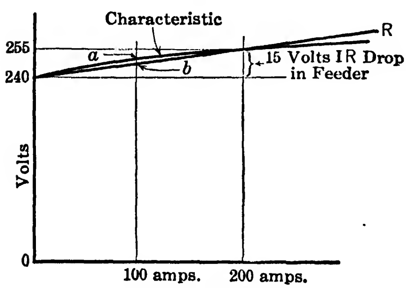



FIG. 60–10. An over-compounded generator may maintain practically a constant voltage on the load at the end of a long feeder.

**Percentage Over-compounding** of a generator is the voltage rise between no-load and full-load currents, calculated as a **percentage of the no-load** voltage. That is:

$$\frac{E\text{ (full load)} - E\text{ (no load)}}{E\text{ (no load)}} \times 100$$

$$= \text{percentage over-compounding.}$$

Note the difference between this calculation and that for the **regulation** of the separately excited or shunt generator in Art. 26.

In Example 12, the percentage over-compounding is:

$$\frac{255 - 240}{240} \times 100 \quad \text{or} \quad 6.25 \text{ per cent.}$$

The dotted portion of the characteristic curves in Fig. 59–10 all show that the terminal voltage of a compound generator falls off somewhere beyond full load. At sufficient overload, the iron of the poles becomes saturated and additional series-field current adds practically no more flux to the magnetic circuit; consequently, the voltage falls. The curves do not bend back, as in

the shunt generator, because the high value of the series-field current keeps the magnetic circuit saturated.

The characteristic curve for a compound generator may be obtained by test in exactly the same way as for the shunt machine, except that this curve is taken with **rated voltage at no load.** Figures 42*a*–10 and 42*b*–10 are the conventional diagrams of connections. Unless otherwise specified, all compound generators in this chapter will be considered as connected **long** shunt.

The characteristic curve may also be determined from the saturation curve and shunt-field-resistance line, as shown in the example below.

**Example 13.** Suppose we add 15 series turns per pair of poles to the 110-volt, 4-kw, 1200-rpm generator of Example 10, which has the magnetization curve of Fig. 57–10. Assume the resistance of the series-field turns to be 0.12 ohm.

The other data for this generator are:

Shunt-field turns per pair of poles = 2000.
Number of armature conductors = 200.
Brush lead = 9°. Number of paths in armature = 2.
Armature resistance (with brush contact) = 0.20 ohm.
Machine is connected long shunt.

**Solution.** Redraw the magnetization curve in Fig. 61–10, which is a copy of Fig. 57–10. Draw the field-resistance line for the no-load voltage crossing the magnetization curve at 110 volts, the rated voltage. This gives a shunt-field current of 0.455 amperes and a resistance of the shunt-field circuit of 242 ohms. From Example 10, the back turns are 10.

At 30 amperes armature current:

Armature back ampere-turns per pair of poles
$= \frac{30}{2} \times 10 = 150$ ampere-turns.
Armature and series-field $IR$ drop
$= 30(0.20 + 0.12) = 9.6$ volts.
Series-field ampere-turns per pair of poles
$= 30 \times 15 = 450$ ampere-turns.

At 15 amperes armature current:

Armature back ampere-turns per pair of poles
$= \frac{15}{2} \times 10 = 75$ ampere-turns.
Armature and series-field $IR$ drop
$= 15(0.20 + 0.12) = 4.8$ volts.
Series-field ampere-turns per pair of poles
$= 15 \times 15 = 225$ ampere-turns.

Note that for 15 amperes, the values above are just half the respective values for 30 amperes. The **ratio** of armature and series-field $IR$ drop to the back ampere-turns and to the series-field ampere-turns is, therefore, constant; that is

$$4.8/75/225 = 9.6/150/450,$$

and we can construct a triangle and obtain values for a characteristic curve in a somewhat similar manner to that used for the shunt generator.

In order to make a triangle of good working size, let us use an armature current of, say 60 amperes. For this current:

Armature back ampere-turns = 300 ampere-turns.
Armature and series-field $IR$ drop = 19.2 volts.
Series-field ampere-turns = 900 ampere-turns.

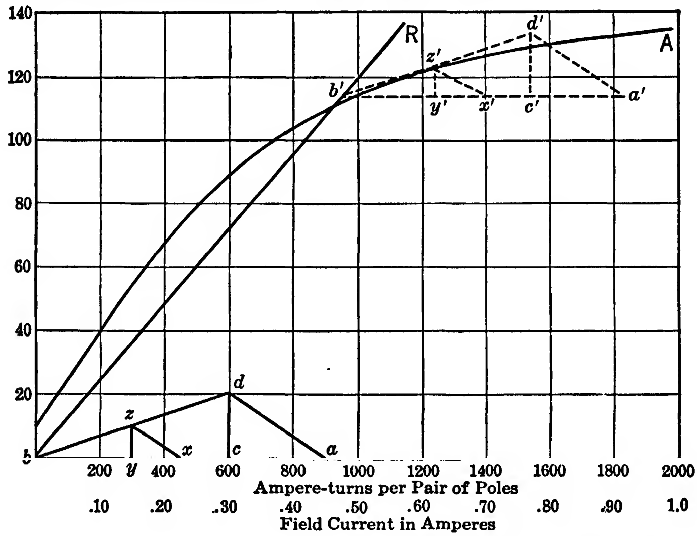



FIG. 61–10. Graphical method of determining the external characteristic of a compound generator.

Lay off the triangle *abd* on Fig. 61–10, making *ba* equal to the series field ampere-turns (900), *ac* equal to the armature back ampere-turns (300) and *cd* equal to the armature and series-field $IR$ drop (19.2 volts). Now connect the points *ad* and *bd*. The proportions of the triangle are the same for any armature current.

To use the triangle to find the generator terminal voltage at, say 30 amperes armature current, we proceed as follows:

The series-field ampere-turns = 450. Lay off 450 ampere-turns from $b$ and secure the point $x$.

The back ampere-turns = 150. From $x$, subtract 150 ampere-turns and get the point $y$.

The armature and series-field $IR$ drop = 9.6 volts. Erect a perpendicular $yz$ equal to 9.6 volts and it will meet the line $bd$ at $z$.

The triangle $bzx$ is exactly similar to the triangle $bda$.

$bx$ represents the series-field ampere-turns for 30 amperes.
$yx$ represents the back ampere-turns for 30 amperes.
$by$ or $(bx - yx)$ then represents the resulting ampere-turns produced by the series field and the armature back ampere-turns.
$yz$ represents the armature and series-field $IR$ drop.

Cut out the triangle *abd* (or preferably construct one to the same scale from another sheet of the same graph paper). Move the point *b* of the triangle up the shunt field-resistance line ($R$) (keeping the base *ba* parallel to the horizontal axis) until the point $z$ falls on the magnetization curve $A$. The ordinate of the point $b'$ is the terminal voltage of the generator at this armature current. Note that the points $y$ and $x$ are not used in locating the terminal voltage, but are used to locate point $z$. The terminal voltage in this case is 113 volts.

The reason that this gives the terminal voltage is as follows: When the terminal voltage is 113 volts (represented by point $b'$) the shunt field strength is 940 ampere-turns as read from the abscissa of point $b'$. The series field adds 450 ampere-turns, $b'x'$, making the excitation now 940 + 450 or 1390 ampere-turns, as seen by the abscissa of point $x'$. But the back armature ampere-turns subtract 150 ampere-turns, $x'y'$, making the resulting field ampere-turns 1240, as seen by the abscissa of point $y'$. This field induces a voltage of 122.6 volts, as seen from the ordinate of point $z'$. But of this voltage, 9.6 volts are used in forcing the 30 amperes through the armature and series-field winding, leaving a terminal voltage of 113 volts.

Similarly, the terminal voltage for all other armature currents can be found and the external characteristic constructed.

Note that while 30 amperes armature current is not quite full load, the generator is over-compounded at this load, as the terminal voltage is 113 volts, or 3 volts above the no-load voltage.

**Prob. 52–10.** An 11-kw, 220-volt, 1350-rpm, compound generator has the following data:

Number of conductors = 420.
Number of poles = 4. Number of paths = 4. Brush lead = 6°.
Shunt-field turns per pair of poles = 1200.
Series-field turns per pair of poles = 10.
Resistance of armature and brush contact = 0.22 ohm.
Resistance of series-field winding = 0.08 ohm.

Construct the external characteristics of the generator, carrying the armature current to 100 amperes. Calculate the percentage over-compounding.

The magnetization curve for this machine at 1350 rpm is given below.

| Ampere-turns per pair of poles | Volts generated | Ampere-turns per pair of poles | Volts generated |
|---|---|---|---|
| 0 | 13 | 1600 | 253 |
| 200 | 63.5 | 1800 | 258.5 |
| 400 | 124.5 | 2000 | 263 |
| 600 | 168.5 | 2200 | 265 |
| 800 | 202 | 2400 | 268 |
| 1000 | 222 | 2600 | 270 |
| 1200 | 235.5 | 2800 | 272 |
| 1400 | 246.5 | | |

**Prob. 53–10.** Construct the external characteristic and calculate the percentage compounding for the generator in Prob. 52–10, if it has 20 series turns per pair of poles. (Resistance of series winding = 0.16 ohm.)

**Prob. 54–10.** Construct the external characteristic and calculate the compounding for the generator in Prob. 52–10, if it has 5 series turns per pair of poles. (Resistance of series winding = 0.04 ohm.)

**Prob. 55–10.** Construct the characteristic and calculate the percentage compounding of Prob. 52–10, if the generator is operated at 1550 rpm. (Same no-load voltage and same number of series turns.) What is the effect on compounding of operating a generator **above** rated speed? Explain why.

**Prob. 56–10.** Repeat Prob. 52–10 when the generator is operated at 1160 rpm. (Same no-load voltage and same number of series turns.) What is the effect on compounding of operating a generator **below** rated speed? Explain why.

**36. Number of Turns Necessary to Produce a Desired Compounding.** To determine the number of turns necessary to produce a desired compounding, a value must be assumed for the resistance which the series turns are to be allowed to have. The resistance of the series winding depends upon the total number of turns on all the poles, the length per turn and the size of wire. The assumed value for this resistance can be adhered to very closely in winding the series coils, provided it is low enough to be sure the wire will carry the required current without over-heating.

**Example 14.** How many series turns are required to flat-compound the generator in Example 10? Assume the machine in this case is a 125-volt generator with a full-load armature current of 30 amperes. All other data remaining the same.

A practical value of the resistance of the series field would be 0.04 ohm.
Resistance of the armature = 0.20 ohm.
Construct again Fig. 62–10, the magnetization curve of the machine.
From abscissa of point $M$, shunt-field current = $\frac{1400}{2000}$ or 0.7 ampere.
Full-load armature current equals the load current plus the shunt-field current at 125 volts or $30 + 0.7 = 30.7$ amperes.
Voltage drop in armature and series field at full load = $30.7(0.20 + 0.04) = 7.4$ volts.

Draw line $MN$ through the no-load voltage point, parallel to the axis. Draw $OP$, parallel to $MN$, at a distance of 7.4 volts above it. Drop the line $ST$ where $OP$ crosses the magnetization curve. Lay off the distance $TV$, making it equal to the armature back ampere-turns at full load, $\frac{30.7}{2} \times 10 = 153$ ampere-turns. $MV$ is the then value of the series ampere-turns per pair of poles necessary to produce flat-compounding, and equals 560 according to scale.

The series turns per pair of poles $= \frac{560}{30.7} = 18.25$ turns. The reason for this construction is:

The abscissa of $M$ is the shunt field ampere-turns. $MV$ is the series field ampere-turns added to $M$. The abscissa of the point $T$ represents the resultant series and shunt field ampere-turns after the armature back ampere-turns $VT$ have been subtracted; this resultant field induces a voltage represented by the ordinate of the point $S$. The terminal voltage is this value minus $ST$, the $IR$ drop in the armature and series field.

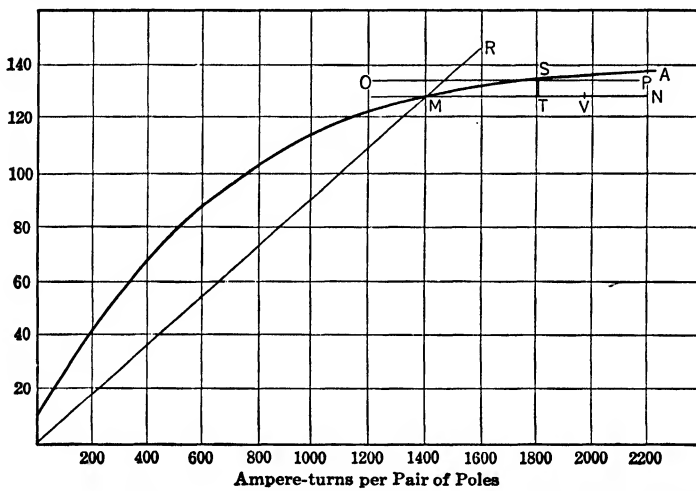



FIG. 62–10. Graphical method of finding the number of series turns necessary to produce some desired compounding.

**Prob. 57–10.** Determine the number of series turns per pair of poles (long-shunt connection) to flat-compound the shunt generator of Prob. 43–10. Allow 0.025 ohm for the resistance of the series field.

**Prob. 58–10.** How many series turns per pair of poles would be required to over-compound the generator in Prob. 43–10, 5 per cent? Allow 0.035 ohm for resistance of series-field winding.

**Prob. 59–10.** Plot the characteristic curve for the generator in Prob. 43–10, using data from Prob. 58–10.

**Prob. 60–10.** How many series turns per pair of poles would be required to over-compound the generator of Prob. 52–10, 7.5 per cent? Allow 0.12 ohm for resistance of series field.

**Prob. 61–10.** How many series turns per pair of poles would be required to over-compound the generator of Prob. 52–10, 12 per cent? Allow 0.2 ohm for resistance of series field.

**37. Adjusting the Compounding of a Generator.** In order to adjust the amount of compounding of a generator, the series field

is generally supplied with a shunt, the resistance of which may be varied. This arrangement is shown in Fig. 63–10. The shunt, *R*, around the series field in large machines is usually composed of grids. In smaller machines, it is in the form of resistance ribbon. For a test machine, a piece of German silver wire is generally used, so arranged that any desired length of it may be connected across the series coils. If the machine is too highly over-compounded, it is merely necessary to cut down the length of the shunt wire and so decrease its resistance. This allows a greater part of the load current to flow through the shunt and less through the series field. The series field, therefore, will be just so much weaker, and the voltage proportionally lower. The approximate resistance which a shunt wire must have may be computed as follows:

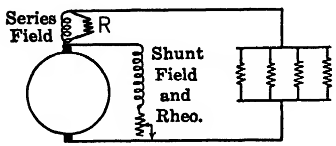



FIG. 63–10. The compounding of a compound generator can be adjusted by means of a resistance shunted across the series field winding.

1. Measure the hot resistance of the series field.

2. Determine the full-load voltage with the series coils cut out and short-circuited.

3. Determine the full-load voltage with the series coils in and unshunted.

4. Compute the total effect of the series coils by subtracting (2) from (3).

5. Compute the ratio of total compounding desired to the total compounding with the series coils unshunted.

6. Put in a shunt of such resistance that the current in the field coils will be the same fraction of the full-load current as computed in (5).

**Example 15.** A compound generator is over-compounded 12 per cent when the series field is unshunted. With the series field cut out, the drop in voltage from no-load value is 18 per cent. The resistance of the series field is 0.02 ohm. What shunt must be used around the series field to reduce the over-compounding to 6 per cent?

Total compounding effect of series coils = 12 + 18 = 30 per cent, or:

Full-load voltage (no series coils) = 100 − 18 = 82 per cent.

Full-load voltage (series coils unshunted) = 100 + 12 = 112 per cent.

Effect of series coils = 112 − 82 = 30 per cent.

Over-compounding desired = 6 per cent or 106 per cent of voltage.

Total compounding desired $= 106 - 82 = 24$ per cent.
Ratio compounding effect desired to unshunted effect $= \frac{24}{30} = \frac{4}{5}$.
Current in shunted series field must equal $\frac{4}{5}$ of full-load current.

Thus $\frac{1}{5}$ of current must go through the shunt wire and $\frac{4}{5}$ through the series field, or 4 times as much through the coils as through the shunt.

This would require, according to the law of parallel circuits, that the shunt be 4 times the resistance of the field coils.

$$\text{Resistance of shunt} = 4 \times 0.02$$
$$= 0.08 \text{ ohm.}$$

This value will not be exactly correct. When the machine has a lower compounding, the terminal voltage is lower and less current flows through the shunt field. Therefore, the series field must be relatively greater to make up for this loss in the shunt field. This would tend to cause the 0.08 ohm shunt wire to be of too low a resistance and take too much current from the series field

However, this value is close enough to allow the approximate length of the needed shunt wire to be determined. The exact length can then be adjusted from a regulation test of the machine. Care should be exercised that the shunt wire is heavy enough to carry the current without excessive heating.

A more precise value of the resistance needed in a shunt around the series coils can be determined from the magnetization curve. First determine the number of series ampere-turns needed, and compute the resistance needed in a shunt to allow enough current to flow through the series turns to produce these ampere-turns.

**Prob. 62-10.** A compound generator delivers 115 volts at no load and 125 volts at full load. With series field cut out, full-load voltage is 98 volts. It is desired to change the compounding so that it will deliver 122 volts at full load. The series field resistance is 0.018 ohm. What must the resistance of the shunt to the series field be (approximately)?

**Prob. 63-10.** Approximately what resistance must be used as a shunt around the series field of the generator in Prob. 62–10 to produce flat-compounding?

**Prob. 64-10.** What precise value of shunt resistance would be needed around the series turns in the generator of Prob. 58–10 to produce 2 per cent over-compounding?

**38. Field Rheostats. Rating.** Rheostats used in series with the shunt fields of generators or motors consist of high-resistance wire or ribbon embedded in porcelain, enamel or other heat-resisting material. In the case of the larger machines, they consist of cast grids of iron alloy, supported in open air. See Fig. 64–10 and Fig. 10–1.

The resistance is varied by means of a movable shoe mounted on an arm that can be rotated. This shoe makes contact with any one of many contact points on the resistance, so that small amounts of resistance may be " cut in " or " cut out " of the field circuit.

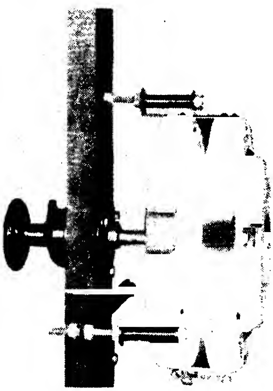


FIG. 64–10. Field rheostat. *General Electric Co.*

The amount of the resistance in the rheostat must be sufficient to vary the field current through rather wide limits. For instance, a rheostat which has a resistance of 10 ohms connected in series with a field winding of 100 ohms resistance would increase or decrease the current approximately only 10 per cent. In general, the resistance in a field rheostat should at least be equal to that of the field winding. With the resistance of the rheostat and the field winding thus proportioned, the value of field current with the rheostat all " cut in " can be increased 100 per cent when the rheostat is all " cut out."

The resistance of field windings of different machines is not the same, and these windings do not carry the same current. A rheostat, which might be satisfactory in the field of one generator, would be burned out, if used in the field of another generator. Shunt-field currents vary from a fraction of an ampere in small machines to 100 amperes and more in large generators. The current-carrying capacity of a field rheostat should be such that, with the normal voltage of the generator across the field circuit, the rheostat will carry the current that flows, either when all the resistance of the rheostat is " cut out " or when it is all " cut in." This means that the current rating of the last few steps, when the resistance is almost " cut out," must be higher than for the other end of the resistance, which carries the smaller current when the resistance is all " cut in." The conductors in a rheostat are therefore tapered with respect to the current they will carry; and the rheostat is often given two current ratings, one for resistance all " cut in," and the other for resistance all, or nearly all, " cut out." Note the example below.

**Example 16.** A certain field rheostat has the following rating:

Minimum field resistance = 100 ohms,
Maximum volts = 320,

Rheostat resistance = 100 ohms,
Current = 3.2 to 1.6 amperes.

This rating means that this 100-ohm rheostat is to be used in a circuit in which the field winding has at least 100 ohms resistance, and that the maximum voltage to be used on the field circuit is 320 volts.

With rheostat resistance all cut in:

Field resistance = 100 + 100 = 200 ohms;
Field current $= \frac{320}{200} = 1.6$ amperes.

With rheostat resistance all, or practically all, cut out,

Field resistance = 100 ohms;
Field current $= \frac{320}{100} = 3.2$ amperes.

FIG. 65–10. A generator frame with commutating poles. *General Electric Co.*

**39. Commutating Poles.** Most modern generators are equipped with "commutating poles," often called "interpoles." They consist of narrow poles set midway between the main poles, as in Fig. 65–10.

We have seen, Art. 16, that the brushes of a generator are shifted ahead in the direction of rotation, so that the short-circuited coil is in the fringe of flux of a following pole. This is the commutating flux which sets up an emf to reverse the current in the coil against the self induction which opposes the reversal.

We have also seen, Art. 22, that when a generator is loaded, the

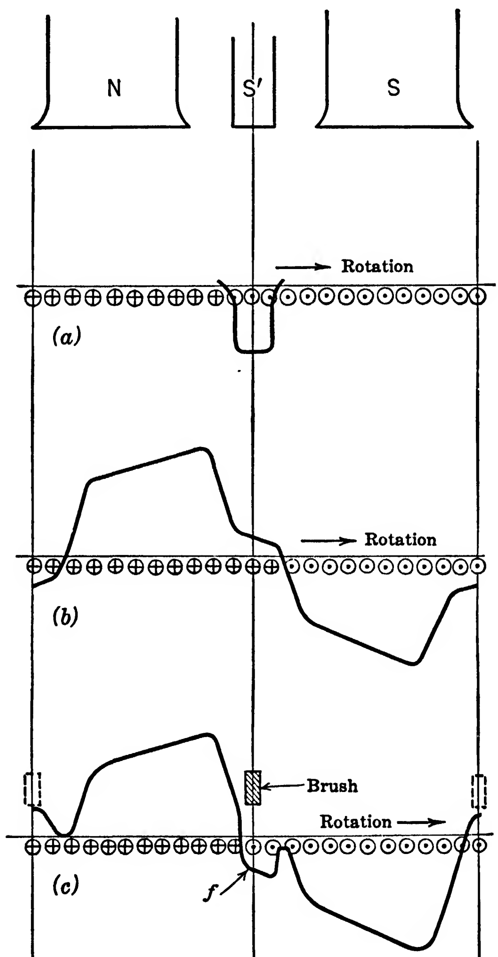



FIG. 66–10. Flux distribution in the air gap of a generator equipped with commutating poles. (a) Flux due to commutating pole only. (b) Flux in air gaps due to both field and armature mmfs when the generator is carrying a load. (c) Combination of the two fluxes in (a) and (b). Note the comparatively wide band of commutating flux.

cross-magnetization of the armature reaction necessitates a still further shift of the brushes to obtain sparkless commutation. Furthermore, the commutating zone or possible position of the brushes is narrowed to the line $pp'$, as indicated in Fig. 44c–10.

The function of the commutating poles is to furnish this commutating flux and provide a wider commutating zone, so that the brushes need not be shifted. This results in better commutation.

Consider Fig. 66–10. The flux curve, due to the interpole $S'$ only, is shown in ($a$). The curve of flux in the air gaps, due to both the field and armature mmf's, is shown in ($b$), and is the same as in Fig. 44c–10. Superposing these two curves, we get the curve of flux in the air gap due to the additional effect of the commutating poles. This is shown in curve ($c$). We now have, if the commutating-pole flux (or mmf) is of the proper value, a comparatively wide zone of commutating flux, shown at $f$ in Fig. ($c$), sufficient to reverse the current in the short-circuited coil and insure sparkless commutation.

Since the voltage, and therefore the flux, necessary to reverse the current in the short-circuited coil increases with the armature current, the commutating-pole flux must increase proportionally. Therefore, the windings on these poles are connected in series with the armature, as shown in Fig. 67–10, and the iron is worked at low flux density on the straight part of the saturation curve.

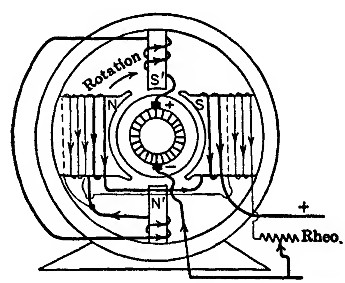



FIG. 67–10. Connections of a compound generator equipped with commutating poles—short-shunt connection.

It is to be noted that the commutating poles do not affect the shift of the flux, due to cross-magnetization, except in the commutating zone, but simply provide the proper commutating flux for the short-circuited coils.

Note also that since the commutating pole supplies an mmf and flux to **reverse** the current in the short-circuited coil, it must, in a generator, be of the **same polarity** as that of the **next** or following main pole in the direction the armature is moving. In a motor, the polarity must be that of the **preceding** main pole, as will be shown in the next chapter.

In large generators, there are as many commutating poles as main poles. In small machines, to save expense, there may be

only half as many of these poles as main poles — depending upon how the armature is wound. See Art. 15, Chap. XIII, page 475.

**40. Compensating Windings.** While machines equipped with commutating poles give satisfactory performance under ordinary service, these poles do not prevent cross-magnetization and flux distortion. Commutating poles alone are generally not satisfactory for machines which are subjected to momentary and repeated heavy over-loads. This applies to high-voltage and high-speed generators, subjected to these over-loads; also, to large motors which must operate through wide ranges of speed in which the direction of rotation must be frequently and rapidly reversed, such as rolling-mill motors.

Cross-magnetization can be prevented by a "**compensating winding,**" set in longitudinal slots in the face of the main poles. This winding, as well as the commutating-pole winding, is also

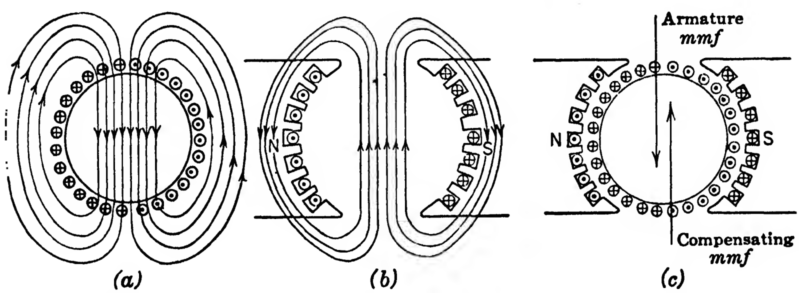



FIG. 68–10. Illustration of the action of a compensating winding in counteracting, or neutralizing, the effect of armature cross-magnetizing mmf, or flux. (a) Cross magnetizing flux due to armature current. (b) mmf, or flux, due to the compensating winding. (c) The combination of the mmfs in (a) and (b) are in opposite directions and neutralize each other.

connected in series with the armature, and carries the armature current. The ampere-turns of the compensating winding are made equal to the magnetizing ampere-turns of the armature. The idea is illustrated in Fig. 68–10. Note that in the figure, the number of compensating conductors in this two-pole, two-path generator is just half the number of conductors on the armature, since the compensating winding carries twice as much current as the armature conductors.

Since this winding is in series with the armature, an increase in cross-magnetization, due to increased armature current, is neutralized by a proportionately increased magnetization in the opposite

direction, due to the compensating mmf, and armature reaction is eliminated. A generator equipped with a compensating winding is illustrated in Fig. 69*a*–10. An enlarged section of this winding is shown in Fig. 69*b*–10.

These windings add greatly to the cost of a generator or motor and consequently are seldom used on small machines.

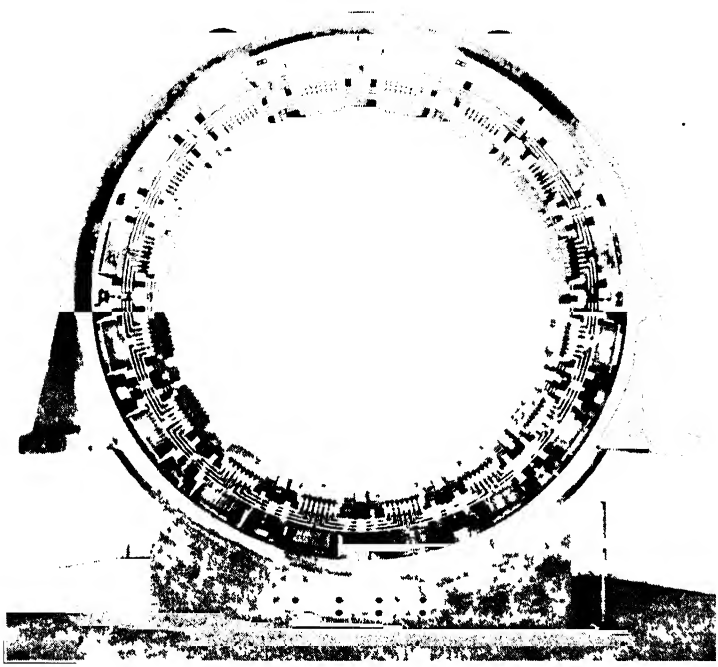


FIG. 69a–10. A field-frame of generator equipped with compensating windings. *Crocker-Wheeler Electric Mfg. Co.*

It is also to be noted that the compensating winding does not provide the commutating flux for reversing the current in the short-circuited coil, and therefore the commutating poles are just as necessary as before.

**41. The Three-Wire Generator.** Many isolated plants, supplying both motor power and electric lighting, have a **three-wire distribution system.** This results in a considerable saving in the amount of copper required in the feeders. Three wires are used in supplying the current for the lamps. See Art. 5, Chapter XV. In this system, one wire is **positive,** one is **negative** and the third

wire is called the **neutral.** Motors are connected in the usual way across the positive and negative (or outside) wires. The lamps are connected between the **neutral** and one or the other of the **outside** wires.

The generator supplying such a system must have three leads connected to it. Ordinary generators have but two sets of brushes, one positive, one negative. In the three-wire generator, the **neutral** lead is obtained by connecting an inductance coil, called the balancing coil, between diametrical taps or points of

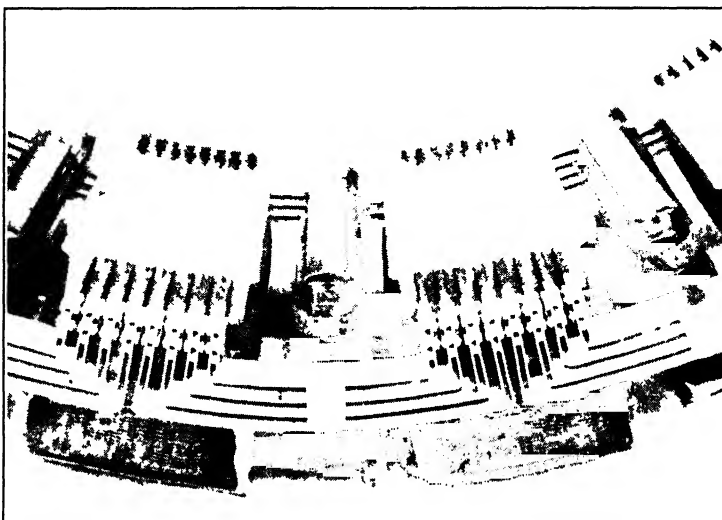


FIG. 69b–10. A close-up showing the details of a compensating winding. *Crocker-Wheeler Electric Mfg. Co.*

maximum potential difference on the armature. The neutral wire is connected to the center of this coil which has an iron core. This coil is sometimes built into the armature spider or mounted on the armature shaft, and its center point connected to a single slip-ring, also mounted on the shaft. Or the coil may be mounted entirely separate from the generator and the two ends connected through two slip-rings to the armature. Figure 70–10 shows an elementary diagram of the second arrangement. The commutator has been omitted for the sake of clearness. Note that half the series-field winding is connected in each outside wire, in order that the compounding may be the same on each side of the neutral.

To obtain this result, there are two series coils on each pole, one in each half of the series winding.

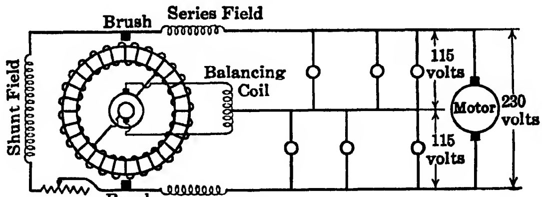



FIG. 70–10. Diagram of a three-wire generator connected to a load.

**42. The Third-Brush Generator.** All generator characteristics which we have so far studied are those of constant-speed machines.

Generators designed to charge batteries and furnish power for lamps on trains and automobiles are driven through a wide range in speed. The requirement of such a generator on a car or automobile is that it shall deliver a normal charging current to the battery when the speed reaches, for example, 10 or 15 miles an hour, and shall not appreciably exceed this normal current at any higher speed. The performance is usually attained by means of the so-called "**third-brush**" generator, illustrated in Fig. 71–10.

The current to charge the storage battery is drawn from the armature through the main brushes $B_1$ and $B_2$. The shunt field is excited by current drawn through brush $B_1$ and the "third-brush" $B_3$, placed somewhat more than 90° from brush $B_1$. The voltage across the field is, therefore, the voltage across the brushes $B_1$ and $B_3$ only.

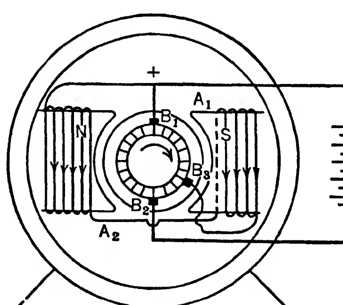



FIG. 71–10. The third-brush generator.

Assume that the train or automobile is traveling at 15 miles per hour and that the generator is charging the battery at normal rate. Now, if the train speed is doubled, doubling the generator speed, the voltage induced between brushes $B_1$ and $B_2$ would be doubled, if the field current remained the same, and a larger current would flow into the battery. But this larger current in the armature windings would set up an increased cross-magnetization of the armature, and would shift or twist the

flux in the air gap in the direction of rotation, as already shown in Fig. 43e–10. This would weaken the pole tips $A_1$ and $A_2$. The weakened field from the pole tip $A_1$ would decrease the voltage between $B_1$ and $B_3$. This would reduce the current delivered to the battery to very nearly the normal value.

The action of this third-brush excitation produces a nearly constant current throughout the full range of speed. However, the action of the coil, short-circuited by the third brush $B_3$, affects the performance of the machine, so that at excessive speeds, the current decreases. The third-brush carries only the very small shunt-field current and, to prevent sparking, is of much higher resistance than the other brushes.

**Shifting** the third-brush, $B_3$, in the **direction of rotation,** increases the voltage across brushes $B_1$ and $B_3$, and thereby increases the voltage across $B_1$ and $B_2$ and **increases** the charging current. Shifting brush $B_3$ in the opposite direction **decreases** the charging current.

If the battery is disconnected from a "third-brush" generator, the cross-magnetization is very much reduced even with the lamps in circuit, and the voltage increases greatly with speed. In this case, either the field winding or the lamps in circuit, or both, may be burned out.

The "third-brush" generator represents an example of a case where use is made of the shift in the field flux caused by the armature reaction to regulate the voltage, and hence the current delivered by a generator. This type of machine has, of course, no compensating winding and no commutating poles.

## SUMMARY OF CHAPTER X

**THE DYNAMO. When a dynamo electric machine is driven by a source of mechanical power to supply electrical energy, it is called a generator. Direct-current generators are rated in kilowatts, speed and the terminal voltage they are to develop without overheating.**

**When the dynamo is supplied with an electric current and is used to drive machinery, cars, etc., it is called a motor. Motors are rated in mechanical horsepower they can supply without overheating, the speed at which they will operate and the voltage of the circuit from which they are to be driven. The dynamo may be used interchangeably, either as a generator or as a motor, except that the rating of the machine in the one case would be slightly different from that in the other.**

**ELECTROMAGNETIC INDUCTION. When an electric "conductor" cuts, or moves across magnetic lines of force, an emf is set up in the conductor proportional to the rate of cutting.**

**DIRECTION OF INDUCED EMF** (Fleming's Right-Hand Rule). If the Thumb, Forefinger and Middle Finger of the Right Hand are all held at right angles to each other, with the Thumb extended in the direction of motion of the conductor, the Forefinger in the direction of the Flux, then the Middle Finger will point in the direction of the Induced EMF.

AMOUNT OF EMF. When $10^8$ magnetic lines are cut per second by a "conductor," one volt of EMF is induced.

MODERN GENERATORS are fundamentally loops of wire, called the armature, revolved so as to cut through a strong magnetic field. An alternating emf is induced in these loops, the curve of which approximates a Sine Curve.

If Collecting Rings are attached to the ends of these loops, or coils, an ALTERNATING CURRENT is delivered by the GENERATOR.

If a COMMUTATOR is used, a DIRECT CURRENT is delivered, as the commutator rectifies the alternating current in the revolving coils. Since the armature coils are connected across a gap between two commutator segments, the brushes must each continually short-circuit two or more coils. A COMMUTATOR is constructed of wedge-shaped copper bars, insulated from each other by strips of mica.

ARMATURES in commercial generators are DRUM WOUND.

BIPOLAR generators have TWO POLES. MULTIPOLAR generators are machines of MORE than two poles, FOUR, SIX, etc.

There are as many SETS OF BRUSHES and as many PARALLEL PATHS in an armature as there are poles (except in multipolar wave-wound armatures). One half the number of brushes are POSITIVE, and half are NEGATIVE.

THE ARMATURE RESISTANCE from plus terminal to negative terminal equals the resistance of all the wire wound on the armature divided by the square of the number of paths. The resistance of the brush contact is generally included in the term Armature Resistance. The brush-contact resistance causes a drop of about 2 volts at all loads.

If the BRUSHES are so placed that with no load on the machine they short-circuit the armature coils at the instant they are not cutting the magnetic flux of the field, they are said to be on the NEUTRAL AXIS. The neutral axis may be defined as a line which bisects the angle between the poles, called the GEOMETRICAL NEUTRAL.

The process of reversing the current in the short-circuited coils as the commutator segments pass under a brush is called COMMUTATION.

Due to the inductive effect under load, each coil, at the instant of short-circuit, must have induced in it A VOLTAGE SUFFICIENT TO REVERSE THE CURRENT. Accordingly, the brushes are so placed ahead of the neutral axis that the coils are commutated while they are cutting some flux from the next pole ahead. This flux is called the COMMUTATING FLUX.

Because of the cross-magnetization, due to the current in the armature, the flux in the air gaps is distorted and shifted ahead. Accord-

ingly, the position of the brushes must be advanced in order to short-circuit the coils while cutting the commutating flux. This position is on a line ahead of the neutral axis, and is called the AXIS OF COMMUTATION.

The angle between the neutral axis and the axis of commutation is called the ANGLE OF BRUSH LEAD.

ARMATURE REACTION is the name given to the magnetomotive forces produced by the armature current flowing through the armature turns. It is divided into two parts as follows:

CROSS-AMPERE-TURNS are the product of the armature current times those armature turns OUTSIDE the double angle of brush shift. These turns produce field distortion and necessitate a forward lead of the brushes to get sparkless commutation. They also have a slight demagnetizing effect.

BACK-AMPERE-TURNS are the product of the armature current times the armature turns INSIDE the double angle of brush lead. These ampere-turns produce demagnetization of the field and cause a drop in terminal voltage.

THE AVERAGE EMF INDUCED IN A D-C ARMATURE equals $\frac{1}{10^8}$ of the number of magnetic lines cut per second, multiplied by the number of conductors in series between any two adjacent brushes; that is, in any single armature path. Expressed as an equation:

$$\text{EMF} = \frac{\Phi PZN}{10^8 \times 60 \times a}$$

when $\Phi$ = flux per pole; P = number of poles; Z = conductors on the armature; N = rpm; a = number of paths in armature.

A SATURATION or MAGNETIZATION CURVE shows the relation between the ampere-turns in the field winding and the flux per pole in a generator driven at constant speed. Curve may be plotted between field current and generated volts.

DIRECT-CURRENT GENERATORS are divided into two classes: first, Separately Excited; second, Self-Excited.

SEPARATELY EXCITED generators have their fields excited by some outside source of current. They are used where the voltage of the armature is not suitable to excite the field. Terminal voltage falls slightly as the load increases. Voltage may be controlled by a field rheostat. Their use is very limited.

SELF-EXCITED generators are of three types according to the method of exciting the field coils: (1) Series; (2) Shunt; (3) Compound.

SERIES. The field coils, which consist of a few turns of heavy wire and low resistance, are connected in series with the line and carry the same current as the armature. Voltage rises rapidly as the load increases. Are used today only as line boosters.

SHUNT. Field winding is connected in parallel with the load circuit. It consists of many turns of fine wire which carry only a small current and have a comparatively high resistance. It is a nearly con-

stant voltage generator, the voltage falling slightly more with increase of load than in the separately excited machine. Voltage may be controlled somewhat by field rheostat.

COMPOUND. The field consists of two sets of coils; one series and the other shunt. When enough series turns are wound on, to offset exactly any fall in terminal voltage due to increased armature drop and armature reaction, the machine is said to be FLAT-COMPOUNDED. When the number of series turns are more than sufficient to offset armature drop and armature reaction, the voltage rises slightly with load, and the machine is said to be OVER-COMPOUNDED. Voltage may be controlled by rheostat in the shunt field.

Percentage VOLTAGE REGULATION of a separately excited or shunt generator is:

$$\frac{\text{Terminal E (no load)} - \text{Terminal E (full load)}}{\text{Terminal E (full load)}} \times 100$$

Terminal voltage at FULL LOAD must be the normal or rated voltage of the machine.

Percentage COMPOUNDING of a compound generator is

$$\frac{\text{Terminal E (full load)} - \text{Terminal E (no load)}}{\text{Terminal E (no load)}} \times 100$$

Terminal voltage at NO LOAD must be the normal or rated voltage of the machine.

RHEOSTATS in the field circuits of generators should have approximately the same resistance as that of the field winding.

Most modern generators are equipped with COMMUTATING POLES set midway between the main poles. The windings on these poles are connected in series with the armature, and supply a commutating flux which is nearly proportional to the armature current. This flux sets up an emf which reverses the current in the short-circuited coils undergoing commutation, and the brushes need not be shifted. These poles do not affect the shift in the main field flux, due to armature reaction, except in the commutating zone.

COMPENSATING WINDINGS, set in slots in the main pole faces and connected in series with the armature, supply a magnetomotive-force which is equal and opposite to that of the armature cross-ampere-turns, and thus prevent the shifting of the field flux in the air gaps. Because of expense, they are used only on large machines subjected to rapidly fluctuating over-loads.

THREE-WIRE GENERATOR supplies current to a three-wire distribution system having a positive, a negative and a neutral wire. The neutral lead from the machine is obtained by connecting the two ends of an induction or balancer coil to points of maximum potential difference in the armature. The neutral wire is connected to the middle point of the balancer coil. If the coil is mounted in the armature structure, the mid-point is connected to the outside lead through one slip-ring. If the coil is installed entirely separate from the gen-

**erator, it is connected through two slip-rings to points on the armature.**

**The THIRD-BRUSH GENERATOR is used for constant current regulation when the speed of the generator varies through wide ranges. As the armature current increases, the cross-ampere-turns weaken the trailing pole tip, the flux of which supplies the voltage between a main brush and the third brush. This is the voltage across the field coils, and hence the field current falls as the armature current rises. This tends to keep the armature current constant.**

## PROBLEMS ON CHAPTER X

**Prob. 65–10.** The field of a 2-pole separately excited generator has 5,000,000 lines. In each armature path, there are 90 conductors in series. If these conductors cut the field 3000 times per minute, how much voltage is induced in each armature path?

**Prob. 66–10.** If the resistance of the armature of Prob. 65–10 is 0.4 ohm and 10 ohms are placed in the external circuit: (*a*) How much current would the generator deliver? (*b*) How much power is consumed in the armature? (*c*) How much in the external circuit? Disregard armature reaction.

**Prob. 67–10.** What horsepower would be required to drive the generator of Prob. 66–10? Disregard friction and other mechanical losses.

**Prob. 68–10.** A 4-pole, 4-path generator has 3,200,000 lines per pole. The armature has 240 conductors and runs at 1200 rpm. What is the no-load voltage?

**Prob. 69–10.** The armature of Prob. 68–10 is wound with 2450 feet of No. 12 B & S copper wire. What is the armature resistance, exclusive of brush-contact resistance?

**Prob. 70–10.** The armature of a bipolar generator has 130 conductors. There are 6,000,000 lines of flux per pole. At what speed must the armature be driven to induce 120 volts?

**Prob. 71–10.** The armature of a 6-pole generator has 1100 conductors. Speed is 900 rpm. Area of each pole face is 80 square inches. Flux density in the air gaps is 60,000 lines per square inch. Number of armature paths is 6. What voltage is induced?

**Prob. 72–10.** A 12-pole, 12-path, 800-kw generator at full load has a voltage across the brushes of 650 volts. There are 5000 feet of No. 4 B & S copper wire wound on the armature. What emf must be induced? Neglect brush-contact resistance.

**Prob. 73–10.** If there are 200 conductors in each path of the generator in Prob. 72–10, what must be the flux per pole at full load? Speed = 125 rpm.

**Prob. 74–10.** The armature of a bipolar generator is wound with 800 turns of No. 8 B & S copper wire. Each turn is 3.5 feet long. What is the armature resistance?

**Prob. 75–10.** Allowing 1000 amperes per square inch of cross section of conductor, how many amperes should the generator of Prob. 74–10 deliver?

**Prob. 76–10.** What emf must be generated in the armature of Prob. 75–10 to maintain a pressure of 110 volts across the brushes?

**Prob. 77–10.** What must be the flux per pole in Prob. 76–10? Rps = 20.

**Prob. 78–10.** An 8-pole, 8-path generator has 820 conductors on the armature, which makes 360 rpm. No-load voltage is 240 volts. Area of each pole face is 140 square inches. What is the flux density?

**Prob. 79–10.** When the generator of Prob. 78–10 is delivering 2000 amperes, the voltage at the brushes is 230 volts. If the armature resistance is 0.004 ohm, what must the flux density be at this load? The speed is unchanged.

**Prob. 80–10.** The resistance of the armature of a 10-pole, 10-path generator is 0.005 ohm. Speed = 200 rpm. When the generator delivers a current of 1200 amperes at 120 volts, the flux per pole = 7,500,000 lines. How many conductors are there on the armature?

**Prob. 81–10.** The resistance of the field coils of a 5-kw shunt generator is 220 ohms. Full-load terminal voltage is 115 volts. (*a*) What current flows in the field circuit? (*b*) How much power is consumed in the field? (*c*) How many turns are there in the field, if the ampere-turns equal 2000? (*d*) What is the armature current?

**Prob. 82–10.** If the generator in Prob. 81–10 has 4 poles, 4 paths, and the armature is wound with 1800 feet of No. 14 B & S wire, what emf must be induced?

**Prob. 83–10.** The resistance of the armature in a bipolar shunt generator is 0.30 ohm. The field resistance is 50 ohms. If the generator delivers 50 amperes to the line at a terminal pressure of 120 volts, what emf must it generate?

**Prob. 84–10.** What current flows in each armature circuit of the generator in Prob. 83–10?

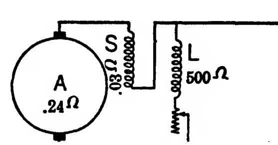



FIG. 72–10. A compound generator connected long-shunt.

**Prob. 85–10.** In the compound generator, Fig. 72–10,

Resistance of armature $A$ = 0.24 ohm;
Resistance of shunt-field circuit $L$ = 500 ohms;
Resistance of series-field $S$ = 0.03 ohm.

When delivering 60 amperes to the line at 550 volts: (*a*) How much current flows through the armature? (*b*) What is the induced emf of the generator?

**Prob. 86–10.** Each resistor in Fig. 73–10 takes 25 amperes at 115 volts.

Armature has 0.04 ohm resistance.
Series field has 0.02 ohm resistance.
Shunt-field circuit has 76 ohms resistance.

Find:

(*a*) The armature current;
(*b*) The emf of the generator at this load.

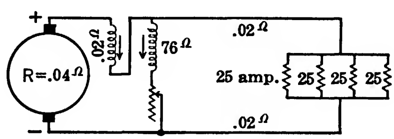



FIG. 73–10. A long-shunt compound generator connected to a lo

**Prob. 87–10.** Find the induced voltage and armature curre t of the machine in Prob. 86–10 when connected as a short-shunt m-pound generator and supplying the same circuit.

**Prob. 88–10.** In designing a generator to have a no-load voltage of 240 volts, the armature is wound with 380 conductors. Two brushes are used. The flux per pole is 2,800,000 lines. How many poles must the generator have in order to run as nearly as possible at a speed of 500 rpm?

**Prob. 89–10.** What would the flux per pole in Prob. 88–10 have to be to produce exactly the 240 volts desired?

**Prob. 90–10.** A compound generator, short shunt, is delivering power to the following equipment in multiple. Eight hundred 60-watt, 115-volt lamps and one 115-volt, 15-horsepower motor, with an efficiency of 87 per cent. The resistance of the shunt-field circuit is 27.5 ohms; of the series field, 0.003 ohm; of the armature, 0.015 ohm; of the line wires, 0.01 ohm.

Find:

(*a*) The shunt-field current.
(*b*) The armature current.
(*c*) The induced emf at this load.

**Prob. 91–10.** If the generator of Prob. 90–10 were connected long shunt, what would be the answers to (*a*), (*b*) and (*c*) of that problem?

**Prob. 92–10.** A 12.5-kw, 240-volt, 1100-rpm shunt generator has the data and magnetization curve given below:

No. of poles = 6. No. of paths = 6. No. of conductors = 180.
Brushes set with 10 degrees forward lead.
Armature resistance (including brushes) = 0.18 ohm.
Turns per pair of poles in shunt field = 700.

MAGNETIZATION CURVE AT 1100 RPM

| Ampere-turns per pair of poles | Volts generated | Ampere-turns per pair of poles | Volts generated |
|---|---|---|---|
| 0 | 5 | 840 | 240 |
| 140 | 58 | 980 | 252 |
| 280 | 118 | 1120 | 261 |
| 420 | 171 | 1260 | 268 |
| 560 | 203 | 1360 | 272 |
| 700 | 224 | | |

Construct the external characteristic of this machine operated as a shunt generator. Determine sufficient points on curve to obtain maximum current. What is the resistance in the shunt-field circuit?

**Prob. 93–10.** What number of series turns must be added to the shunt generator of Prob. 92–10 to produce 7 per cent over-compounding? Use 0.03 ohm as the resistance of the series-field and long-shunt connection.

**Prob. 94–10.** If 15 series turns per pair of poles are added to the generator of Prob. 92–10, what amount of resistance must be shunted around the series field to produce flat compounding? Resistance of series field = 0.05 ohm.

**Prob. 95–10.** Plot the external characteristic for the generator of Prob. 94–10.

**Prob. 96–10.** (*a*) If the generator of Prob. 92–10 is run at 1200 rpm, how much resistance must be inserted in the field to bring the no-load voltage down to 220 volts? (*b*) By cutting out this added resistance to the field, for how great an armature current can the terminal voltage be maintained at 220 volts?

**Prob. 97–10.** A group of motors, situated 1200 feet from the power-house, requires a voltage of 230 volts when using a current of 50 amperes. Also the no-load voltage must not rise above 230 volts. The line wire is No. 2 B & S copper. It is proposed to use the generator of Prob. 92–10 with a fixed resistance in series with the shunt field to produce a no-load voltage of 230 volts, and sufficient series turns (resistance to be 0.03 ohm) added to keep the voltage at the motors 230 volts. Long-shunt connection.

(*a*) What value must the resistance have which is added to the shunt field?

(*b*) How many series turns must be added?

(*c*) Plot a curve between voltage at the motors and the current up to 50 amperes; determine the greatest percentage voltage deviation from 230 volts.

**Prob. 98–10.** It is proposed to meet the conditions of Prob. 97–10 using the generator of Prob. 92–10 as follows: the resistance of the shunt-field circuit is to be adjusted to 200 ohms and held constant, while the no-load voltage of 230 volts is to be obtained by lowering the speed. The 230 volts at the motors when drawing 50 amperes is to be secured, as before, by adding series turns to the generator.

(*a*) At what speed must the generator be run?

(*b*) How many series turns must be added (resistance = 0.03 ohm)?

(*c*) What is the greatest percentage voltage variation at the motors from no load to 50 amperes?

**Prob. 99–10.** The generator described in Prob. 30–10 is to be used as a shunt generator, using the same field coils. The no-load voltage desired is 260 volts.

(*a*) What resistance must be inserted in series with the field?

(*b*) What will be the full-load voltage under these conditions?

**Prob. 100–10.** (*a*) What resistance must be inserted in the field circuit of Prob. 99–10 to produce a no-load voltage of 250 volts? (*b*) What series turns must be added (long shunt) to produce flat compounding? Resistance of series field = 0.1 ohm.

# CHAPTER XI

## MOTORS

It was stated in Chapter X that a dynamo can be used interchangeably, either as a generator or as a motor. A dynamo which operates satisfactorily as a generator will generally operate equally well as a motor. In practice there are some differences in construction. Generators are usually of an open type with the field coils and armature exposed; while motors are often built with the frame partially enclosed to protect the windings from injury. Where motors must operate in dirty or dusty locations, they may be entirely enclosed.

The **operation** of the motor, however, is quite different from that of the generator. The generator is always driven by a "prime mover," or source of mechanical power, and generally at a constant, or nearly constant speed. The main function of a generator is to **generate an electromotive force** by rotating the armature conductors through a magnetic field. The motor has no source of mechanical power, but instead is operated by electrical power supplied at a constant, or nearly constant voltage. The main function of a motor is **to produce turning effort, or torque** sufficient to maintain a mechanical rotation. This turning effort is called **motor action.**

In order to understand this motor action, we must recall the effect of a magnetic field upon a conductor carrying a current.

**1. Force Exerted by a Magnetic Field on a Wire Carrying Current.** It will be remembered that a wire carrying current has a circular magnetic field about it like that in Fig. 1–11. If this wire is placed in a uniform magnetic field at right angles to the lines of force, a resultant field is obtained as shown in Fig. 2–11. Here the wire carries a current flowing **in** and has **a** clockwise magnetic flux, or field, about it. This field acts **with** the main field above the wire, and acts **against** the main field **below** the wire. The result is to deflect and crowd the lines, or increase the flux density above the wire; and to reduce the flux density, or thin out the lines below the wire. Since magnetic lines can be thought

of as acting like stretched rubber bands, the lines above the wire tend to contract or shorten. Thus they exert a **downward** push on the wire, as indicated by the arrow in the figure. If the current in the wire of Fig. 2–11 were in the **opposite** direction, the magnetic lines would be crowded **below** the wire, and would exert an **upward** push on the wire. The force exerted by the magnetic field is

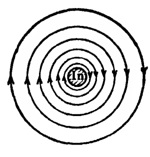


FIG. 1–11. The magnetic field about a wire carrying an electric current.

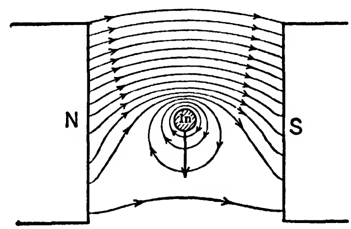



FIG. 2–11. The effect of placing the wire of Fig. 1–11 into a uniform parallel magnetic field.

actually exerted on the electric current, but since the current cannot leave the wire, the force is transmitted to the wire.

Note that the direction of the force, the direction of the field and the direction of the current are all at right angles to one another.

**2. Fleming's Left-Hand Rule.** In Art. 3, Chapter X, the relations of the direction of the magnetic field, the motion of the conductor and the direction of the **induced electromotive force** and resulting current are given by Fleming's **Right-Hand Rule.**

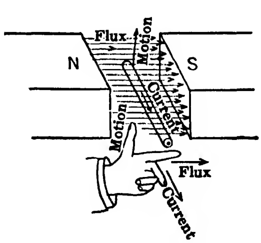



FIG. 3–11. Fleming's Left-Hand Rule for motors.

Similarly, the relations of the direction of the magnetic field, the direction of the current in the conductor and the direction of the force and resulting motion are determined by Fleming's **left-hand** rule for the motor, which is stated as follows:

Point the forefinger of the **left hand** in the direction of the magnetic lines, the middle finger in the direction of the current, and the thumb will point in the direction of the force, or in the direction the conductor tends to move. This is illustrated in Fig. 3–11. Apply this rule to Fig. 2–11.

**3. Calculation of the Force Exerted on a Conductor in a Magnetic Field.** The force acting on a conductor in a magnetic field is due to three factors — the flux density of the field; the current

flowing in the conductor; and the length of the conductor in the field. An increase in flux density of the field results in the deflection of more magnetic lines and a greater flux density above or below the conductor, and hence a greater side thrust on the wire. An increase of current in the conductor also results in the deflection of more lines and a greater side thrust on the wire. An increase in the length of the conductor in the magnetic field increases the side thrust proportionately.

This force expressed in dynes (cgs units) is written as:

$$F = \frac{BlI}{10}, \tag{1}$$

where $B$ = flux density in lines per square centimeter,
$l$ = length of conductor in the field in centimeters,
$I$ = current in the conductor in amperes.

The force in pounds acting on the conductor may be computed from the equation

$$F = \frac{8.85\,BlI}{10^8}, \tag{2}$$

where $F$ = force in pounds acting on the conductor,
$B$ = flux density of the field in lines per square inch,
$l$ = length of the conductor in inches,
$I$ = current in the conductor in amperes.

**Example 1.** If the wire in Fig. 2–11 is 24 inches long and is carrying a current of 100 amperes in a field with a flux density of 25,000 lines per square inch, what force is acting on the wire?

**Solution.**

$$F = \frac{8.85 \times 25{,}000 \times 100 \times 24}{10^8} = 5.31 \text{ pounds.}$$

**4. Torque, Single-Coil Armature.** The torque applied to a shaft may be defined as the effort tending to rotate it

When, for example, a tangential force, $F$, is applied by the belt on the rim of the pulley in Fig. 4–11, it exerts a turning effort or a torque on the shaft, $S$, tending to produce rotation. This torque is measured not only by the amount of the force, $F$, but by the product of this force and its perpendicular distance from the center of the shaft, which in this case is the radius of the pulley, $r$.

Torque is a moment or a leverage and is measured in **pound-feet** (not in foot-pounds, which measure work). It can be expressed

by the equation

$$T = Fr \tag{3}$$

where $T$ is the torque in pound-feet,
$F$ is the force in pounds,
and $r$ is the radius, or distance in feet at which the force acts.

The distance $r$ is called the arm of the force.

In the case of a generator, the torque is supplied by the prime mover which drives it.

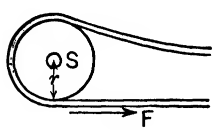



FIG. 4–11. The pull of the belt exerts a force, F, at the rim of the pulley, thereby exerting a torque on the shaft S.

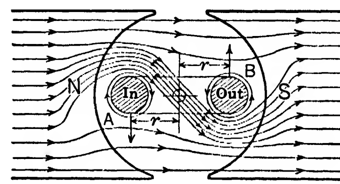



FIG. 5–11. The forces on a coil carrying a current in a parallel magnetic field.

In the case of a motor, the torque is developed within the machine itself.

Consider Fig. 5–11, which shows a cross section of a single-coil armature carrying current in a magnetic field. The conductors are mounted on the armature core at a distance $r$ from the center of the shaft. The current is flowing **in** on conductor $A$ and **out** on conductor $B$, as marked, and the flux density is increased **above** $A$ and **below** $B$. Thus there is a force pushing **down** on conductor $A$, and also a force pushing **up** on conductor $B$. This tends to cause the coil to rotate counter-clockwise, as marked. This tendency to rotate, or torque, is measured by the total force on the two wires multiplied by the distance of either wire from the center of the shaft — the radius $r$.

The force exerted by each conductor is computed as in equation (2), and the torque exerted by each conductor is expressed as,

$$T = Fr = \frac{8.85\ BlI \times r}{10^8} \text{ pound-feet,} \tag{4}$$

where $r$ is the arm of the force in feet.

**Example 2.** Each wire in Fig. 5–11 is 24 inches long, is carrying 100 amperes and is in a uniform field of 25,000 lines per square inch.

The diameter of the loop is 8 inches. What is the torque on each wire when the loop is in this position?

**Solution.**

$$T = Fr = \frac{8.85 \times 25{,}000 \times 24 \times 100}{10^8} \times \frac{4}{12}$$

$$= 5.31 \text{ pounds} \times 0.333 \text{ feet} = 1.77 \text{ pound-feet.}$$

Torque is often expressed as the number of pounds at the end of an arm of 1 foot. Thus 1.77 pounds with a 1-foot arm would produce the same torque (1.77 × 1) as 5.31 pounds with a 0.333-foot arm (5.31 × 0.333).

Of course the torque on both wires of the loop is 2 × 1.77 = 3.54 pound-feet.

**Prob. 1–11.** A wire 16 inches long lies at right angles to the lines in a magnetic field. The force on the wire is 1.8 pounds when 30 amperes flow through it. How strong is the magnetic field?

**Prob. 2–11.** A 10-inch wire is urged at right angles to the flux in a magnetic field of 31,000 lines per square inch by a force of 2.5 pounds. What current is flowing in the wire?

**Prob. 3–11.** (*a*) Indicate the resultant field if the poles and current are as marked in Fig. 6–11.

(*b*) Indicate direction in which loop will tend to turn.

(*c*) If flux is 25,000 lines per square inch, length of conductors is 14 inches each and current through the loop is 120 amperes. What force in pounds is exerted on each wire?

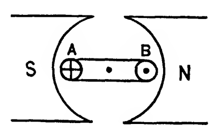


FIG. 6–11. A coil carrying a current in a parallel magnetic field.

**Prob. 4–11.** The loop in Prob. 3–11 has a diameter of 15 inches. What torque does the loop develop?

**Prob. 5–11.** If the field strength in Prob. 4–11 were increased to 35,000 lines per square inch, what would be the torque?

**5. Torque of a Motor.** The single-coil armature of Fig. 5–11 cannot be used to produce rotation, for when it reaches the position shown in Fig. 7–11, the conductors can move no farther, except to spread the loop. If an armature with a comparatively large number of conductors is used, as indicated in Fig. 8–11, a steady torque to produce rotation can be obtained. Note, in the figure, that the conductors under the faces of the south poles are carrying currents **in**, and according to the "left-hand rule," develop a torque tending to rotate the armature clockwise as shown by the arrows. And those conductors under the faces of the north poles are carrying currents **out**, and by the same rule,

also tend to rotate the armature clockwise, or with the arrows. Thus the current in all the conductors develops a torque in the same direction. If the force on one conductor be computed, and the radius or distance to the center of the shaft be measured, the torque developed by one conductor can be calculated. Multiplying this torque by the number of active conductors (conductors carrying current in the magnetic field), the total torque exerted on the shaft of the machine may be obtained.

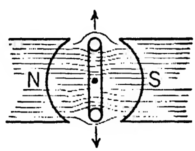



FIG. 7-11. Force exerted on a single coil in this position does not produce rotation, but merely tends to spread the loop.

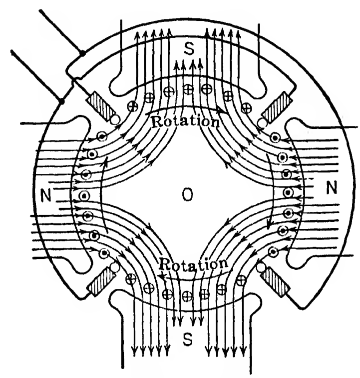



FIG. 8-11. The conductors of an armature in a four-pole field. Left-hand rule shows that all are urged in a clockwise direction.

**Example 3.** Assume that each conductor in Fig. 8-11 is carrying 50 amperes, and that the distance from the center of the armature is 6 inches. Let the length of the armature be 10 inches, and the flux density in the air gaps be 60,000 lines per square inch. What torque does the armature develop?

**Solution.**

$$\text{Torque due to each conductor} = \frac{8.85\,BlI}{10^8} \times r$$

$$= \frac{8.85 \times 60{,}000 \times 10 \times 50}{10^8} \times \frac{6}{12} = 1.33 \text{ pound-feet.}$$

Number of active conductors under pole faces = 28.

Total torque developed in the armature
$= 28 \times 1.33 = 37.3$ pound-feet.

This torque is equivalent to a force of 37.3 pounds acting at the rim of a two-foot pulley; i.e., acting one foot from the center of the pulley.

To develop a continuous torque in one direction, it is seen from Fig. 8-11 that the direction of the current in the conductors under a south pole must be opposite from that in the conductors under a north pole. Therefore, as the armature revolves, the current

must be reversed in the coils as they move through the neutral position, or position of zero torque. Thus a commutator is just as necessary in the d-c motor as in the d-c generator.

It is also to be noted that the torque developed by a motor with a considerable number of armature conductors is a steady or continuous one, as compared to the pulsating torque developed by a reciprocating engine.

The armatures of modern machines are wound on toothed cores like that in Fig. 9–11, and the force is not actually on the wires, but on the teeth. The conductors lie in slots where there is little flux, most of the magnetic lines passing through the teeth.

Note that the current in the windings between the points $A$ and $B$ on the armature tends to make a south pole on this quadrant of the surface of the armature. Fig. 10–11 is a reproduction of the portion $A$ to $B$ of the armature of Fig. 9–11. It is evident how this

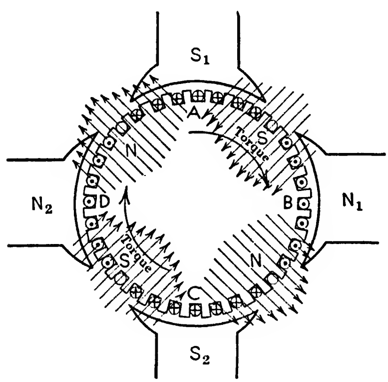



FIG. 9–11. Diagram showing the motor action of a slotted core armature.

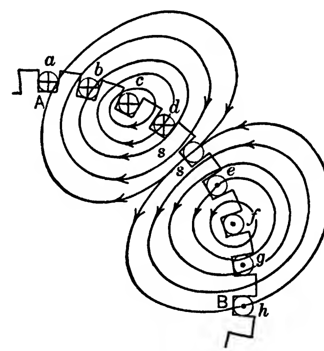



FIG. 10–11. An enlarged view of the field on that part of the surface between the points A and B of the armature in Fig. 9–11.

area tends to become a south pole. The current is flowing **in** in conductors $a$, $b$, $c$ and $d$; and **out** in conductors $e$, $f$, $g$ and $h$. These conductors act then like a flat coil. By applying the right-hand rule for a coil, it can be seen that there is a tendency to set up a flux in the direction indicated, which will make this area a south pole.

This quadrant of the armature would therefore be repelled by the south pole $S_1$, Fig. 9–11, and attracted by the north pole $N_1$. Similarly, the current in the conductors, lying between the points $B$ and $C$ on the armature, tends to set up a north pole on this

quadrant of the armature, which is attracted to the south pole $S_2$, and repelled by $N_1$. In like manner, the quadrant $CD$ becomes a south pole and is attracted by $N_2$ and repelled by $S_2$. The quadrant $DA$ becomes a north pole and is attracted by $S_1$ and is repelled by $N_2$.

It is fortunate that the force in large machines does not come upon the conductors, but on the teeth, as it would be very difficult to provide insulation which would not be crushed by the enormous forces developed by modern electrical machinery.

In computing the torque of these machines, however, the equation that has been given can be used, if the value of the flux density, $B$, is taken as the average flux density in the air gap.

**Prob. 6–11.** The armature of a motor is 10 inches long, 9 inches in diameter and has 120 conductors under the pole faces. Average flux density in the air gap is 52,000 lines per square inch. If the current per conductor is 40 amperes, what torque does the armature develop?

**Prob. 7–11.** What force would the armature of Prob. 6–11 exert on a belt at the rim of a 7.5-inch pulley fitted to the shaft?

**Prob. 8–11.** The armature of a bipolar motor has a diameter of 6 inches, a length of 12 inches and 180 active conductors under the pole faces. It draws 30 amperes from the line, and the torque developed is 32.3 pound-feet. What is the average flux density in the air gap?

**Prob. 9–11.** A 6-pole, 6-path motor is to be used to drive a shaft on which there is a 12-inch pulley. The force required at the rim of this pulley is 50 pounds. What current must be supplied to the armature if it is 18 inches long, 20 inches in diameter and has 246 active conductors under the pole faces? The average flux density in the air gap is 48,000 lines per square inch.

**Prob. 10–11.** A 4-pole, 4-path motor has an armature, 12 inches in diameter and 10 inches long, with 184 active conductors. The flux density in the air gap averages 45,000 lines per square inch. The armature takes 80 amperes. With what diameter pulley must it be fitted to exert a pull of 72 pounds on a belt?

**6. Torque Relations in Any Given Motor.** The torque developed in a motor by a single conductor has been expressed in Art. 4, as $T = \frac{8.85\ BlIr}{10^8}$ and shows that torque is proportional to flux density in the air gap and to the current per conductor. But flux density equals the flux per pole divided by the area of a pole face, or $B = \frac{\Phi \text{ per pole}}{\text{area of pole face}}$. And the current per conductor

equals the total armature current divided by the number of paths, or $I$ per conductor $= \dfrac{\text{total } I_a}{\text{no. of paths}}$.

Substituting these values of $B$ and $I$ in the equation above, we have

$$\text{Torque} = \frac{8.85 \times \Phi \times l \times I_a \times r}{10^8 \times (\text{area of pole face}) \times (\text{no. of paths})}.$$

Since, for a particular machine, all the terms in the above equation, excepting $\Phi$ and $I_a$, are constant, the torque equation may be written

$$T = K\Phi I_a. \tag{5}$$

The relation expressed by this equation is very important, as its use helps to analyze the performance of a motor under various loads. The load on a motor is mechanical, and is applied at the armature shaft, or at the rim of a pulley connecting the motor to the load. Thus, any increase in load requires a greater turning effort, or torque, on the part of the motor. From the equation above, it is easily seen that increased torque must be obtained, either by an increase in armature current, $I_a$; or by an increase in flux, $\Phi$; or by an increase in both.

**7. Calculation of the Horsepower of a Motor.** It has been shown that the torque developed by a motor, which causes the armature to rotate, is equal to the side push, or force, exerted by all the active conductors multiplied by the distance at which this force acts — the radius of the armature.

If the peripheral speed of the conductors is known, the output of the motor in foot-pounds per minute or in horsepower may be calculated.

For if $F$ = force in pounds exerted by all the active armature conductors,

$r$ = radius of the armature in feet,

and $N$ = revolutions of the armature per minute;

then $2\pi r$ = the distance in feet through which the force acts in one revolution,

$2\pi rN$ = the distance through which the force acts in one minute,

and $2\pi rFN$ = foot-pounds of work done in one minute,

or $\dfrac{2\pi rFN}{33{,}000}$ = horsepower of the motor.

But, $Fr = T$ = torque in pound-feet.

Therefore,

$$\text{horsepower} = \frac{2\,\pi TN}{33{,}000}. \tag{6}$$

It should be noted that in the actual motor the torque available at the shaft is somewhat less than that developed by the current in the armature conductors, due to the fact that some torque is used to overcome losses in the machine itself. However, if $T$ in the equation above is the net torque at the shaft, or pulley, this equation expresses the horsepower output of the motor.

**Prob. 11–11.** A motor develops a net torque of 40 pound-feet at the shaft and runs at 900 rpm. What horsepower does it develop?

**Prob. 12–11.** What torque must a 15-hp motor develop if it delivers full load at 1200 rpm?

**Prob. 13–11.** If the motor in Prob. 12–11 operates at 1800 rpm what torque will it have to develop?

**Prob. 14–11.** A 20-hp motor, equipped with a 14-inch pulley, supplies full load at 1200 rpm through a belt. What pull is exerted on the belt?

**8. Types of Motors.** It was stated in Art. 21, Chapter X, that motors, as well as generators, are classified according to the method of exciting the field windings. Direct-current motors, therefore, are either shunt, series or compound wound.

In a shunt motor, the field winding with the field rheostat is connected directly across the mains, exactly as in the shunt generator, as illustrated in Fig. 41*b*–10. **The field** of the **shunt generator** is in **parallel with the external circuit,** or the load; hence part of the armature current is diverted and flows through the field winding, and part flows on through the line to the load; so $I$ (arm) = $I$ (line) + $I$ (field). On the other hand, the field of the shunt motor is in **parallel with the armature;** and hence part of the current supplied to the motor flows through the armature and part through the field; and $I$ (arm) = $I$ (line) − $I$ (field). This is illustrated in the example below.

**Example 4.** (*a*) The armature of the shunt generator, Fig. 11*a*–11, is supplying 21.5 amperes at 110 volts. The shunt field takes 1.5 amperes. Therefore 21.5 − 1.5 or 20 amperes is supplied to the load. (*b*) In Fig. 11*b*–11, the shunt motor is drawing 21.5 amperes from the line. If the field circuit takes 1.5 amperes, the armature current is 21.5 − 1.5 or 20 amperes.

In the normal operation of a shunt motor, the resistance of the field circuit is constant. Since this circuit is connected directly across constant-pressure mains, the field current is constant and the motor operates automatically with **nearly constant flux** regardless of the change in armature current, or load on the machine.

The field winding of the series motor, like the series generator, is connected in series with the armature as in Fig. 41*c*–10. Thus

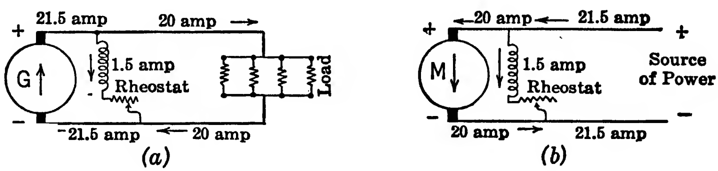



FIG. 11–11. (a) Currents in the armature and field of a shunt generator. (b) Currents in the armature and field of a shunt motor.

any change in armature current, due to variation in the load, changes the field current and also the flux.

The arrangement of the field windings of the compound motor is similar to that of the compound generator, Fig. 41*d*–10, the connection being either long or short shunt. The motor operates under the combined flux of the two fields; the shunt-field flux which is nearly constant and independent of the load; and the series-field flux which varies with the load.

**9. Motor Action in a Generator — Why it Requires Power to Drive an Electric Generator.** The fact that there is a force on a wire in a magnetic field, when it carries current, explains why it takes power to drive a generator when it is delivering current; and why it **does not take power** when the generator is not delivering current, even though a voltage is generated.

If the generator of Fig. 12–11 is driven mechanically in a clockwise direction, an **electromotive force** will be induced in the conductors in the direction indicated. At no load, practically no current flows in the armature, and the prime mover has to overcome only the friction and other losses in the armature core. When the generator supplies a load, **current** flows in the armature. The direction of this current in the conductors is the same as that of the induced electromotive force, or in the direction indicated in the figure. These conductors are now carrying current in a magnetic field; and applying the **left-hand rule,** we see that there

is a force acting on these conductors, retarding their motion. This force tends to make the armature revolve in the opposite direction. Hence, the prime mover must supply an additional force to overcome this retarding action.

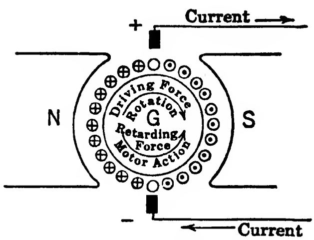



FIG. 12–11. Motor action in a generator. Shows direction of induced voltage and current in the conductors for clockwise rotation; also retarding action against rotation due to current in the conductors.

In other words, neglecting the losses, the torque developed in a motor, by a given armature current, is exactly equal to the torque which must be supplied by the driving engine when the machine is driven as a generator to supply the same armature current.

**Example 5.** Let us assume the following data concerning the motor shown in Figs. 8–11 and 9–11, and compute the power necessary to drive it as a generator delivering a certain current. If we disregard the losses, the mechanical power put into it should equal the electrical power delivered by it.

Let us assume that each circle, Figs. 8–11 and 9–11, represents a bundle of 9 conductors.

Total conductors then $= 9 \times 32 = 288$.
Diameter of the armature = 12 inches.
Length of each conductor = 10 inches.
Axial length of pole face = 10 inches.
Fraction of armature covered by poles $= \frac{7}{8}$.

$$\text{Peripheral length of pole face} = \frac{7}{8}\,\frac{(12 \times 3.14)}{4} = 8.25 \text{ inches.}$$

Average flux density in the air gap under pole faces = 50,000 lines per square inch.

Let us find how fast we must drive the armature to generate 120 volts.

Area of pole face $= 10 \times 8.25 = 82.5$ square inches.
Flux per pole $= 50{,}000 \times 82.5 = 4{,}130{,}000$ lines.
Conductors per armature path $= 8 \times 9 = 72$.

$$\text{Voltage} = \frac{\text{lines cut per second}}{10^8} \times \text{number of conductors in series.}$$

$$\text{Therefore, } 120 = \frac{4{,}130{,}000 \times 4 \times 72 \times N}{10^8 \times 60}$$

where $N$ = revolutions per minute.

$$N = \frac{120 \times 10^8 \times 60}{72 \times 4{,}130{,}000 \times 4} = 605 \text{ rpm.}$$

Let us drive this machine 605 rpm in a counter-clockwise direction and deliver 100 amperes which would flow in the direction marked in Figs. 8–11 and 9–11. Assuming no loss, we would deliver 100 × 120 or 12,000 watts.

But there would be $\frac{100}{4}$ or 25 amperes flowing in each conductor in the direction marked, which would set up a force in the opposite direction to the motion.

On each conductor under the poles there would be a force equal to

$$F = \frac{25 \times 50{,}000 \times 10 \times 8.85}{10^8} \text{ or } 1.107 \text{ pounds.}$$

Since $\frac{7}{8}$ of the conductors are active (that is, are under the pole faces and cutting flux at any one time), the force on $\frac{7}{8}$ of 288 conductors = $\frac{7}{8} \times 288 \times 1.107 = 279$ pounds.

$$\text{The work per minute} = 279 \times \frac{12 \times 3.14}{12} \times 605$$

$$= 530{,}000 \text{ foot-pounds per minute.}$$

$$\text{Horsepower} = \frac{530{,}000}{33{,}000} = 16.05.$$

$$\text{Watts} = 16.05 \times 746 = 12{,}000 \text{ watts.}$$

In this method of calculating by neglecting the losses, we see that it takes 12,000 watts motor power to generate 12,000 watts generator power.

**Prob. 15–11.** At what speed would the motor of Fig. 8–11 have to run to develop 30 horsepower, if a current of 140 amperes was supplied to the armature? Use other data from Example 5, and neglect losses.

**Prob. 16–11.** What voltage would have to be applied to the motor of Prob. 15–11? Assume no losses.

**Prob. 17–11.** If the field of the motor of Prob. 15–11 is weakened so that the flux density in the air gap is only 40,000 lines per square inch, at what speed would it run to supply 30 horsepower when taking a current of 140 amperes?

**Prob. 18–11.** What voltage would have to be applied to the motor of Prob. 17–11?

**10. Counter Electromotive Force.** In the previous article, it was shown that there is a **motor effect in a generator** when it is carrying a load, because the current in the armature conductors produces a retarding effect, or force opposing the motion of the armature.

Similarly, there is a **generator effect in a motor** known as the **counter or back electromotive force.** When a voltage from an

outside source is impressed upon a motor armature, a current flows in the conductors in the direction of this impressed voltage. This develops a torque to produce rotation. In the motor of Fig. 13–11, the direction of the impressed voltage and resulting current is in a direction to produce clockwise rotation (apply **left**-hand rule). Now if conductors are rotated in a magnetic field, an electromotive force will be induced in these conductors. It does not matter whether this rotation is obtained through a driving engine or any other source. In the case of the motor, the rotation is produced by the current in the conductors. Thus in the machine of Fig. 13–11, a voltage will be induced in the conductors in a direction in accordance with Fleming's **right**-hand rule for a generator. But the generator in Fig. 12–11, with the polarity of the fields the same as in Fig. 13–11 and rotating in the **same** direction, induces a voltage in the conductors in the direction indicated. This is **opposite** to that of the current in Fig. 13–11. Therefore, there is a voltage induced in the conductors of the motor of Fig. 13–11 opposite in direction to the current. This induced voltage, which always opposes the flow of current in the armature, is called the **counter electromotive force.** It is so called because it is in the opposite direction to the voltage which is impressed across the armature, and which causes the machine to run as a motor.

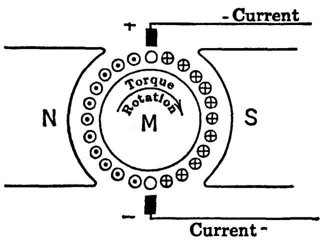



FIG. 13–11. Generator action in a motor. Shows direction of impressed voltage and current in the conductors of a motor for same direction of rotation and same polarity of field poles as in Fig. 12–11.

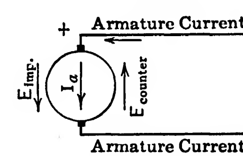



FIG. 14–11. Arrows represent the relative directions of impressed voltage, armature current and counter, or induced, voltage in a motor.

Thus there are two voltages acting in opposite directions in the windings of a motor when it is running — the voltage impressed on the armature from an outside source, $E$ (imp), and the induced or counter electromotive force, $E_c$, set up by the rotation. The relative direction of these two voltages and the armature current is shown by the arrows in Fig. 14–11.

In order that current may flow in the armature in the proper direction to give motor action and cause the armature to rotate, the impressed voltage must be **greater** than the counter or induced voltage by an amount necessary to overcome the armature resistance. Therefore, the difference between the impressed voltage and counter voltage must be equal to the armature $IR$ drop. $E_c$ must always be less than $E$ (imp) in any d-c motor. If $E_c$ were equal to $E$ (imp), no voltage would be available to force current through the resistance of the armature. If $E_c$ were greater than $E$ (imp), the armature current would flow in the direction of $E_c$ (induced voltage) and the action would be that of a generator.

The relation between $E$ (imp), $E$ (counter) and armature $IR$ drop in any d-c motor is expressed by the following equation which must always be fulfilled under any condition of operation.

$$E \text{ (imp)} = E \text{ (counter)} + I_a R_a \qquad (7)$$

where $I_a$ = armature current
and $R_a$ = armature resistance.

This is a very important equation — so important that it is often called the fundamental equation of the motor.

As the resistance of any armature is comparatively small, the armature $IR$ drop is small. Therefore, $E_c$ approaches closely to the value of $E$ (imp) even at full load on the motor.

As $E_c$ is an induced voltage it may be expressed by Equation (1), Chapter X, as,

$$E \text{ (counter)} = E \text{ (generated)} = \frac{\Phi PZN}{10^8 \times 60 \times a}.$$

Since the quantities $P$, $Z$ and $a$ are constant for any given motor, the equation can be expressed as,

$$E_c = K'\Phi N \qquad (8)$$

Note that induced voltage, or $E_c$, for any given motor, **varies directly with flux and with speed.**

**Armature current.** When a motor is running, the **current** in the armature **is not determined** by the resistance of the armature and the impressed voltage. If the fundamental equation be transposed, it can be written

$$I_a = \frac{E \text{ (imp)} - E_c}{R_a}. \qquad (9)$$

If the motor is operated from a constant voltage circuit, both $R_a$ and $E$ (imp) are constant; thus the armature current is determined by the value of $E_c$, and therefore, by the speed and the flux. It can be shown that since armature resistance is very small compared to the other quantities in the equation above, a very small change in the value of $E_c$ makes a very considerable change in the value of $I_a$.

**Prob. 19–11.** The impressed voltage on a motor armature is 125 volts; resistance of the armature is 0.18 ohm. What is the counter electromotive force, if the armature draws 15 amperes?

**Prob. 20–11.** At no load, the armature current of the motor in Prob. 19–11 is 3.5 amperes. What is the counter electromotive force?

**Prob. 21–11.** The impressed voltage across a motor armature is 115 volts; the counter electromotive force is 112 volts and the current is 16 amperes. What is the resistance of the armature?

**Prob. 22–11.** It is desired to find the counter voltage in a motor armature when the impressed voltage is 112 volts and the current is 16 amperes. Armature resistance = 0.24 ohm.

**Prob. 23–11.** If the speed of the motor in Prob. 22–11 were reduced one half and the field remained the same, what armature current would flow?

**Prob. 24–11.** Suppose the armature in Prob. 22–11 were stopped altogether; what current would flow?

**Prob. 25–11.** The resistance of a motor armature is 0.16 ohm. If it has a counter electromotive force of 113.4 volts when running on a 115-volt circuit, what will be the armature current?

**Prob. 26–11.** If the counter electromotive force of the motor in Prob. 25–11 drops to 112.4 volts, what will be the armature current?

**Prob. 27–11.** (*a*) What is the percentage decrease in counter voltage in Prob. 26–11 from that in Prob. 25–11? (*b*) What is the percentage increase in current?

**Prob. 28–11.** A shunt-motor armature has 224 conductors, 2 paths and a resistance of 0.05 ohm. Motor has 4 poles and a flux of 2,000,000 lines per pole. If the armature draws 150 amperes and runs at 1500 rpm, what is the impressed voltage?

**11. Motor Action under Changing Load — Constant Flux.** Assume that a motor is operating from constant-voltage mains, and that the magnetic flux in the air gap is also constant.

When the motor is running without load, the retarding force tending to stop the motor is small, and the only torque required is that necessary to overcome friction and other losses in the arma-

ture. Hence from the equation, $T = K\Phi I_a$, it is obvious that only a small value of armature current is required to keep the armature revolving at a fixed speed.

If now an external mechanical load is applied to the pulley, the motor, if it continues to operate, must develop an increased torque sufficient to balance the increased retarding torque of the load. Since the flux is constant, the armature current must be increased. But $I_a = E$ (imp) $- E_c$; so the armature current can be increased only by decreasing the counter voltage, $E_c$, since the impressed voltage, $E$ (imp), is constant. But $E_c = K'\Phi N$, and $\Phi$ is constant; therefore, the speed, $N$, must decrease. Thus, when a load is applied to the motor, the increased retarding force is greater than the no-load torque, and the motor speed decreases, thereby decreasing $E$ (counter) and increasing the armature current. The speed continues to drop until the armature current reaches such a value that the developed torque just balances the retarding force of the load. The motor again operates at a constant or fixed speed somewhat below the no-load speed.

If now the load on the motor is **decreased**, there is more torque than is required. The motor speeds up until $E$ (counter) is increased sufficiently to reduce the armature current, $I_a$, and consequently the torque, until it just balances the retarding torque of the decreased load. Again the motor operates at a fixed speed somewhat higher than before.

In the above discussion, the flux was assumed to be constant. This generally is not the actual case, due to armature reaction. However, it may be said that in any d-c motor, the change in armature current required for varying loads is obtained by a change in $E$ (counter), which in turn is obtained by a change in speed.

**12. Motor Equations.** For convenience, the four important motor equations already discussed are repeated. The relations which govern the action of a d-c motor are more readily visualized if they are listed together.

$$\text{The Torque Equation, } T = K\Phi I_a \tag{5}$$

$$\text{The Voltage Equation, } E \text{ (imp)} = E_c + I_a R_a \tag{7}$$

$$\text{The Counter-emf Equation, } E_c = K'\Phi N \tag{8}$$

$$\text{The Current Equation, } I_a = \frac{E \text{ (imp)} - E_c}{R_a} \tag{9}$$

**13. Armature Reaction.** Armature reaction occurs in a d-c motor in a manner similar to that in a generator. See Art. 22, Chapter X. However, for the same polarity of field poles and the same direction of rotation, the direction of the current in the armature conductors of a motor is reversed from that in a generator. Note Figs. 11–11, 12–11 and 13–11.

Figure 15*a*–11 shows the direction of the field mmf, or flux, in a two-pole motor when there is no current in the armature. The brushes are shown on the mechanical, or geometrical, neutral axis, *xy*. The field flux is in a direction parallel to the axis of the poles, and at 90° to the axis, *xy*.

Figure 15*b*–11 shows only the magnetomotive-force, or flux, set up by the armature conductors when they are carrying current in a direction to produce clockwise rotation. As in the case of the generator, the conductors act like the turns of a coil and set up a magnetomotive-force and a flux, shown by the arrows **upward** in the figure, and at 90° to the field flux. There are also in the motor, as well as in the generator, two mmf's or fluxes acting on the magnetic circuit at the same time, when the machine carries a load. However, the armature mmf in the motor is reversed from that in the generator, as a comparison of Figs. 15*b*–11 and 43*c*–10 shows.

These two fields set up a resultant field displaced from the axis of the field poles, as shown by the vectors in Fig. 15*c*–11. The resultant flux in the air gaps has now shifted, becoming more dense in the leading pole tips and less dense in the trailing pole tips, as indicated in Fig. 15*d*–11. Note that the neutral axis under load has now shifted backward to the position *mn*, in the **opposite** direction to the rotation, and the brushes correspondingly must be shifted back to the axis *mn*, or given a "backward lead," as it is called.

However, the proper position of the brushes is not on the axis *mn*, as just stated. Due to the necessity of counteracting the electromotive force of self induction in the short-circuited coils by the reversal of current, the brushes for sparkless commutation must be shifted **ahead** to the axis *bb′* as shown in Fig. 15*d*–11.

Note that in both the motor and the generator, it is necessary to shift the brushes **ahead** of the load neutral to counteract this emf of self induction.

Due to the brush shift, the armature mmf is now parallel to the axis *bb′* and the relation between the field and armature mmf's is

shown by the vectors in Fig. 15e–11. It is to be noted that the resultant mmf or field strength is less than before.

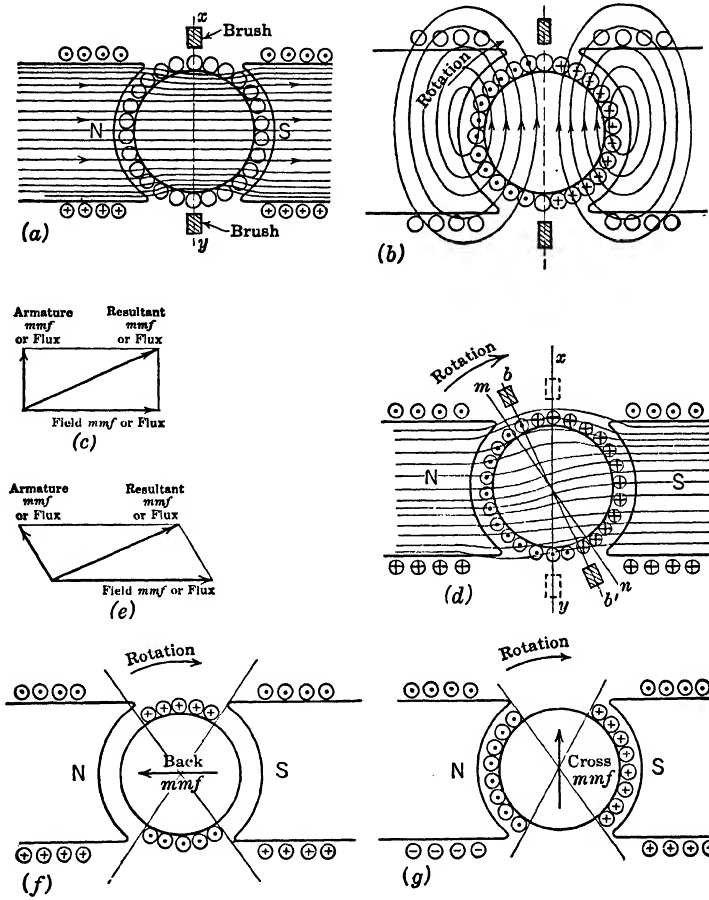



FIG. 15–11. Armature reaction in a motor. (a) Flux, or mmf, due to field ampere-turns only. (b) Flux, or mmf, due to armature ampere-turns only. (c) Resultant of both armature and field mmfs. (d) Distortion of flux in the air gap due to current in both armature and field windings. (e) Resultant of armature and field mmfs after brushes have been shifted. (f) Demagnetizing armature ampere-turns. (g) Cross-magnetizing armature ampere-turns.

As in the generator, the armature ampere-turns can be separated into two parts: those within the double angle of brush shift, which comprise the demagnetizing, or back ampere-turns, Fig.

15*f*–11; and those outside the double angle, which comprise the distorting or cross-magnetizing ampere-turns, Fig. 15*g*–11.

The effect of armature reaction, then, in both the motor and the generator, is to distort and weaken the flux in the air gap. The weakening of the flux, or the field, in a generator affects the voltage, while the weakening of the field in a motor affects the speed.

**14. Direction of Rotation.** The direction of rotation of a motor is in the direction of the torque and is found by Fleming's **left-hand rule** as already described in Art. 2.

To **change** the direction of rotation of a motor, it is necessary to **change the relation** between the direction of the field flux and the direction of armature current. If the direction of **both** the field and armature currents is reversed, direction of the rotation is not changed.

To reverse the direction of rotation of a motor, then, **either** the direction of the armature current **or** the direction of the field current must be reversed, **not both.** This is done by reversing either the connections to the field circuit, or the connections to the armature.

**15. Performance Curves.** In the choice of a motor for any specified service, it is necessary to know the speed at which the motor will operate for any load put upon it. This performance can best be shown by curves, called speed-load curves, or speed-load characteristics. Since the current taken by a motor varies with the load, these curves are generally plotted between speed as ordinates, and load current as abscissae. The curves may also be plotted between speed and horsepower output, or even between the speed and the torque developed.

Note the difference between the performance curves of the generator and the motor. The generator is driven by mechanical means at a constant speed, and the curve, plotted between terminal volts and load current, shows the **variation in voltage** with **change in load**; while the motor is driven electrically from a constant-voltage circuit, and the curve, plotted between speed and load current, shows the **variation in speed** with **change in load.**

These curves, in addition to others which will be discussed later, give us the data we must have about the performance of the machine.

**16. The Shunt Motor. Running Performance. Speed-Load Characteristics. Regulation.** As the load on a shunt motor is varied, the speed will decrease slightly as the load is increased. The general shape of the speed-load curve of the shunt motor is

shown in Fig. 16–11. Note that the speed-load curve of the shunt motor is very similar to the load-voltage characteristic of the shunt generator. However, the speed of the shunt motor does not fall off quite as rapidly as does the voltage of the shunt generator. The decrease in speed between no load and full load on the motor is relatively small. For this reason the shunt motor is called a "constant-speed" machine.

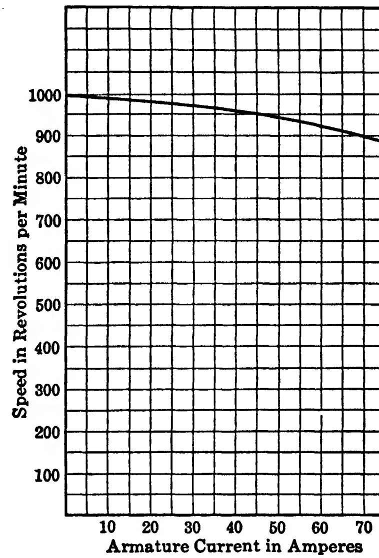



FIG. 16–11. The speed-load curve of a shunt motor.

The motor is connected, as in Fig. 11b–11, across a constant-voltage line of proper value. The field winding is also connected across this constant-voltage line, and the resistance in the field rheostat left at the no-load setting. The field current, therefore, is constant and the flux should be constant. Assume for the present that this is so.

It has already been shown in Art. 11 that an increased mechanical load on the motor requires that it develop an increased torque, which with constant flux, can be obtained only by an increased armature current ($T = K\Phi I_a$). In turn, increased current with constant impressed voltage can be obtained only by a decrease in counter voltage $\left(I_a = \dfrac{E_{\text{imp}} - E_c}{Ra}\right)$. But with constant flux, the counter voltage, $E_c$, can be decreased only by a decrease in speed ($E_c = K'\Phi N$). Therefore, under the assumption of constant flux, the change in the speed of the motor is directly proportional to the change in counter voltage. Consider the example below.

**Example 6.** A 5-hp, 115-volt shunt motor runs at 1030 rpm at no load from 115-volt mains. Resistance of field circuit, including rheostat, is 69.7 ohms. Resistance of armature, including brushes, is 0.18 ohm. The motor takes 4.85 amperes at no load. The full-load current is 41.65 amperes. What is the full-load speed?

**Solution.**

$$\text{Field current, } I_f, = \frac{115}{69.7} = 1.65 \text{ amperes.}$$

At no load, armature current, $I_a$, = 4.85 − 1.65 = 3.2 amperes.
At full load, armature current, $I_a$, = 41.65 − 1.65 = 40 amperes.
$E$ (counter) at no load = 115 − 0.18 × 3.2 = 114.4 volts.
$E$ (counter) at full load = 115 − 0.18 × 40 = 107.8 volts.
Speed at no load = 1030 rpm.

$$\text{Then } \frac{\text{speed at full load}}{\text{speed at no load}} = \frac{E_c \text{ at full load}}{E_c \text{ at no load}}.$$

$$\text{Speed at full load} = 1030 \times \frac{107.8}{114.4} = 970 \text{ rpm.}$$

But it was shown in Art. 13 that when the brushes of a motor are set for sparkless commutation, they are given a "backward-lead," and the armature current sets up demagnetizing ampere-turns. Therefore, although the field current in the shunt motor is constant, the flux is **not** constant, as was assumed above, but decreases slightly as the armature current increases. And the full-load speed determined in Example 6 above is not correct, but is somewhat lower than the true value.

The speed of the motor at any load can be more accurately determined from the magnetization curve, the turns in the field winding, the brush lead and the number of armature conductors. Here, again, use is made of the fact that the counter or back voltage of a motor depends upon the strength of the field, the flux and the speed of the armature.

The magnetization curve is obtained by driving the motor as a generator at any convenient speed (see Art. 19, Chapter X), and plotting it as shown in Fig. 17–11. The generated voltage is then divided by the speed (at which magnetization curve was taken), and a second scale is added to the ordinates. This scale shows the relation of the volts generated **at a speed of one revolution per minute** to the field strength. It is necessary now merely to find the counter voltage of the motor at any particular load (armature) current, and divide it by the volts per revolution per minute induced by the field strength at this load. The result will be the speed at which the motor must run to generate the computed counter voltage.

**Example 7.** The curve of Fig. 17–11 is the magnetization curve of the 5-hp, 115-volt motor of Example 6 taken at 1000 rpm. The second scale of ordinates indicates volts per rpm, and the abscissae are field ampere-turns per pair of poles. This motor has 240 conductors on the 2-path armature, 1600 field turns per pair of poles and the brushes are set with 4° backward lead. Other data, as given in Example 6,

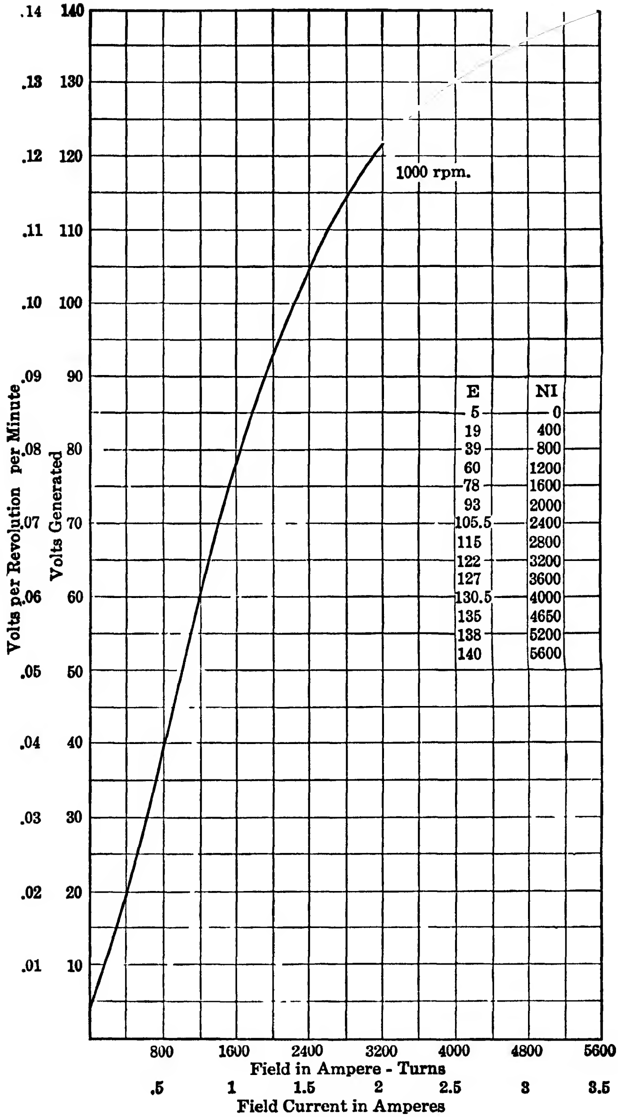



FIG. 17–11. The magnetization curve of the motor of Example 7.

are: armature resistance including brushes = 0.18 ohm; resistance of field circuit = 69.7 ohms; full-load current = 41.65 amperes; impressed voltage = 115 volts.

At what speed will the motor run at full load?

**Solution.**

Field current, $I_f$, as before $= \dfrac{115}{69.7} = 1.65$ amperes.

Full-load armature current, $I_a$, $= 41.65 - 1.65 = 40$ amperes.

Back turns $= \dfrac{2 \times 4 \times 240}{360} = \dfrac{16}{3}$.

Back ampere-turns $= \dfrac{16}{3} \times \dfrac{40}{2} = 107$.

Resulting field ampere-turns $= (1.65 \times 1600) - 107 = 2533$.

From curve, Fig. 17–11, 2533 ampere-turns generate 0.1087 volts per rpm.

$IR$ drop in armature at full load $= 40 \times 0.18 = 7.2$ volts.

Counter voltage $= 115 - 7.2 = 107.8$ volts.

If 1 rpm generates 0.1087 volts, it will take as many rpm to generate 107.8 volts as 0.1087 is contained in 107.8

or $\dfrac{107.8}{0.1087} = 991$ rpm, speed at full load.

Note that the full-load speed in Example 7 is considerably higher than that in Example 6. The difference in speed, 991 − 970 or 21 rpm, is due to the effect of the armature back ampere-turns. It is thus seen that armature reaction tends to raise the speed, and keep the full-load speed in a shunt motor more nearly equal to the no-load speed.

**Regulation.** The change in speed of a motor between no load and full load, calculated as a percentage of the full-load speed, is called the **percentage regulation.** Note that in a motor, regulation refers to a change in **speed;** while in a generator, it refers to a change in **voltage.**

**Example 8.** The no-load speed of the motor in Example 7 is 1030 rpm. The full-load speed is 991 rpm.

The percentage regulation =

$$\frac{1030 - 991}{991} \times 100 = 3.94 \text{ per cent.}$$

**Note.** In solving the following problems, the saturation curve of Fig. 17–11 should be plotted on fine coordinate paper — preferably divided to $\frac{1}{20}$ of an inch.

**Prob. 29–11.** At what speed will the motor of Example 7 run when the load on it is such that the armature is carrying 20 amperes?

**Prob. 30–11.** If the motor in Example 7 is overloaded so that the armature takes 60 amperes, at what speed will it run?

**Prob. 31–11.** If the no-load armature current of the motor in Example 7 is 3.2 amperes, determine the no-load speed from the magnetization curve of Fig. 17–11.

**Prob. 32–11.** Determine the speed of the motor in Example 7 at armature currents of 10 and 30 amperes; and from the results of this and the previous problems, plot the speed-load curve of the motor.

**Prob. 33–11.** If 12 ohms is added to the resistance of the field circuit of the motor of Example 7, at what speed will it run at full load; and at no load? Motor is driven from a 115-volt circuit and the no-load armature current is 3.2 amperes. What effect has weakening the field upon the speed of the motor?

**Prob. 34–11.** Calculate the regulation of the motor in Prob. 33–11, and explain why it differs as it does from that in Example 8.

**Prob. 35–11.** What resistance must be cut out of the field circuit in the motor of Example 7, so that the speed at full-load armature current will be 925 rpm?

**Prob. 36–11.** Calculate the no-load speed and the percentage regulation of the motor in Prob. 35–11. Armature $I$ at no load = 3.2 amperes. Explain why the regulation differs as it does from that in Example 8.

**Prob. 37–11.** If the motor in Example 7 is run on a 125-volt circuit, with the same resistance in the field circuit, what will be the full-load and no-load speeds and the percentage regulation? Assume the no load, $I_a$, is the same as in Example 7.

**17. Starting Resistance.** It has already been shown that when a motor is running, the armature current depends upon the difference between the impressed voltage and the counter emf. The counter emf depends upon the speed. Therefore when the motor is started from standstill, the counter voltage at the beginning is zero and the armature current depends upon the impressed voltage and armature resistance. Since armature resistance is very small, an excessive current would flow if full voltage were thrown on an armature which has not started to rotate.

**Example 9.** In the 5-hp motor of Example 6, the resistance of the armature is 0.18 ohm. If the armature were thrown directly across the 115-volt line while the armature was standing still, the armature current would be $\frac{115}{0.18}$ or 640 amperes. Full-load armature current in this motor is 40 amperes. The starting current then is 16 times full-load value, if the motor is started by throwing full voltage directly across the armature.

In order to avoid this excessive current on starting, a **starting resistance** is introduced into the armature circuit, which limits the current to a safe value. This starting resistance is generally of such a value that it permits about 1.5 times full-load current to flow. As soon as the armature begins to rotate, the counter electromotive force rises and the armature current decreases. In order to maintain enough current to accelerate the motor to full speed, the resistance is gradually cut out as the speed increases, until full-line voltage is across the armature. It is safe to have full voltage across the armature as soon as the speed is sufficient to set up a high counter voltage.

Figure 18–11 is a simple diagram of the starting resistance used with a shunt motor. When the line switch is closed and arm *C* swung to point *A*, the shunt field is put in circuit and the machine has full field flux. Then when arm *C* is swung to point *B*, the starting resistance *SR* is connected in series with the armature across the line, and the armature begins to accelerate. As the armature speed rises, the arm *C* is gradually swung to point *P*, cutting out the resistance *SR* and putting the armature directly across the line. This is the running position.

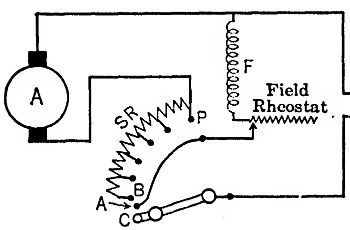



Fig. 18–11. A shunt motor with a starting resistance.

The arm *C* should not be left on any intermediate point more than a few seconds, as the resistance is not designed to carry the load current for a longer period.

Small motors of around ½-hp capacity may, without danger, be put directly across the line without a starting resistance. Armature resistance of these small motors is comparatively high, and they accelerate ra[illegible]dly.

[illegible]es are called " starters " or " starting boxes." [illegible]ed either by hand or automatically by means [illegible]tors." The two most common types of hand [illegible] field release " and the " no-voltage release," [illegible]d later.

[illegible]e. When a shunt motor is started, the field [illegible]rectly across the constant-voltage supply, as [illegible]nd the field flux is at its normal or full value,

except for the slight effect of armature reaction. The torque then is directly proportional to the armature current ($T = K\Phi I_a$). This relation is shown by the curve $A$ in Fig. 19-11, which is practically a straight line. The armature current on starting is determined by the starting resistance only, $SB$, Fig. 18-11, and not by the speed or counter voltage. Since the field flux is the same whether the armature is turning or not, the starting torque is directly proportional to the starting current. If this starting current is the normal full-load value, the starting torque is the same as the running torque with full-load current. Thus a shunt motor, which starts, for instance, with 20 per cent above full-load armature current, starts with 20 per cent above full-load torque. The starting torque of a shunt motor then can be made equal to any desired value, within the limit of safe starting current, by properly designing the value of the starting resistance.

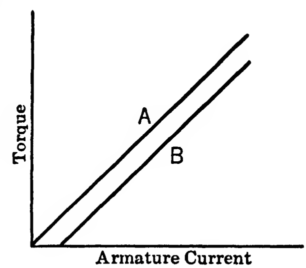


FIG. 19-11. Torque curves of the shunt motor.

The **running torque at the pulley** of the motor, for the same armature current, would be somewhat less than the developed starting torque, due to the losses in the machine. It would, however, be practically a straight line, as shown by curve $B$, Fig. 19-11.

**19. Speed Control of the Shunt Motor.** The speed of a shunt motor can be **controlled** by changing either the magnetic flux of the field or the voltage impressed upon the armature.

(1) An **increase** or **decrease** of the field flux will respectively **decrease** or **increase** the speed (the impressed voltage remaining constant).

(2) An **increase** or **decrease** of the voltage impressed across the armature will respectively **increase** or **decrease** the speed (the voltage across the field circuit remaining constant).

**20. Speed Control by Adjusting the Magnetic Field.** The usual method of speed adjustment, or speed control, of a shunt motor is by varying the resistance of a field rheostat placed in the field circuit, as indicated in Fig. 18-11.

Consider a shunt motor, operating from constant-voltage supply mains, under load conditions requiring a fixed armature current. Then from the equation, $E$ (imp) $= E_c + I_aR_a$, it is seen that the

counter voltage must be constant. If it is desired to increase the motor speed, the resistance in the field rheostat is increased, thereby decreasing the flux, $\Phi$, and the counter voltage ($E_c = K'\Phi N$). This momentarily increases the armature current, $\left(I_a = \dfrac{E\ (\text{imp}) - E_c}{R_a}\right)$. It has been shown that a small decrease in counter voltage results in a correspondingly large increase in current. Thus the torque is increased; and the motor automatically speeds up until the counter voltage increases, and the armature current decreases to their former values. The motor now operates at a higher fixed speed than before, the increase in speed depending upon how much the flux has been decreased.

If it is desired to decrease the motor speed, resistance is cut out of the rheostat, thereby increasing the field current and the flux. The counter voltage is momentarily increased and the armature current falls, thereby decreasing the torque; the motor slows down until the counter voltage decreases and the armature current rises to their former values. The motor now operates at a lower fixed speed than before.

The relations discussed above can be readily seen by expressing them as an equation, as follows:

If equation (7) is transposed, it may be written as,

$$E_c = E\ (\text{imp}) - I_aR_a. \tag{10}$$

Substituting the value of $E_c$ from equation (8) in equation (10), it may be written,

$$K'\Phi N = E\ (\text{imp}) - I_aR_a$$

and

$$N = \frac{E\ (\text{imp}) - I_aR_a}{K'\Phi}. \tag{11}$$

It is thus apparent that with constant impressed voltage, the speed of the motor varies **inversely** with the field flux.

On first thought, it may be argued that with the same armature current and a reduced flux, the torque is necessarily reduced; therefore the motor cannot carry the same load. However, the horsepower of a motor depends upon **both** speed and torque, as shown by equation (6) below,

$$\text{Horsepower} = \frac{2\pi TN}{33{,}000}.$$

With constant load (horsepower output) and the motor running at a higher speed, less torque is actually needed, as shown in the example below.

**Example 10.** (*a*) In a motor running at 1000 rpm and developing a torque at the pulley of 120 pound-feet,

$$\text{the output} = \frac{2\,\pi \times 120 \times 1000}{33{,}000} = 22.9 \text{ hp.}$$

(*b*) If the load condition is such that the armature current ($I_a$) remains fixed and the flux is reduced 10 per cent, the counter voltage must remain the same and the speed must be 110 per cent of what it was before, or 1100 rpm.

Since the flux is decreased 10 per cent, the torque would now be $\frac{120}{1.10}$ or 109.1 pound-feet and

$$\text{the output} = \frac{2\,\pi \times 109.1 \times 1100}{33{,}000} = 22.9 \text{ hp, as before.}$$

When the speed of a shunt motor is adjusted by field control, the speed-load curve is about the same shape, and the regulation is not much different at different values of field current. In other words, the motor operates at almost constant speed both above and below normal field flux. However, the limits to the changes in speed which can be effected by this means are about 15 to 20 per cent.

(1) It is not feasible to increase the flux density in the magnetic circuit much beyond saturation (the knee of the curve).

(2) On the other hand, when the flux density is greatly reduced, armature reaction is proportionately larger, and it is difficult to obtain sparkless commutation under load. When a wide range of speeds is desired by means of field control, motors are generally equipped with commutating poles. Large motors may be equipped with both commutating and compensating windings. By this means, a speed variation of as great as 6 to 1 can be obtained.

**Example 11.** In order to increase the speed of the motor in Example 7, a resistance of 10 ohms is introduced into the field. Other data as in Example 7. The impressed voltage is 115 volts and no-load armature current is 3.2 amperes.

(*a*) What will be the no-load speed?

(*b*) What will be the full-load speed?

(*c*) How much has increased field resistance affected the speed regulation? Compare with Example 8.

**Solution.**

(*a*) No-load speed.

$$\text{Field current} = \frac{115}{69.7 + 10} = 1.44 \text{ amperes.}$$

Field ampere-turns = 1600 × 1.44 = 2304 ampere-turns per pair of poles.

$$\text{Back ampere-turns} = \frac{16}{3} \times \frac{3.2}{2} = 8.5 \text{ ampere-turns.}$$

Resulting ampere-turns per pair of poles = 2304 − 8.5 = 2296.

Volts per rpm from Fig. 17–11 = 0.102 volts/rpm.

$E$ (counter) = 115 − 3.2 × 0.18 = 114.4 volts.

$$\text{No-load speed} = \frac{114.4}{0.102} = 1124 \text{ rpm.}$$

(*b*) Full-load speed.

Back ampere-turns as in Example 7 = 107 ampere-turns.

Resulting ampere-turns = 2304 − 107 = 2197 ampere-turns.

Volts per rpm from Fig. 17–11 = 0.099 volts/rpm.

$E$ (counter) as in Example 7 = 107.8.

$$\text{Speed at full load} = \frac{107.8}{0.099} = 1080 \text{ rpm.}$$

$$\text{Percentage regulation } \frac{1124 - 1080}{1080} \times 100 = 4.07 \text{ per cent.}$$

(*c*) 4.07 − 3.94 = 0.13 of 1 per cent.

**Prob. 38–11.** (*a*) What resistance must be inserted in the field circuit of the motor in Example 7 in order to produce a full-load speed of 1200 rpm when run on a 115-volt circuit? (*b*) What is the speed regulation of the motor under these conditions? Assume armature takes 3.2 amperes at no load.

**Prob. 39–11.** (*a*) What resistance must be cut out of the field circuit of the motor in Example 7 in order that it may run at full load at 900 rpm on a 115-volt line? (*b*) What is the speed regulation? Armature current at no load is 3.2 amperes.

**Prob. 40–11.** What resistance must be inserted or cut out of the field circuit of the motor in Example 7 if it is run with full load at a speed of 1100 rpm on a 125-volt circuit?

**21. Speed Control by Changing the Voltage Across the Armature.** If a resistance or rheostat be placed in series with the armature, the voltage on the armature and therefore the speed can be changed by varying the value of this resistance. The connec-

tions are shown in Fig. 20–11. Since the field is connected directly across the supply mains, the flux is constant (neglecting armature reaction). If the load requirement is such as to demand a constant-armature current, and if the flux is assumed constant, the speed varies directly with the voltage across the armature. An **increase** in the resistance lowers the voltage on the armature and **decreases** the speed.

Should the **starting** resistance be designed to carry the load current of the armature continuously, the handle of the starting box may be held on an intermediate point and the motor will run at a lower speed.

This method of speed control has the disadvantage of wasting energy. Furthermore, even a slight change in load on the motor changes the drop in the resistance and results in a very material change in speed; thus this method produces poor speed regulation.

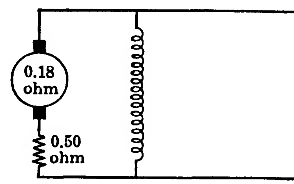



FIG. 20–11. The 0.50 ohm resistance is used to cut down, or decrease the voltage across the armature.

**Example 12.** It is desired to lower the speed of the motor in Example 7 by inserting a resistance of 0.50 ohm in series with the armature, as in Fig. 20–11. What will be the speed at no load and at full load? Assume no-load armature current is 3.2 amperes. What will be the regulation of the motor under these conditions?

**Solution.**

No-load speed.

Back ampere-turns as in Example 11 are 8.5.

Resulting ampere-turns $= 2640 - 8.5 = 2632$.

Volts per rpm from Fig. 17–11 $= 0.111$.

Voltage applied to the armature $= 115 - (3.2 \times 0.5) = 113.4$ volts.

Counter voltage $= 113.4 - (3.2 \times 0.18) = 112.8$ volts.

$$\text{Speed at no load} = \frac{112.8}{0.111} = 1011 \text{ rpm.}$$

This is $1030 - 1011$ or 19 rpm less than in Example 7.

Full-load speed.

Back ampere-turns as in Example 7 are 107.

Resulting ampere-turns $= 2640 - 107 = 2533$.

Volts per rpm from Fig. 17–11 $= 0.1087$.

Volts applied to the armature $= 115 - (40 \times 0.5) = 95$ volts.

Counter voltage $= 95 - (40 \times 0.18) = 87.8$ volts.

$$\text{Speed at full load} = \frac{87.8}{0.1087} = 808 \text{ rpm.}$$

This is $991 - 808$ or 83 rpm less than in Example 7.

The regulation in this case is

$$\frac{1011 - 808}{808} \times 100 = 25.1 \text{ per cent.}$$

The regulation of the motor in the above example is too high for most purposes, and the motor no longer behaves as a constant-speed machine. Without this extra resistance in the armature circuit, the regulation of the motor in Example 7 was only 3.94 per cent (Example 8).

Note also that the power taken by the motor at full load in the example above is $95 \times (40 + 1.65)$ or 3957 watts, while the power consumed by the resistance is $40^2 \times 0.5$ or 800 watts. This is $\frac{800}{3957} \times 100$ or approximately 20 per cent of the power supplied to the motor.

**Prob. 41–11.** What will be the speed of the motor in Example 12 when it is drawing 20 amperes armature current?

**Prob. 42–11.** What would be the speed regulation of the motor in Example 12, if instead of the 0.50-ohm resistance in the armature circuit, a voltage of 95 volts is applied direct to the armature? Voltage across the field remains 115 volts.

**Prob. 43–11.** How much resistance must be placed in the armature circuit of the motor of Example 7 to obtain a full-load speed of 750 rpm on a 115-volt circuit?

**Prob. 44–11.** What voltage must be applied to the armature of the motor of Example 12 (with the 0.50-ohm resistance removed) in order to raise the full-load speed to 1100 rpm? Voltage across the field remains at 115 volts.

**Prob. 45–11.** A resistance of 0.02 ohm is added to the armature circuit and a resistance of 8 ohms to the field circuit of the motor in Example 7. If the motor operates on a 120-volt line, what will be the full-load speed?

**22. Comparison of the Two Methods of Speed Control.** Speed adjustment of the shunt motor by field control is accomplished by means of a small, low-capacity rheostat. Fairly wide ranges in speed may be obtained with good speed regulation, and the efficiency of this method is high. **By this method, speed may be increased,** but cannot be decreased below that obtained when all the resistance in the rheostat is cut out.

**Speed adjustment by armature control can be used only to lower the speed.** This method is wasteful and inefficient, and the speed regulation of the motor is poor.

To avoid the poor regulation and the power wasted by armature control, a multivoltage system of power supply may be used, in which the motor armature may be connected as desired to two or three different feeders having different voltages. Or an auxiliary motor-generator set may be employed, and the voltage on the motor armature varied by adjusting the voltage of the generator, as in the Ward-Leonard System. In these systems, in addition to varying the voltage on the motor armature, the motor field also may be controlled by rheostat; and wide ranges in speed with good regulation and high efficiency may be obtained. However, these systems are very expensive.

**23. "No-Field" Release. Three-Point Starting Box.** What would happen, while the motor was running, if the current in the field circuit of a shunt motor **were reduced to zero,** and only the residual magnetism were left in field poles?

It has been shown that the armature increases in speed as the field decreases in strength $\left(N = \frac{E\ (\text{imp}) - I_a R_a}{\Phi K'}\right)$. We would thus expect the armature speed to become excessive if the field became very weak. And this is just what happens to a shunt motor when the field is broken, and the load is light. It immediately races and destroys the armature windings, the centrifugal force pulling them away from the core. Note from the magnetization curve in Fig. 17–11, that when the current in the field circuit is reduced to zero, the volts per revolution per minute are only 0.004. To generate a counter voltage of 114.4 volts at no load (Example 7), the speed must be $\frac{114.4}{0.004}$ or 28,600 rpm. Long before this speed is reached, the windings would be pulled out of the slots in the armature core.

Of course, if the machine is heavily loaded, it stops; and the greatly increased current in the armature causes it to burn out. This event of "no field" must be guarded against by some device which will automatically break the armature circuit as soon as the field circuit is broken and thus avoid the racing or burning out of the armature.

Figure 21–11 shows a device of this kind. The field current is led through a small electromagnet $M$ on the starting box. The movable arm $C$ has a soft iron keeper $K$ attached to it. When the arm has come into the running position and all the starting resistance $SR$ is cut out, the keeper $K$ comes into contact with the

electromagnet *M* which holds the arm in this position, acting against the tension in the spring *S*.

If anything happens to break the current in the field coil, the current in the electromagnet is also broken, and the movable arm

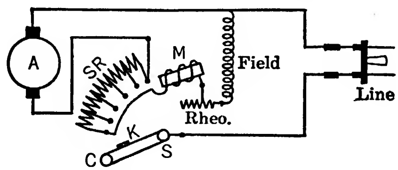



Fig. 21–11. A shunt motor connected to a starting box with no-field release coil, M.

is released and pulled away by the spring *S*. This action breaks the armature circuit and thus stops the motor.

A starting box equipped with a "no-field" release is called a "three-point box" because it has three studs from which leads are carried to the line and to the motor. Figures 22–11 and 23–11 are

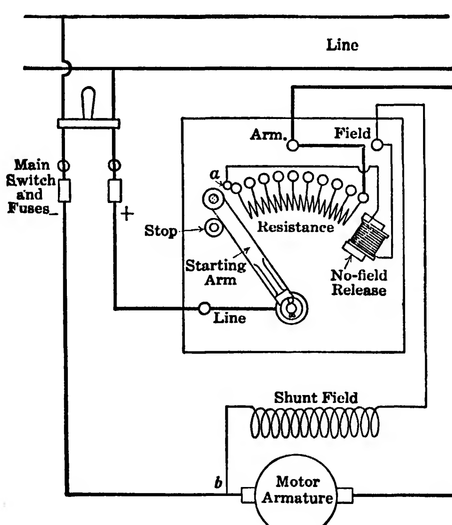



Fig. 22–11. A three-point starting box for a shunt motor.

diagrams of such boxes. Note that one lead goes from the box to the **field circuit** of the motor, another to the **armature** and the third to **one side** of the line. The other field and armature termi-

nals are **connected together**, and then connected to the **other side** of the line.

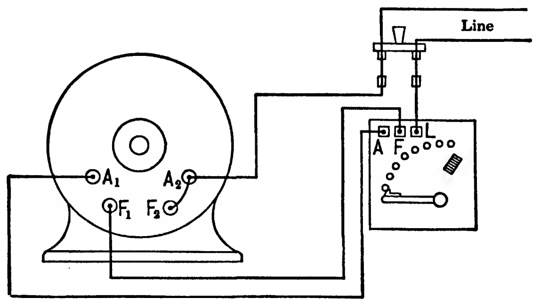



FIG. 23–11. A shunt motor connected to a three-point starting box.

**24. "No-Voltage" Release. Four-Point Starting Box.** Besides the danger that the field of a shunt motor may be opened, there is also the danger of the voltage going off the supply line, with the consequent slowing down and stopping of the motor. If the arm of the starting box remains in the running position after the voltage has gone off, and the motor slows down or stops, the motor is likely to burn out when the voltage is again thrown on,

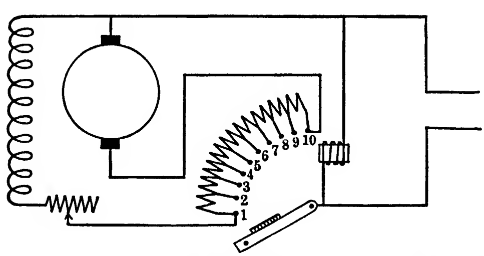



FIG. 24–11. A starting box with a no-voltage release.

since all the starting resistance is cut out with the arm in this position. To take care of this emergency, the small retaining magnet instead of being in series with the field is connected directly across the line. When the voltage on the line lowers to a dangerous extent, the current through the magnet decreases and the magnet is weakened. This releases the arm, which is pulled back by a spring, and cuts the motor from the line. This device is called a "**no-voltage release**" and the box is called a "Four-point Starting Box." Figure 11–1, page 10, shows one of these starting boxes. Figures 24–11 and 25–11 are diagrams showing the connec-

tions of a four-point box with the no-voltage release. Note that one lead from the box is brought out to the **armature,** another to the **field** and the other two to the **line.** If the motor does not start when the arm is thrown to the first point, transfer the lead from the point $x$ on Fig. 25–11 to the other side of the line.

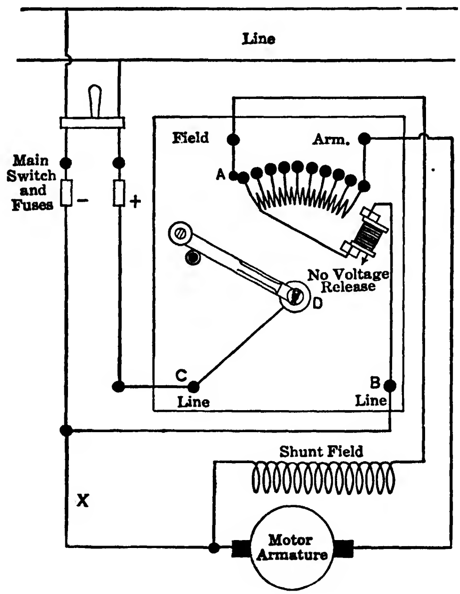



FIG. 25–11. The connections of a shunt motor to a four-point starting box.

Since the current in the retaining magnet is not affected by any changes in the resistance of the field circuit, the no-voltage type of release is generally preferred.

Many types of starters operate automatically by means of a push-button switch. The shunt field and the starting resistance are first thrown into circuit. Then the starting resistance is automatically cut out at the proper rate until full voltage is across the armature. These starters are somewhat complicated and beyond the scope of this text. For more complete information, the student is referred to the various Handbooks on Electrical Engineering.

**Example 13.** The motor of Example 7 is equipped with a three-point starting box similar to that shown in Figs. 21–11 and 22–11. This starter is designed so that on a 115-volt line the current taken by the armature when the arm is first thrown to each point is $1\frac{1}{2}$ times full-load armature current. The arm is left on each point, until due to the acceleration of the armature, the armature current drops to full-load

value, and then is thrown to the next point. The resistance of the "no-field-release" coil is 5 ohms. There are seven contact points on the starting resistance. The data for the motor in Example 7, as previously given, are

Armature resistance = 0.18 ohm;
Number of armature conductors = 240;
Full-load armature current = 40 amperes;
Angle of brush lead = 4°;
Rated voltage of the motor = 115 volts;
Resistance of shunt field, including rheostat = 69.7 ohms;
Number of shunt-field turns per pair of poles = 1600;
Number of armature paths = 2.

Figure 17–11 is the magnetization curve of this machine.

(*a*) What must be the value of the total starting resistance?

(*b*) What speed should the motor attain before the arm is shifted from point 1 to point 2?

(*c*) What must be the starting resistance from point 2 to point 6?

**Solution.**

(*a*) When the armature is not revolving, counter voltage is zero and the resistance of the armature circuit alone limits the armature current. In order to limit the armature current to 1.5 × 40 or 60 amperes:

Total resistance in armature circuit $= \frac{115}{60} = 1.917$ ohms,
Total starting box resistance $= 1.917 - 0.18 = 1.737$ ohms.

(*b*) Arm to remain on point 1 until armature current drops to 40 amperes.

$$\text{Current taken by the field} = \frac{115}{69.7 + 5} = 1.54 \text{ amperes.}$$

Field ampere-turns $= 1600 \times 1.54 = 2464$.
Back ampere-turns at full-load current (Example 7) $= 107$.
Resultant ampere-turns per pair of poles $= 2464 - 107 = 2357$.
Volts per rpm at this field (Fig. 17–11) $= 0.104$.
Voltage drop through armature and starting resistance $= 40 \times 1.917 = 76.68$ volts.
Armature counter voltage $= 115 - 76.78 = 38.32$ volts.

$$\text{Speed to produce 38.32 volts} = \frac{38.32}{0.104} = 368 \text{ rpm.}$$

The motor armature should therefore attain a speed of 368 rpm before the arm is shifted to point 2.

(*c*) The initial current at each point = 60 amperes. The motor is producing a counter voltage of 38.32 volts when the arm is shifted to point 2. The voltage drop through armature and remaining resistance (points 2 to 6) must be $115 - 38.32 = 76.68$ volts.

Total resistance now in armature circuit must equal $\frac{76.68}{60} = 1.276$ ohms.

Resistance of starter (points 2 to 6) $= 1.276 - 0.18 = 1.096$ ohms.

**Prob. 46–11.** (*a*) What speed should the motor in Example 13 attain before the starter arm is shifted to point 3? (*b*) What should be the resistance of the starter points 3 to 7?

**Prob. 47–11.** (*a*) Find the resistance in the starter in Example 13, points 1 to 2, 2 to 3, 3 to 4, 4 to 5 and 5 to 6, etc. (*b*) What speed should the motor attain at each point?

**Prob. 48–11.** A 4-point starting box similar to Figs. 24–11 and 25–11 is to be used with the motor of Example 7. The resistance of the "no-voltage release" coil is 920 ohms. Answer (*a*) and (*b*) of Prob. 47–11. Allow $1\frac{1}{2}$ full-load current for initial current on each point and full load as final current on each point.

**25. Over-load Release.** Many starting boxes are supplied with an **over-load** release as well as a "no-voltage" or "no-field" release. This arrangement prevents over-loading the motor. Figure 26–11 shows an over-load release applied to a four-point box. If the current taken by the motor reaches a value which is likely to harm the armature windings, the plunger $P$ is drawn up into the coil. This short-circuits the no-voltage release coil and allows the starter arm to be thrown to the "**off**" position by the spring. The position of the plunger $P$ can be adjusted so that it will operate at any desired current. The same arrangement can be applied to a three-point box.

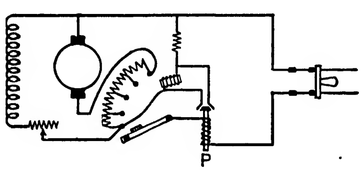



FIG. 26–11. A starting box with a no-voltage and an overload release connected to a shunt motor.

**26. Directions for Starting and Stopping a Shunt Motor.** Start slowly, moving the starter arm slowly until the motor comes up to speed. Do not leave it on any intermediate point for more than 3 or 4 seconds, or the starting resistance may be burned out.

Always stop the motor by pulling the line switch, allowing the starter arm to snap back when the electromagnet releases it. **Never pull** the starter arm back, as this causes bad arcing at the first point, which roughens the copper contacts.

**27. Summary of Shunt Motor Performance. Applications.** From the conclusions of the previous articles, it may be said that the shunt motor is practically a constant-speed machine. It may be made to operate at various speeds by field control, but maintains its constant-speed characteristics at any of the several speeds. Thus the speed can be fixed by the operator at any value between a maximum and a minimum; but when once set, will remain at practically a constant value. This fact makes the motor appli-

cable, for instance, for machine-tool drive, where the machine must be operated at a constant speed, but in which the character of the work in the machine determines the particular speed employed.

The starting torque of the shunt motor is fairly high, enabling it to start loads for which the torque required is not excessive.

**28. The Series Motor. Running Performance.** The conventional diagram of the series motor is shown in Fig. 27–11. The field winding of this motor, like that of the series generator, carries the armature current, and is composed of a relatively few turns of comparatively large wire. See Fig. 41c–10. Any change in armature current, due to variation in load, changes the value of the field current, and also of the field flux. Therefore, as the load on a series motor changes, the speed also changes; an increase in load causes a decided drop in speed, and a decrease in the load a large increase in speed. Because of the wide change in speed with change in load in the series motor, it is called a variable-speed machine.

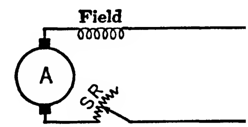


FIG. 27–11. Conventional diagram of a series motor with a starting resistance.

Consider a series motor operating from constant-voltage mains and carrying a mechanical load applied to the armature shaft. If this load be increased, the armature current must increase to develop additional torque to balance the increased retarding action of the load. But the armature current can only be increased by a decrease in counter voltage, for $I_a = \dfrac{E\text{ (imp)} - E_c}{R_a}$. Since the field current increases with the armature current, the flux also increases approximately with the armature current (to the saturation point). Therefore, since $E_c = K'\Phi N$, and $E_c$ must decrease while the flux, $\Phi$, increases, it follows that, as additional load is applied to the motor, the speed must decrease to a greater extent than that by which the flux is increased.

For instance, if it be assumed for the moment that the flux varies directly with the armature current, an increased load requiring a **doubled** armature current would also **double** the flux. But with increased load, $E$ (counter) must **decrease**; therefore the **speed must drop to less than half** its former value. Conversely, if the load on the motor be **decreased** until the required current is **halved**, the flux will be **halved**. And since $E$ (counter) must increase to reduce the armature current, and

must do so with **half** the flux, the **speed** must rise to **more than twice** its former value.

In the actual motor, the exact relations just cited in the preceding paragraph do not hold for heavy loads, because of the saturation of the magnetic circuit and the effect of armature reaction. They are approximately true for lighter loads, when the magnetic circuit is being worked on the straight part of the saturation curve.

The series motor, unlike the shunt motor, has no definite " no-load " speed. At no load, there is little or no opposing torque; and the speed tends to increase until the counter voltage is practically equal to the impressed voltage. But this high counter voltage decreases the current in the armature and field, and therefore the flux, to a very low value. Consequently, a counter voltage, practically equal to the impressed voltage, must be set up by a very weak field and high speed. Thus the motor " races "; and the centrifugal forces so set up may tear the armature conductors from the slots in the core and wreck the machine.

Therefore, a series motor is used only where the load is attached directly to the shaft, as for a fan or blower; or is connected through gearing, as in a railway car or an electric crane. The series motor should never be connected to its load by means of a belt. Very small series motors ($\frac{1}{10}$ horsepower and less) generally have friction and other losses and resistance drop, high enough at no load to keep the speed down to a safe limit.

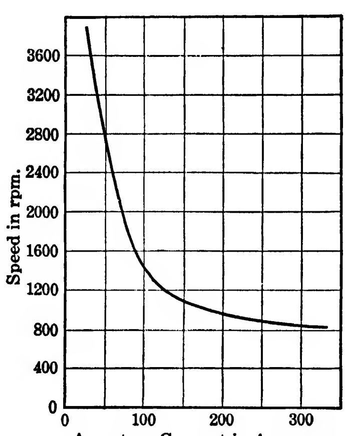



FIG. 28–11. The speed-load curve of a series motor.

**29. Speed-Load Characteristic of the Series Motor.** The relation between the speed and the load of a series motor is shown in Fig. 28–11. Note that the no-load speed is too high to be plotted; that the speed falls rapidly with the first increases in load; and then more gradually as full load and saturation of the magnetic circuit of the motor are approached.

The speed-load curve of a series motor may be obtained by applying a mechanical load which is gradually decreased from

at least full load to the smallest load that the motor will carry without a dangerous increase in speed. Readings of speed are taken for each decrement of load.

The speed-load curve may also be obtained from the magnetization curve of the machine, the field ampere-turns, brush shift, etc.

The magnetization curve is obtained by running the motor as a separately excited generator (at any convenient speed), as described in Chapter X.

**Example 14.** Figure 29–11 represents the magnetization curve for a certain series motor taken at 1200 rpm. The armature resistance including brush contacts is 0.048 ohm; the resistance of the series field winding is 0.012 ohm. There are 60 turns in the field per pair of poles and 200 conductors in the armature. The brushes are set with an 8° backward lead. Construct the speed-load characteristic up to 300 amperes armature current, on a 125-volt circuit. Two paths in armature.

At no load the back emf practically equals the impressed emf. The volts per revolution per minute at zero ampere-turns in the field from Fig. 29–11 = 0.005 volts/rpm.

$$\text{Speed to produce 125 volts} = \frac{125}{0.005} = 25{,}000 \text{ rpm.}$$

This cannot be represented on the plot (Fig. 28-11).

At 50 amperes:

$$\text{Back ampere-turns} = \left(\frac{8 \times 2}{360} \times 200\right)\left(\frac{50}{2}\right) = 222 \text{ ampere-turns.}$$

Field ampere-turns = 60 × 50 = 3000 ampere-turns.
Resultant field = 3000 − 222 = 2778 ampere-turns.

From Fig. 29–11 these will produce 0.045 volt per rpm.

Voltage $IR$ drop in series field and armature = (0.048 + 0.012) × 50 = 3 volts.

Back voltage generated = 125 − 3 = 122 volts.

At 0.045 volts per rpm,

$$\text{the speed} = \frac{122}{0.045} = 2710 \text{ rpm.}$$

At 100 amperes:

$$\text{Back ampere-turns} = \left(\frac{8 \times 2}{360} \times 200\right)\left(\frac{100}{2}\right) = 444 \text{ ampere-turns.}$$

Field ampere-turns = 60 × 100 = 6000 ampere-turns.
Resultant field = 6000 − 444 = 5556 ampere-turns.
Volts/rpm (Fig. 29–11) = 0.0833 volt/rpm.
Field and armature $IR$ drop = (0.048 + 0.012) × 100 = 6 volts.
Back voltage generated = 125 − 6 = 119 volts.

$$\text{Speed} = \frac{119}{0.0833} = 1450 \text{ rpm.}$$

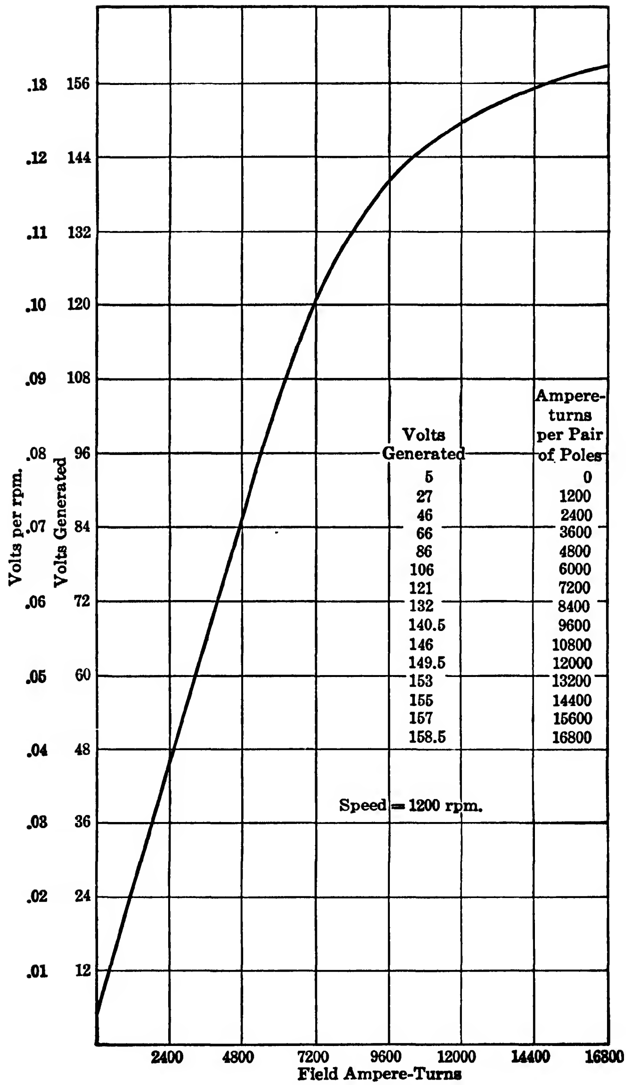



FIG. 29–11. The magnetization curve of the series motor of Example 14.

At 200 amperes:

Back ampere-turns = 888 ampere-turns.
Field ampere-turns = 12,000 ampere-turns.
Resultant field turns = 11,112 ampere-turns.
Volts/rpm (Fig. 29–11) = 0.122 volt/rpm.
Series field and armature *IR* drop = 12 volts.
Back voltage = 125 − 12 = 113 volts.

$$\text{Speed} = \frac{113}{0.122} = 928 \text{ rpm.}$$

At 300 amperes:

Back ampere-turns in armature = 1332 ampere-turns.
Field ampere-turns = 60 × 300 = 18,000 ampere-turns.
Resultant field = 18,000 − 1332 = 16,668 ampere-turns.
Volt/rpm (Fig. 29–11) = 0.1325 volt/rpm.
Armature and field *IR* drop = 18 volts.
Back voltage = 125 − 18 = 107 volts.

$$\text{Speed} = \frac{107}{0.1325} = 808 \text{ rpm.}$$

**Prob. 49–11.** Compute the speed at which the motor in Example 14 will run when carrying (*a*) 75 amperes armature current; (*b*) 150 amperes; (*c*) 400 amperes.

**Prob. 50–11.** Plot the speed-load characteristic for the series generator of Prob. 39–10 when run as a series motor on a 600-volt line. Carry the computation from 2 amperes to 30 amperes armature current.

**Prob. 51–11.** Replot the speed-load curve of the series motor of Prob. 50–11 run on a 500-volt line. What effect does the line voltage have on the speed of a series motor?

**30. Starting Resistance.** To start a series motor, a starting box or starting resistance, *SR*, Fig. 27–11, must be used to limit the current as in the case of the shunt motor; since at standstill, there is no counter voltage generated in the armature. Starting boxes for series motors may be equipped with either an "over-load" release which cuts the motor out of circuit, if the load on it draws too great a current; or with an "under-load" release to cut the motor out of circuit, when a decrease in load allows the motor to approach a dangerous overspeed. The series motor is started in exactly the same way as a shunt motor. Small series motors below 5 horsepower may be started without injury by throwing them directly across the line.

**Prob. 52–11.** Draw a wiring diagram of a series motor equipped with a starting box, having a "no-load" and an "over-load" release. Show inside wiring of box and motor.

**31. Starting Torque.** It has been shown that the starting torque of the shunt motor is practically proportional to the armature current, since the flux is almost constant at all loads; and therefore we may write

Torque varies with $I_a$.

In the series motor, the field current and therefore the flux, $\Phi$, changes with the armature current, until the magnetic circuit approaches saturation. Since the torque, $T$, equals $K\Phi I_a$, and $\Phi$ varies nearly with $I_a$, a doubling of the armature current, due to increase in load, nearly doubles the flux, $\Phi$; we may write, for small loads

Torque varies nearly with $I_a^2$.

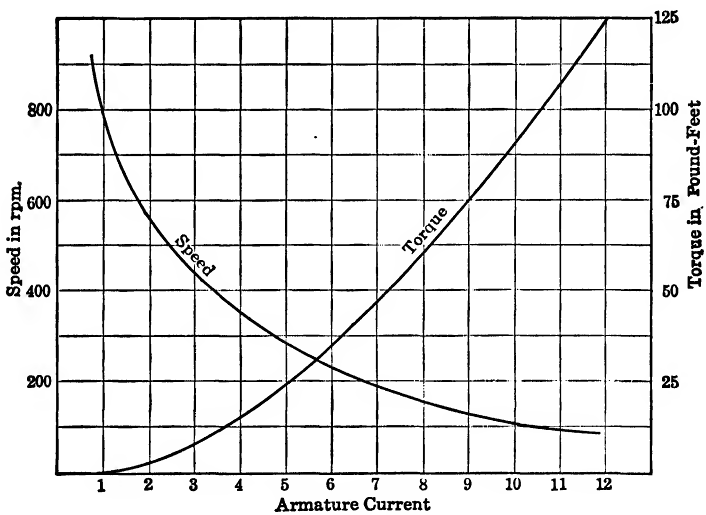



FIG. 30-11. Curves showing the relation of the speed and torque of a series motor to the armature current.

Therefore, the starting torque of the series motor increases more rapidly with increase in current than does the shunt motor. The general shape of the torque curve of a series motor is shown in Fig. 30-11. Note that at first the torque increases more rapidly than the current and then becomes almost a straight line, due to saturation of the magnetic circuit at the higher values of current.

In any motor taking full-load current at starting, the developed torque is the same as when the motor is carrying full-load current

at normal speed. Therefore, torque relations under load can be considered in discussing torque at starting, as is done below.

Two motors, one shunt and the other series, with the same horsepower rating, the same full-load speed and the same efficiency, will develop the same torque; since for each motor all the terms in the equation

$$\text{Horsepower} = \frac{2\,\pi T \text{ rpm}}{33{,}000}$$

are the same.

However, if an increased load be put upon each motor, calling for the **same increase in torque,** the series motor will develop this increased torque with **less** current than the shunt motor. Therefore, the required horsepower output of the series motor, with this increased torque, will be **less** than that of the shunt motor. This is so because the series motor has dropped considerably in speed, while the shunt motor must develop the increased torque at practically the same speed.

Note the following examples:

**Example 15.** Consider two motors, one a shunt and the other a series, having the same rating as to voltage, horsepower output and speed at full load — hence the same full-load torque. The full-load armature current of 50 amperes will develop 100 per cent torque in each motor.

Assume that

(*a*) Torque varies with $I_a$ in the shunt motor.

(*b*) Torque varies with $I_a^2$ in the series motor.

What current will be necessary to start each motor with 150 per cent of full-load torque?

**Solution.**

Current necessary to develop 150 per cent torque in the shunt motor under assumption (*a*) above

$$\frac{I_a}{50} = \frac{150}{100} \quad \text{or} \quad I_a = \frac{50 \times 150}{100} = 75 \text{ amperes,}$$

or 1.5 times full-load current.

Current necessary to develop 150 per cent torque in the series motor under assumption (*b*) above

$$\frac{I_a^2}{50^2} = \frac{150}{100} \quad \text{or} \quad I^2 = \frac{50^2 \times 150}{100} = 3750$$

and $$I_a = \sqrt{3750} = 61.2 \text{ amperes.}$$

This is only $\dfrac{61.2}{50}$ or 1.22 times full-load current.

**Example 16.** Since starting boxes are generally designed to limit the armature starting current to 1.5 times the normal full-load value, what will be the percentage starting torque with this current in each of the motors of Example 15, under the assumptions (*a*) and (*b*) of that example?

**Solution.**

150 per cent of full-load current = 75 amperes.

For the shunt motor,

$$\frac{\text{per cent } T}{100 \text{ per cent}} = \frac{75}{50}, \quad \text{or} \quad T = 150 \text{ per cent of full-load torque.}$$

For the series motor,

$$\frac{\text{per cent } T}{100 \text{ per cent}} = \frac{75^2}{50^2}, \quad \text{or} \quad T = \frac{100 \times 75^2}{50^2} = 225 \text{ per cent of full-load torque.}$$

The results obtained in the examples above do not strictly apply to the commercial motor, because of the saturation of the magnetic circuit at the higher values of current, as has been stated. They do show, however, that the series motor requires less current for a given starting torque, or develops greater starting torque for a given current than the shunt motor.

In the actual series motor, the current required for 150 per cent torque is probably from 1.25 to 1.35 times full-load current. Herein lies the advantage of the series motor for starting heavy loads for which constant speed is not required.

**32. Speed-Torque Characteristics.** It has been shown that the torque of a series motor varies greatly with the speed, being greatest when the speed is slowest. So when a heavy load is thrown on a series motor, the speed decreases and both the armature current and flux increase, with the result of a great gain in torque; and the motor can carry the load with a minimum of additional power output.

On the other hand, when a heavy load is thrown on the shunt motor, the flux and the speed are approximately constant and the required torque must be obtained entirely by increased current in the armature. Since the speed is constant, the additional power required will increase with the torque. Consequently, the shunt motor will have to be much larger than the series motor to take care of the same load. The shunt motor cannot slow down and carry the heavy load at a slower rate, but must carry it at about the speed it maintains on light load. Figures 30–11 and 31–11 show the relation of speed to torque in a series and in a shunt motor.

When the speed is not an important factor, but high torque is required, a series motor is preferable, as has been said. Consider, for instance, the most common use of a series motor, electric railway work. If a shunt motor were used the car would have to climb the hills at about the same rate it travels along a level, because the shunt motor is very nearly a constant-speed machine. This would require a much larger motor than the series motor used. The series motor, however, decreases its speed and thus drives the car up the grade at a slower speed, with largely increased

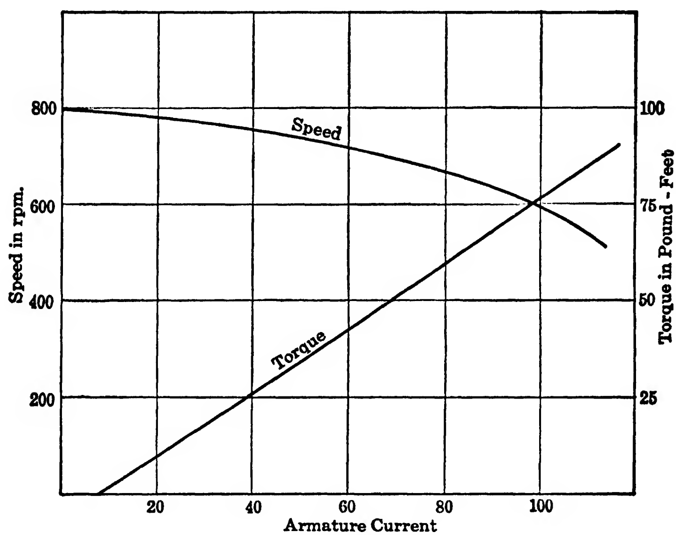



FIG. 31–11. Curves showing the relation of the speed and torque of a shunt motor to the armature current.

torque to overcome the large opposing torque offered by the grade. There are also other important advantages in using series motors to drive electric cars. See "Elements of Electricity," by W. H. Timbie.

On the other hand, a series motor would not be suitable to run a lathe in a machine shop. Every time the load changed, the speed would change; and the most efficient cutting speed could not be maintained. Moreover, when the load was taken off, the motor would "race."

**33. Series-Parallel Speed Control for Electric Cars.** Most electric cars have at least two motors. The controller shown in

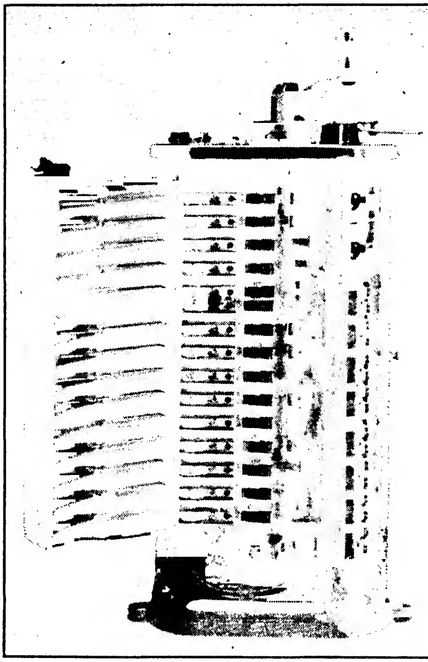


FIG. 32–11. A controller for an electric car. *General Electric Co.*

Fig. 32–11 is typical of that in many cars and is operated by the motorman as follows:

When the controller handle is advanced to the first notch, it places the two motors *A* and *B*, Fig. 33–11, in series with each other and in series with the starting resistance *SR*. As the handle is advanced, it gradually cuts out the starting resistance. When the resistance is all cut out, the controller is said to be on the first running point; and the car reaches a speed of 10 or 12 miles an hour on the level. Then the next notch puts the two motors in parallel with each other and again in series with the resistance *SR*, as in Fig. 34–11. To reach the full or second running position, the resistance is again cut out by advancing the handle.

The scheme of putting the two motors in series at the start allows the car to be started on half the current it would take to start with them in parallel. This arrangement also accelerates the car more evenly and wastes less power. It really makes one motor act as a starting resistance for the other, at the same time helping it to supply the tractive effort.

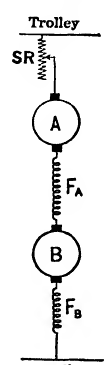



FIG. 33–11. The connections of the two motors as the car starts.

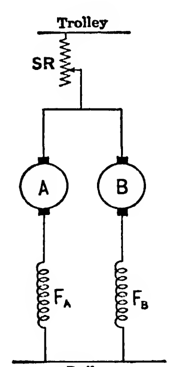



FIG. 34–11. The connections of the two motors when the car is at full speed.

**34. The Compound Motor. Running Performance. Speed-Torque Characteristics.** In the compound motor, the shunt and series-field windings are generally so connected that the ampere-turns of the two windings **aid each other,** as shown in Fig. 35*a*–11; and the machine is called a "**cumulative**" compound motor. If the two field windings are so connected that the ampere-turns of the series winding **oppose** those of the shunt winding, as shown in Fig. 35*b*–11, the machine is called a "**differential**" compound motor.

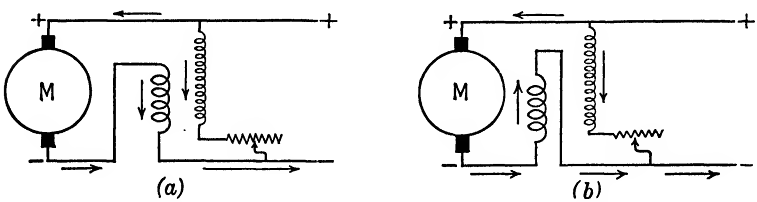



FIG. 35–11. Conventional diagram of the compound motor; (a) cumulative; (b) differential.

The shunt-field ampere-turns at full load are generally considerably greater than those of the series winding.

In the **cumulative** motor, the flux increases with load, due to the series field; and therefore the speed drops off more rapidly than does the shunt motor. The compound motor has a greater starting torque than the shunt motor. The greater the proportion of series ampere-turns to shunt ampere-turns, the more nearly does its performance approach that of the series motor, in both decrease in speed and increase in torque with increased load. At no load, the flux in the magnetic circuit is practically that of the shunt field alone; and therefore the motor has a definite no-load speed and does not "race" as does the series motor.

Since the flux increases with increase in load, the torque increases in greater proportion than does the armature current. So the cumulative compound motor requires less current than the shunt motor for a given increase in torque, though the difference is not so marked as in the series motor.

The cumulative motor is used where large starting torque is required and considerable variation in speed is not objectionable; and where the load may be thrown off with safety. Direct-current rolling-mill motors, and motors for certain types of hoists, are generally of this type.

In some cases, the motor is started as a compound motor to

obtain the larger starting torque; and after normal speed is reached, the series-field winding is automatically short-circuited and the machine operates as a shunt motor with its closer speed regulation.

In the **differential** motor, the flux decreases with increase in load, due to the opposing action of the series field. Thus the speed does not drop off with increase in load, even as much as does the shunt motor. Its regulation is closer; and it may, with proper proportion of series-field turns, operate at the same speed at both no load and full load. In fact, if the number of series-field turns is sufficiently increased, it is possible to cause the speed to rise with increase in load.

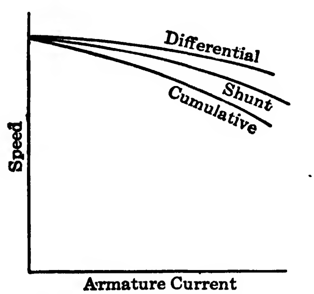



FIG. 36–11. Speed-load characteristics of the shunt, cumulative and differential compound motor.

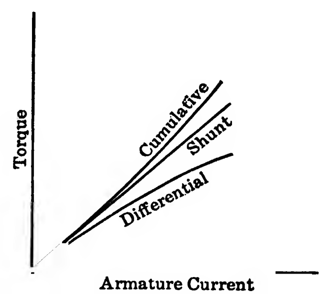



FIG. 37–11. Torque-load characteristics of the shunt, cumulative and differential compound motor.

Since the flux is reduced with increase in armature and series-field currents, the torque increases in smaller proportion to the armature current. So the differential motor requires **more** current than the shunt motor for a given increase in torque.

The speed of the differential motor tends to become unstable at heavy loads; for if it is sufficiently loaded, the increased series-field ampere-turns may approach the value of the shunt-field ampere-turns. This greatly reduces the flux and the counter voltage and further increases the armature current. For, since the torque does not increase in proportion to the armature current, the motor cannot " race " under its load even with decreased flux; and a heavy over-load current flows through the armature and series field. This may reverse the polarity of the field poles and the direction of rotation.

The differential motor, then, has exaggerated shunt motor characteristics; the starting torque is comparatively low — lower

than the shunt motor; the speed regulation is exceedingly good — better even than that of the shunt motor; but it may reverse its direction of rotation under heavy load. Thus the only characteristic to recommend it is the fact that it can be made to operate at constant speed, which makes it desirable for certain applications. However, because of its other undesirable features, it is not extensively used.

A comparison of the speed-load and torque-load curves of the shunt, the cumulative and the differential compound motor is shown in Figs. 36–11 and 37–11.

Speed control of the compound motor is obtained, as in the shunt motor, by adjusting the resistance in a rheostat connected in series with the shunt-field winding.

**Prob. 53–11.** Plot the speed-load characteristic of the compound generator of Prob. 52–10 to 60 amperes armature current, when run as a long-shunt cumulative-compound motor on a 220-volt line. Use 5 series turns per pair of poles. Armature takes 5 amperes at no load. Resistance of shunt-field circuit is 240 ohms. Brushes are set with 3° backward lead. Resistance of series-field winding is 0.04 ohm.

**Prob. 54–11.** Plot the speed-load characteristic of the compound generator of Prob. 52–10 to 60 amperes armature current, when run as a long-shunt differential-compound motor. Resistance of shunt-field circuit, no-load armature current, brush lead and other data same as in Prob. 53–11.

**Prob. 55–11.** How many series turns per pair of poles must be added to the shunt motor of Examples 6 and 7 in order that it may run at the same speed at full load as at no load (40 amperes armature current) on a 115-volt line? Allow 0.02 ohm for resistance of series winding.

**Prob. 56–11.** At what speed will the compound motor of Prob. 55–11 run at half load (20 amperes armature current)?

**Prob. 57–11.** If the series turns on the motor of Prob. 55–11 are reversed, what will be the full-load speed and the regulation? Compare the results with those in Examples 7 and 8.

**35. Commutating Poles in Motors.** Most d-c motors are equipped with commutating or interpoles. Their use was discussed with reference to the generator in Art. 39, Chapter X. It was there shown that the function of commutating poles is to set up the proper commutating flux for the short-circuited armature coils, while the flux in the air gaps is shifted, due to heavy currents in the armature. Commutating poles serve the same purpose in the motor; and in addition, allow the motor to be operated through a wider range in speed by means of field rheostat control.

To obtain considerably increased speed, the field is very materially weakened. Even at moderate loads on the motor, the armature ampere-turns are thus relatively increased with respect to the field ampere-turns, which have been decreased, and the flux distortion in the air gaps is increased. Accordingly the flux in the commutating zone is changed; and sparking at the brushes occurs. By use of commutating poles, the necessary commutating flux is maintained, even at very small values of field flux; and sparkless commutation at greatly increased speeds is obtained. As was mentioned in Art. 39, Chapter X, speed ranges of 5 to 1 may be obtained in variable-speed motors equipped with interpoles. Large variable-speed motors may, in addition, be equipped with a compensating winding, as discussed in Art. 40, Chapter X.

However, a low-speed motor weighs more than a high-speed motor of the same horsepower and voltage rating. In fact, the weight can be said to vary inversely with the speed. Therefore, in variable-speed motors designed for wide ranges in speed, the motor must be large enough to carry its rated load at the lowest speed; and would thus weigh considerably more than a motor of the same rating designed to run only at the higher speed.

The construction of interpoles in motors is practically the same as that in generators. However, for the same polarity of main poles and same direction of rotation, the direction of the armature current in a motor is reversed from that in a generator. Therefore, the polarity of the commutating poles in a motor is reversed to set up a reversed commutating flux; so the sequence of the commutating poles in a motor is opposite to that in a generator. Figure 38–11 shows a two-pole motor equipped with commutating poles. Note that these are reversed from those shown for the generator in Fig. 67–10 for same polarity of main field and same direction of rotation.

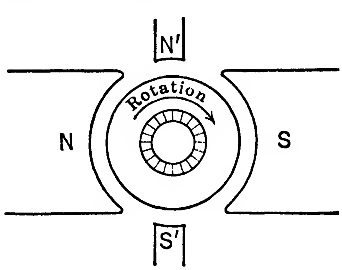



FIG. 38–11. Sequence of commutating poles with respect to main poles in a motor.

Where motors are equipped with compensating windings, the mmf, or direction of current, in these windings is reversed from that in generators, since the armature mmf is reversed, as explained above. Compensating windings in motors, as in generators, counteract or eliminate the effect of the cross ampere-turns.

## SUMMARY OF CHAPTER XI

**FORCE ON WIRE IN A MAGNETIC FIELD.** When a wire carrying a current lies in a magnetic field at right angles to the lines, there is a force on the wire due to the reaction between the circular field about the wire and the field in which the wire lies. The amount of this force in pounds can be calculated from the equation:

$$F = \frac{8.85\ BlI}{10^8}$$

**TORQUE.** The torque developed in the armature of a motor is the turning effort exerted on the shaft, and is proportional to the above force on all the active conductors multiplied by the radius of the armature. The torque in pound-feet for one conductor is computed from the following equation:

$$T = Fr = r\left(\frac{8.85\ BlI}{10^8}\right)$$

where I = current in armature conductor in amperes;
B = flux density of the field in lines per square inch perpendicular to the wire;
l = length of active conductor in inches;
r = radius of armature in feet;
T = torque in pound-feet.

**TORQUE IN ANY GIVEN MOTOR** can be expressed as:

$$T = K\Phi I_a$$

where $\Phi$ = flux per pole; $I_a$ = total armature current;
and K = a constant which depends upon the radius of the armature; area of poles; length of conductors; etc. Has a different value for different motors.

**THE HORSEPOWER OF A MOTOR** depends upon both the torque and the speed, and can be computed from the following equation:

$$\text{Horsepower} = \frac{2\pi TN}{33{,}000}$$

where T = torque in pound-feet;
N = revolutions per minute.

**MOTOR EFFECT IN GENERATORS.** This torque developed in the armature of a motor is the opposing torque which the driving engine has to overcome, when the machine is running as a generator with the same field strength, and delivering the same armature current.

**GENERATOR EFFECT IN MOTORS. COUNTER EMF.** The armature conductors of a motor, due to their rotation, cut lines of force and generate an emf, opposite in direction to the impressed emf. As in the generator, the emf may be computed from the equation:

$$E\ (\text{gen}) = E\ (\text{counter}) = \frac{\Phi PZN}{10^8 \times 60 \times a}$$

**COUNTER EMF FOR ANY GIVEN MOTOR** can be expressed by the equation: E (counter) $= K'\Phi N$, where $K'$ is a constant for each particular motor and depends upon the number of poles; number of conductors; number of armature paths, etc. Counter emf is directly proportional to flux and speed. It is always LESS than the impressed voltage by the amount of the armature resistance drop, ($I_aR_a$). The relation is expressed by the following equation, which must always be fulfilled in any d-c motor:

$$E\ (\text{imp.}) = E\ (\text{counter}) + I_aR_a$$

**THE CURRENT IN A MOTOR ARMATURE** is, therefore, under a pressure equal to the difference between the impressed voltage and the counter emf. The value of the current is expressed by the equation:

$$I_a = \frac{E\ (\text{imp}) - E_c}{R_a}$$

In any d-c motor, the CHANGE IN ARMATURE CURRENT REQUIRED FOR VARYING LOADS is obtained by a change in counter emf, which in turn is obtained by a change in speed, or a change in flux.

**ARMATURE REACTION. BACK AMPERE-TURNS** are produced by the armature current in a motor as in a generator; and, per pair of poles, are equal to the current in the armature winding, multiplied by the turns included within twice the angle of brush shift. The effect of the back ampere-turns is to weaken the field flux, thereby tending to raise the speed. CROSS AMPERE-TURNS are produced by the current in the remaining armature coils, outside the double angle of brush shift. Since the current in the armature of a motor is reversed from that of a generator, for the same direction of rotation and same polarity of field poles, the mmf of the Cross Ampere-Turns is also reversed and the field flux is shifted in the opposite direction to that in a generator.

**AXIS OF COMMUTATION. BACKWARD LEAD.** Because of the reversed direction of the Cross Ampere-Turns, the position of the brushes of a motor for sparkless commutation is on an axis, BACK of the neutral axis, instead of ahead, as in a generator.

**DIRECTION OF ROTATION.** Extend the thumb, forefinger and middle finger of the LEFT hand at right angles to one another. When the middle finger points in the direction of the current flow and the forefinger in the direction of the flux, the thumb will indicate the direction of rotation.

To REVERSE the direction of rotation of a motor, the direction of the current in EITHER the armature OR the field winding must be reversed — NOT BOTH.

**STARTING BOXES.** A motor must be started slowly by means of a starting box, which puts resistance in series with the armature to prevent an excessive armature current, until a counter electromotive force is set up by the rotation of the armature. Starting boxes are generally designed to limit the starting current to 150 per cent of normal. Types of starting boxes are generally the "no-field" release 3-point,

or the "no-voltage" release 4-point, operated either by hand or automatically. Automatic magnetic-contactor starters are also used.

TYPES OF MOTORS. D-C motors are divided into three general types.

(1) SHUNT: Field shunted around the armature;
- Nearly constant speed;
- Fair starting torque;
- Races when field circuit is broken, thus the advantage of "no-field" release;
- Speed controlled by varying field strength, by means of a rheostat connected in the field circuit.

(2) SERIES: Field winding in series with the armature;
- Speed varies — decreasing with added load;
- Large torque at low speeds, thus large starting torque;
- Races at "no-load"; thus the necessity of having the load permanently connected.

(3) COMPOUND: Equipped with both shunt and series-field windings.
- (a) CUMULATIVE; Series-field coils aiding shunt coils;
  - Series characteristics;
  - Will not race at "no-load";
  - Large torque at low speeds, hence high starting torque;
  - Speed decreases with added load.
- (b) DIFFERENTIAL; Series-field coils bucking shunt coils;
  - Exaggerated shunt characteristics;
  - Low starting torque;
  - Constant speed at all loads within limits;
  - May reverse direction of rotation under heavy over-loads.

THE SPEED OF A MOTOR is increased by weakening the field, as shown by the following equation:

$$N = \frac{E\,(\text{imp}) - I_a R_a}{K'\Phi}$$

TO DETERMINE THE SPEED at which a motor will run under given load, add or subtract the ampere-turns in the series field to those of the shunt field and subtract the armature back ampere-turns. On the magnetization curve (obtained by running the machine as a generator) find volts generated by these resulting ampere-turns. Divide volts generated by the speed at which magnetization curve was taken. Result is volts per revolution per minute for this field strength. Subtract armature IR drop from impressed voltage to obtain counter, or generated, voltage. Divide generated voltage by volts per revolution per minute to obtain speed in revolutions per minute.

COMMUTATING POLES. Most motors are equipped with commutating poles which furnish the necessary commutating flux for the short-circuited coils; and enable the motor to operate with sparkless commutation without shifting the brushes. By means of these poles, motors may be designed to operate through wide ranges in speed by shunt-field-rheostat control.

## PROBLEMS ON CHAPTER XI

**Prob. 58–11.** Draw the resultant field if poles and current are as marked in Fig. 39–11. Indicate direction of force on each side of loop and direction of tendency of rotation.

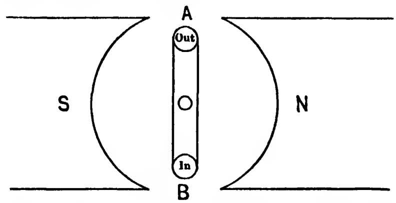



FIG. 39–11. A coil in a magnetic field.

**Prob. 59–11.** There are 420 conductors on a motor armature of which 90 per cent lie in a magnetic field of 46,000 lines per square inch. If each conductor is 9 inches long and carries a current of 25 amperes, how many pounds are acting on the armature?

**Prob. 60–11.** What torque would be developed in a 6-pole, 6-path motor taking 72 amperes, if there are 500 active conductors on the armature which lie in a magnetic field of 40,000 lines per square inch? Length of each conductor is 12 inches and the diameter of the armature is 18 inches.

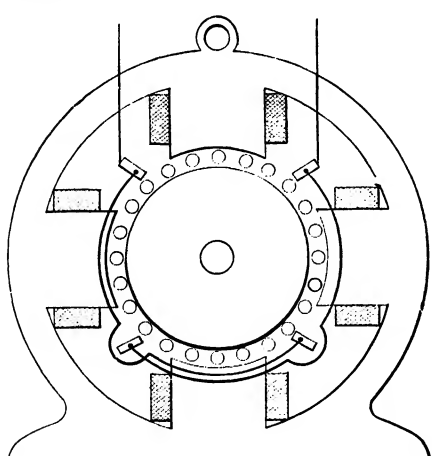


FIG. 40–11. A four-pole motor

**Prob. 61–11.** The voltage impressed upon a shunt motor is 230 volts. (*a*) What is the armature resistance, if the counter voltage is 224 volts and the armature current is 12 amperes? (*b*) What will be the armature current, if the counter voltage drops to 220 volts? (*c*) If the speed of the motor in (*a*) is 1200 rpm, what will be the speed under the conditions in (*b*), assuming the flux constant?

**Prob. 62–11.** There are 600 conductors on the armature of Fig. 40–11, 80 per cent of which lie in the magnetic field. Diameter of the armature is 12 inches. Width of pole face parallel to the shaft is 12 inches. Average intensity of the magnetic field is 35,000 lines per square inch. Armature takes a total current of 60 amperes. What is the force in pounds on each active conductor?

**Prob. 63–11.** (*a*) What is the total force on the active conductors in Prob. 62–11? (*b*) What torque in pound-feet is developed?

**Prob. 64–11.** (*a*) If speed of motor in Prob. 62–11 is 1200 rpm, what horsepower is transmitted to the pulley, allowing 10 per cent for constant losses? (*b*) What voltage must be applied to the armature, if the field takes 2 amperes and the efficiency is 85 per cent?

**Prob. 65–11.** In Prob. 62–11 the width of the pole face parallel to the shaft is 12 inches. Assume width in the other direction to be 7.5 inches. What is the counter emf at the speed given?

**Prob. 66–11.** A shunt motor takes a total current of 80 amperes from 115-volt mains. The resistance of the armature is 0.04 ohm. Resistance of the field is 60 ohms. (*a*) What current does each take? (*b*) What power is used in heating the field? the armature? (*c*) What is the counter emf?

**Prob. 67–11.** It requires 2500 feet of No. 26 (B & S Gauge) copper wire for the field coils of a 4-pole, 4-brush shunt motor. The armature is wound with 800 feet of No. 18 copper wire. If the motor takes a total current of 20 amperes at 120 volts: (*a*) What is its counter emf? (*b*) What power is lost in heating the armature and field?

**Prob. 68–11.** (*a*) What terminal voltage would the motor in Prob. 67–11 have, if run as a generator at the same speed, and 20 amperes were flowing through the armature? Field separately excited to same degree of magnetization and forward lead of brushes made equal to backward lead in motor. (*b*) What power would it be delivering to the line?

**Prob. 69–11.** A 15-kw, 115-volt shunt generator has a speed of 900 rpm at full load. Field resistance is 35 ohms; armature resistance is 0.06 ohm. (*a*) What are the armature and field currents at full load? (*b*) What is the emf of the generator?

**Prob. 70–11.** The generator in Prob. 69–11 is driven as a motor from 115-volt mains with full-load armature current and field excited to same degree of magnetization. Backward lead of brushes is equal to the forward lead, when machine is operated as a generator. At what speed does it run?

**Prob. 71–11.** A 2-pole, 230-volt shunt motor takes 50 amperes at full load. The field resistance is 100 ohms. Armature consists of 200 conductors and has a resistance of 0.36 ohm. Flux per pole is 4,000,000 lines. At what speed does it run?

**Prob. 72–11.** (*a*) If the motor in Prob. 71–11 took a current of 7 amperes at no load and all other data were the same (flux assumed constant), what would be the no-load speed? (*b*) What would be the speed regulation?

**Prob. 73–11.** If the motor in Prob. 71–11 were a 4-pole, 4-brush machine, and all other data were the same, what would be the full-load speed?

**Prob. 74–11.** If the motor in Prob. 71–11 were a 4-pole, 2-brush machine, and all other data were the same, what would be the full-load speed?

**Prob. 75–11.** A 550-volt series motor, having 4 poles and 2 brushes requires 180 kilowatts at full load. The field has a resistance of 0.024 ohm. There are 350 conductors on the armature, which has a resistance, including brush contact, of 0.092 ohm. Speed is 750 rpm. What is the flux per pole?

**Prob. 76–11.** The data for the magnetization curve of a 230-volt shunt motor taken at 1200 rpm are as follows:

| Field current in amperes | Generated voltage | Field current in amperes | Generated voltage |
|---|---|---|---|
| 0.0 | 14 | 3.50 | 231 |
| 0.5 | 56 | 4.00 | 243 |
| 1.00 | 100 | 4.50 | 252 |
| 1.50 | 140 | 5.00 | 260 |
| 2.00 | 173 | 5.50 | 268 |
| 2.50 | 199 | 6.00 | 274 |
| 3.00 | 217 | 6.50 | 280 |

No. of conductors = 250.
Shunt-field turns per pair of poles = 1000.
Shunt-field resistance = 43.1 ohms.
Armature resistance = 0.15 ohm.
Brush backward lead = 5°.
No. of armature paths = 2.
Full-load armature current = 50 amperes.
No-load armature current = 4 amperes.

A rheostat resistance of 23.6 ohms is inserted in the field.

Compute the speed regulation of this motor when operated on a 230-volt line.

**Prob. 77–11.** What would be the speed regulation of the motor of Prob. 76–11 if used on a 220-volt line without the field rheostat? No-load armature current is 4.5 amperes.

**Prob. 78–11.** Design a "no-field-release" starting box for the motor of Prob. 76–11 used without field rheostat. The resistance of the no-field-release coil is 8.00 ohms. The maximum current on each point of the starting resistance is to be $1\frac{1}{4}$ full-load armature current, and the starter arm is to be shifted when armature current becomes $\frac{3}{4}$ full-load value. Divide the starting resistance into 8 steps and state the resistance between steps 1 and 2; 2 and 3; 3 and 4; 4 and 5; 5 and 6; 6 and 7; 7 and 8.

State the no-load speed of the motor when the starting arm has remained on point 8 long enough for the current to become steady.

**Prob. 79–11.** If, by mistake, the starting box designed for the motor of Prob. 78–11 were used with the motor of Example 13 on 115 volts, compute

(*a*) The maximum armature current when starter arm came into contact with point 1.

(*b*) The maximum speed on first point, if the starter arm is allowed to remain there until the armature current becomes 3.2 amperes.

**Prob. 80–11.** Design a "no-voltage-release" starting box for the motor of Prob. 76–11. The conditions are the same as in Prob. 78–11. Compute the resistance between steps 1 and 2; 2 and 3; 3 and 4; 4 and 5; 5 and 6; 6 and 7; 7 and 8. Resistance of no-voltage-release coil is 900 ohms.

State the no-load speed of motor when the starter arm has remained on point 8 until the current has become steady.

**Prob. 81–11.** (*a*) What current will flow in the armature of the motor in Prob. 76–11 when the starter arm is turned to point 1, if, by mistake, the starter of Prob. 48–11 is used with this motor on 230 volts? (*b*) What field current will flow?

**Prob. 82–11.** The data for the magnetization curve of a 115-volt shunt motor, when driven at 1200 rpm, are as follows:

| Field current in amperes | Generated voltage | Field current in amperes | Generated voltage |
|---|---|---|---|
| 0 | 4 | 4 | 120 |
| 1 | 40 | 5 | 129 |
| 2 | 78 | 6 | 136.5 |
| 3 | 105 | 6.5 | 140 |

Armature resistance including brush contacts = 0.139 ohm.
Shunt-field resistance including rheostat = 24.5 ohms.
Shunt-field turns per pair of poles = 500.
No. of conductors on the armature = 120.
Brush backward lead = 8°. No. of armature paths = 2.
Full-load armature current = 72 amperes. No-load armature current = 7 amperes. What is the full-load speed?

**Prob. 83–11.** What is the no-load speed and the percentage regulation of the motor in Prob. 82–11?

**Prob. 84–11.** Compute the speed of the motor in Prob. 82–11 at a sufficient number of loads to plot the speed-load characteristic to 100 amperes armature current.

**Prob. 85–11.** How many differential-series turns must be added to the motor of Prob. 82–11 in order that it may operate at full load at the same speed as in Prob. 83–11? Allow 0.02 ohm for resistance of series field. Machine is connected long shunt.

**Prob. 86–11.** If the motor in Prob. 82–11 is operated as a cumulative compound motor, with the number of series turns as found in Prob. 85–11, what will be the full-load and no-load speeds and the percentage regulation? Compare these values with those found in Probs. 82–11 and 83–11.

**Prob. 87–11.** If the motor of Prob. 82–11 is rewound as a series motor to operate at 1080 rpm at full load with the same impressed voltage and same brush shift, how many series turns will be required? Assume resistance of series field to be 0.12 ohm.

**Prob. 88–11.** How many series turns will be required in order that the motor in Prob. 87–11 shall operate at 900 rpm at full load? Assume the resistance of the series field to be 0.15 ohm.

**Prob. 89–11.** A shunt motor on 125-volt mains has a speed of 1200 rpm when the armature current is 10 amperes. The armature resistance is 0.5 ohm. What will be the speed, if the field flux is decreased 10 per cent, a resistance of 1.0 ohm is connected in series with the armature and the armature current is increased to 15 amperes?

**Prob. 90–11.** Connect up the motor, starting box and line shown in Fig. 41–11.

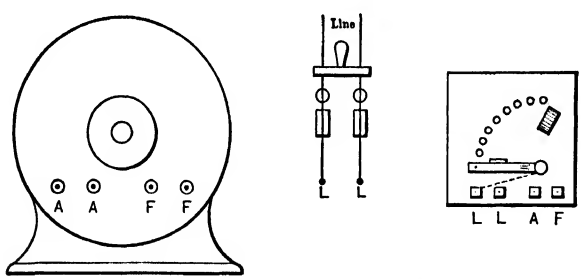



Fig. 41–11. A shunt motor to be connected to a four-point starting box.

**Prob. 91–11.** Show change you would make in your connections of Prob. 90–11, if motor did not start when starter handle was moved up to contact points.

# CHAPTER XII

## EFFICIENCY AND LOSSES IN MOTORS AND GENERATORS. RELATION OF VOLTAGE TO EFFICIENCY OF POWER TRANSMISSION. PARALLEL OPERATION OF D-C GENERATORS

**1. Efficiency.** If the electrical power put into a motor is measured, and **at the same time** the mechanical power output is measured, the commercial efficiency of the machine may be computed by finding **the ratio of the power output to the power input.**

That is,

$$\text{efficiency} = \frac{\text{output}}{\text{input}}.$$

In calculating efficiency, both output and input must be expressed in the same units. The input to a motor is usually measured in watts and the output in horsepower; so it is necessary to reduce either the watts input to horsepower, or the horsepower output to watts. Thus, in a small motor which requires an input of 3 kw to produce an output of 3 hp, the efficiency must be computed in the same units.

Either, both in horsepower,

$$\text{efficiency} = \frac{3 \text{ hp}}{3 \times 1.34 \text{ hp}} \times 100 = 74.6 \text{ per cent};$$

or, both in kilowatts,

$$\text{efficiency} = \frac{3 \times 0.746 \text{ kw}}{3 \text{ kw}} \times 100 = 74.6 \text{ per cent}.$$

In the same way, the efficiency of a generator may be determined, except that, in this case, the efficiency is the **ratio** of the **electrical power output to the mechanical power input.** It is somewhat difficult to measure mechanical power input to a generator directly; and therefore efficiencies of generators are usually obtained by a determination of the losses. This is discussed in Art. 8.

Note that in all commercial tests for efficiency, the measurements

must be taken when the machine is "hot," or has risen to constant temperature.

**2. Importance of High Efficiency.** The efficiency of an electrical machine, especially one of large capacity, is very important.

Consider a 200-hp motor, having an efficiency of 92 per cent, used to furnish power for a factory. Assume this motor operates at full load for 8 hours per day, and the cost of energy is $0.03 per kilowatthour. The input to the motor is $\frac{200}{0.92}$ or 217 hp. This is $217 \times 0.746$ or 162 kw. The cost of operating the motor one day is $162 \times 8 \times 0.03 = \$39.00$. The cost for one month of 26 days is $1010.

If the efficiency of this motor had been only 85 per cent, the input would be $\frac{200}{0.85}$ or 235 hp. This is $235 \times 0.746$ or 175.5 kw. The cost of operating the motor one day is now $175.5 \times 8 \times 0.03$ or $42.00; and for one month, $26 \times \$42.00$ or $1095.00. Thus the higher-efficiency motor in this case makes a saving of some $80.00 per month, or about $1000 per year.

It is to be noted that in the illustration above, the saving is based on the continuous operation of the motor at full load.

However, the first cost of a high-efficiency machine is usually greater than for one of a lower efficiency having the same power and speed rating. And where loads are intermittent, or are used only for short periods, the saving in first cost may outweigh the advantage of high efficiency, and make a lower-efficiency machine the more desirable in certain cases. This is particularly true for very small motors.

**3. Measured Efficiency. Brake Horsepower.** If a motor is not too large (under 200 hp), the efficiency can be measured by loading it.

The input is measured by an ammeter in the line and a voltmeter across the terminals, the product of the amperes and the volts being the total watts input. The output, or brake horsepower, is measured by means of a "prony brake" applied to the pulley.

A simple form of prony brake is shown in Fig. 1–12. A rope is wound around the pulley and two spring balances *A* and *B* are attached, one to each end of the rope. The balances should be spaced from each other at a distance equal to the diameter of the pulley plus the diameter of the rope, so that the forces exerted on

the balances are parallel to each other. The tension on the rope is adjusted by swivels, until the desired load on the motor is secured, the resistance to motion consisting of the friction of the rope on the pulley. The resistance, or force used by the motor to drive the pulley in the given direction, must be the difference between the readings of the two balances; because the force indicated by balance *B* aids the motion, and that indicated by balance *A* resists the motion of the pulley. The speed of the pulley in rpm is taken at the same time at which the balances are read. The mechanical horsepower output is then (Equation 6, Chapter XI),

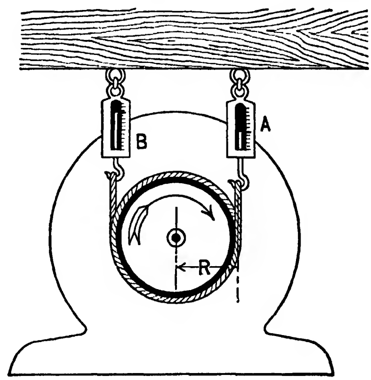



Fig. 1–12. A prony brake for measuring output of a motor.

$$\text{Horsepower output} = \frac{2\ \pi RF \text{ rpm}}{33{,}000},$$

where $R$ = radius of pulley plus that of the rope.

The distance to the center of the rope is taken because it is there that the balances are measuring the forces; $F$ is the difference between the readings of the two balances, *A* and *B*.

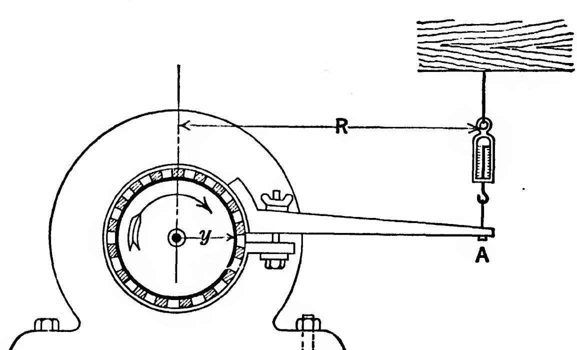



Fig. 2–12. The beam type of prony brake.

Figure 2–12 shows another form of prony brake. A band of belting, with wooden cleats riveted to it, is connected to a wooden or metal lever arm, and clamped around the pulley. The reading

($B_1$) of the balance placed at $A$ is taken with the motor still. This gives the weight of the lever arm. Then a reading ($B_2$) of the balance is taken with the motor under a load controlled by the hand screw. The force $F$ exerted by the motor is then the difference between the two balance readings ($B_2 - B_1$). This force is exerted at a distance $R$ from the center of the pulley. Applying again the equation for mechanical horsepower:

$$\text{Horsepower output} = \frac{6.28 \times R(B_2 - B_1)\,\text{rpm}}{33{,}000}.$$

Of course, the force exerted by the motor at the rim of the pulley does not act at the point $A$ in Fig. 2-12, but it is measured at $A$. If the force were measured at the rim where it acts, it would measure $\frac{R}{y}$ times as large, or equal to $\frac{R}{y}(B_2 - B_1)$. The torque measured at the rim of the pulley is then $y\frac{R(B_2 - B_1)}{y}$. But this equals $R(B_2 - B_1)$, which is the torque measured at point $A$.

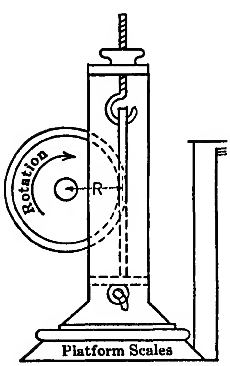


FIG. 3-12. Another form of prony brake for small motors.

Another form of prony brake, for motors of 10 horsepower and below, is that shown in Fig. 3-12, in which a frame resting on platform scales supports a rope wound around the motor pulley. The weight $W_1$ of frame and rope is first measured. Then the rope is adjusted on the pulley, and with the motor running, the tension on the rope is adjusted by the hand wheel until the desired load is secured and the frame balanced on the scales, giving reading $W_2$. The difference in scale readings is the force opposing the motion of the pulley. This force acts at a distance $R$ (radius of the pulley plus that of the rope), and mechanical horsepower =

$$\frac{2\,\pi R\,(W_2 - W_1)\,\text{rpm}}{33{,}000}.$$

Mechanical power absorbed by any brake is converted into heat; and therefore pulleys for brake tests should be flanged on the inside rim, so that water may be applied to the inside surface to

carry away the heat. In larger motors, arrangements must be made to furnish a continuous supply of cold water to the inside of the pulley, and at the same time take away the hot water.

**Example 1.** A motor takes 25 amperes at 230 volts. It is rigged with a prony brake like that in Fig. 1–12, and runs at 1200 rpm with a reading on balance $B$ of 3 pounds, and on balance $A$ of 78 pounds. The distance $R$ is 4.5 inches. What is the efficiency?

**Solution.**

$$R = 4.5 \text{ inches} = 0.375 \text{ foot.}$$

$$\text{Output} = \frac{6.28 \times 0.375\ (78 - 3) \times 1200}{33{,}000}$$

$$= 6.43 \text{ horsepower.}$$

$$\text{Input} = \frac{230 \times 25}{746} = 7.70 \text{ horsepower.}$$

$$\text{Efficiency} = \frac{6.43}{7.70} \times 100 = 83.4 \text{ per cent.}$$

**Prob. 1–12.** A motor is rigged with a prony brake of the form shown in Fig. 2–12. When the motor is standing still, the balance at $A$ reads 3.2 pounds. When running at a speed of 1050 rpm, the balance at $A$ reads 29 pounds. The distance $R$ measures 3 feet, 6 inches. The motor takes 72 amperes at 230 volts. What is the efficiency?

**Prob. 2–12.** In applying a brake to measure the horsepower of a motor, the form shown in Fig. 3–12 is used. The brake frame weighs 14 pounds. When the motor draws 26 amperes at 230 volts and runs at a speed of 1300 rpm, the scales balance at 79 pounds. The diameter of the pulley is 8.5 inches, and that of the rope $\frac{7}{8}$ inch. What is the efficiency of the motor?

**Prob. 3–12.** (*a*) What torque has the motor in Example 1? (*b*) What torque has the motor in Prob. 1–12? (*c*) What torque has the motor in Prob. 2–12?

**Prob. 4–12.** (*a*) From the speed-torque curves of the shunt motor of Fig. 31–11, page 395, find the horsepower output at an armature current of 80 amperes. (*b*) What is the efficiency of this motor at this load, if it operates on a 115-volt line? Resistance of the field circuit is 88 ohms.

**Prob. 5–12.** What efficiency has the series motor of Fig. 30–11, page 392, when drawing a current of 10 amperes from a 230-volt line?

**Prob. 6–12.** What is the efficiency of the motor of Fig. 30–11 when drawing 5 amperes from the 230-volt line?

**4. Efficiency from Name-Plate Data.** The **approximate** efficiency of a motor at full load may be determined from the rating given on the name plate. The name-plate rating of a motor,

according to the rules of the A. I. E. E., shall give the voltage, the full-load current, speed and horsepower output for which the machine is designed.

**Example 2.** The rating of a certain motor from the name plate is 115 volts, 40.5 amperes, 5 hp. What is the efficiency?

**Solution.**

$$\text{Watts } \textbf{output} \text{ at full load} = 5 \times 746 = 3730;$$

$$\text{Watts } \textbf{input} \text{ at full load} = 115 \times 40.5 = 4650;$$

$$\text{Efficiency at full load} = \frac{3730}{4650} \times 100 = 80.2 \text{ per cent.}$$

**5. Losses in a Motor or a Generator.** A certain part of the power delivered to a motor or a generator is lost, or wasted, in the various parts of the machine itself. The greater this loss is in relation to the input, the lower are the output and the efficiency. Furthermore, this energy lost is converted into heat and raises the temperature of the machine, thereby limiting its output. Therefore, excessive losses result in high temperatures dangerous to the insulation of the windings, and in reduced output. These losses, then, must be kept as low as is practicable.

The losses in a motor or a generator consist of: (*a*) the "copper losses," or $I^2R$ losses, in the copper circuits of the machine; and (*b*) the mechanical losses, or losses due to the rotation of the armature. These losses may be tabulated as follows:

| | | |
|---|---|---|
| Copper, or Electrical Losses | Armature Loss | Armature $I^2R$ |
| | Field Loss | Shunt field $I^2R$ |
| | | Series field $I^2R$ |
| | | Commutating field $I^2R$ |
| Rotational, or Mechanical Losses | Iron, or "Core" Losses | Hysteresis loss |
| | | Eddy-current loss |
| | Friction Losses | Bearing friction |
| | | Brush friction |
| | | Windage (air friction) |

**Armature Copper Loss.** When current is forced through the resistance of an armature winding, a certain amount of power is lost which is proportional to the square of the current. In addition, there is a small electrical loss in the brushes and brush contacts, which is practically proportional to the current. If the armature resistance be taken to include the resistance of the

brushes and brush contact, both losses may be figured with very little error, as proportional to the square of the current; or $P_a = I_a^2R_a$ where $P_a$ = armature copper loss, $I_a$ = armature current and $R_a$ = resistance of armature, brushes and brush contact. This loss then is **a function of the load** — **a doubling** of the armature current increases this loss **four** times.

**Series-Field Copper Losses** are also **a function of the load** and $P_s = I_s^2R_s$ where $P_s$ = series-field copper loss, $I_s$ = series-field current and $R_s$ = series-field resistance. Copper losses in commutating and compensating windings are also computed in the same way.

**Shunt-Field Copper Loss.** The copper loss in the shunt field is **approximately constant and independent of the load,** since the field circuit is connected across the machine terminals and carries practically a constant current, $I_f$. This loss is calculated as $P_f = E_tI_f$ or $P_f = I_f^2R_f$. The shunt-field losses should include those in the shunt-field rheostat.

**Hysteresis Loss.** Since the armature core revolves in a magnetic field, it becomes magnetized, first in one direction, and then in the other. It was shown in Art. 21, Chapter VIII, that when iron is taken through a magnetic cycle, a loss results which is proportional to the area of the hysteresis loop. This loss has been expressed by the equation

$$P = KfB^{1.6},$$

where $P$ = hysteresis loss,
$f$ = cycles or double reversals of magnetism per sec.,
and $B$ = flux density in lines per square inch.

In a two-pole machine, the armature core in one revolution would be taken through one complete cycle or hysteresis loop. Figure 4*a*–12 shows the polarity of a section, $A$, of the armature core of a two-pole machine. In Fig. 4*b*–12, after one-half a revolution, the position of the section $A$ is reversed and the direction of the flux through the section and its magnetism is also reversed. In one revolution, then, the iron in section $A$ goes through one complete hysteresis loop. Thus, practically all the iron in the armature core is carried through a loop in every revolution of the armature. In a four-pole machine, the armature core would be carried through two double reversals of flux in one revolution.

Hysteresis loss in the pole cores is negligible since they are

magnetized always in the same direction and practically to the same field intensity.

Hysteresis loss is **independent of the load** on the machine, but is **a function of the flux density and the speed.***

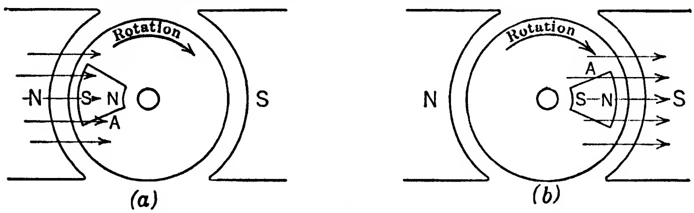



FIG. 4–12. The direction of the magnetic flux in section A of the armature core is reversed every half revolution. This is true for practically the entire core.

**Eddy-Current Loss.** It has been shown that currents flow in the copper conductors of an armature, due to the emf induced in them as they rotate in a magnetic field, if the external circuit is closed. These conductors are wound on an iron core which also rotates in the same magnetic field. Therefore, voltage will also be induced in certain parts of this core. As the iron itself is a conductor of electricity, and since the current paths are short and of large cross section, large currents called **Foucault,** or **Eddy Currents,** flow in the armature core, as shown in Figs. 5*a*–12 and 5*b*–12.

These currents heat the core and also the copper conductors wound on it, which is objectionable. Furthermore, it is always true that there is required an expenditure of energy to set up an electric current. Since these currents flow in closed circuits in the core itself, they cannot be used and the power required to generate them is wasted in $I^2R$ losses in the iron.

It is therefore important that these eddy currents be reduced to as small a value as possible. This is done by "laminating" the armature core; that is, building it up of thin sheets of iron, as illustrated in Fig. 5*c*–12, insulated from each other by japan or some form of lacquer. See Fig. 22–10, page 271. This does not greatly affect the magnetic circuit, or field flux, but cuts down these electric currents, since the laminations are "transverse" to

* It should be noted here that while the equation above is a fairly accurate expression for hysteresis loss in a stationary iron core, it does not hold in the case of rotating machines in which the reversals of flux take place in a toothed core. It is true, however, that hysteresis loss depends upon the speed and upon some exponential power of the flux density which is not constant, but changes with varying flux densities.

the direction in which the currents tend to flow. The thinner the laminations, the more effective is this method in decreasing this loss. In the commercial machine, laminations may be approximately 0.014 inch in thickness.

The amount of the induced voltage in any single lamination is proportional to its thickness. In Fig. 5c–12 one lamination, *l*, is purposely shown one-half the thickness of the others. The voltage induced in this is just one-half that induced in the others;

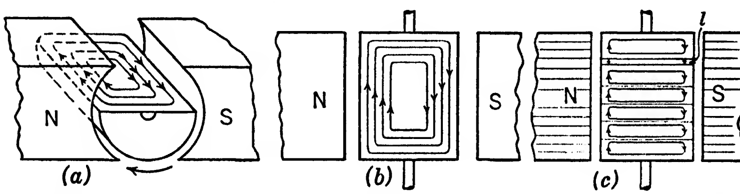



FIG. 5–12. (a) and (b) Eddy currents in a solid armature core. (c) Laminated core and pole shoes. The laminations break up the current paths and reduce the induced voltage in each path in the core, thereby reducing the eddy currents.

and yet the length of the current path and its resistance is approximately the same. Therefore, the current is reduced one-half, and the $I^2R$ loss is reduced **four times.** The eddy-current loss then varies with the **square** of the thickness of the laminations. Eddy-current loss also varies with the **square** of both the flux density and the speed. For, doubling either the flux or the speed would **double the induced voltage** in each lamination, thereby **doubling the current** and increasing the loss **four times.** Therefore, we can write

$$P \text{ (eddy loss) varies with } t^2, B^2 \text{ and speed}^2;$$

where

$$t = \text{thickness of the laminations,}$$
$$B = \text{flux density.}$$

Note that eddy-current loss is independent of the load.*

* Eddy-current losses are generally expressed by the equation, $Pe = kf^2B^2t^2$, but for rotating machines, the use of this equation gives results which are too low for low speeds and too high for high speeds. Furthermore, due to manufacturing methods, the laminations are imperfectly insulated in the commercial machine, and eddy-current losses are correspondingly high. In practice, results calculated by use of the above equation must be multiplied by a factor of from 1.5 to 2.

In commercial machines with slotted armature cores, the flux enters and leaves the armature in tufts through the teeth, as shown in Fig. 6*a*–12. As these tufts of flux move across the pole face, they produce pulsations, or waves, of the magnetic lines, as they emerge or enter the face of the pole. These pulsations induce small emf's near the face of the poles and parallel to the armature shaft, and therefore eddy currents are set up in the pole faces as shown in Fig. 6*b*–12. Poles are therefore usually laminated, as shown in Fig. 5–10, page 259.

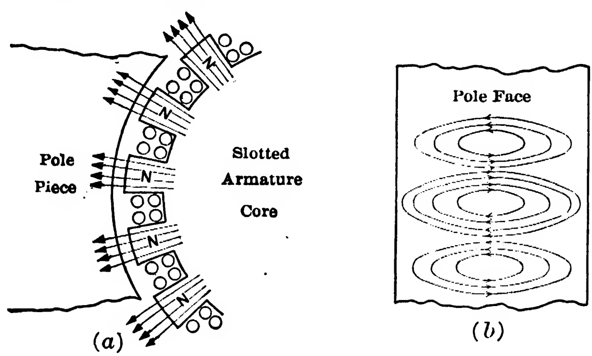



FIG. 6–12. The slotted armature core sets up eddy currents in the pole faces.

**Friction Losses.** These losses are due to bearing friction and to the friction of the brushes on the moving commutator. Bearing friction may be reduced in special cases by the use of roller or ball bearings. Brush friction can be kept at a minimum, if the surface of the commutator is smooth and the brushes are well fitted to the surface with just enough pressure to give good contact. These losses vary approximately with the speed.

**Windage (Air Friction).** The revolving armature sets up a fanning action and resulting air currents which consume energy. This loss is generally small. However, in some cases, one end of the armature spider is fitted with fins or blades (Fig. 12–3, page 72) to set up a definite circulation of air through the armature structure. This carries away the heat and thus increases the capacity of the machine. This increases the windage loss. **Windage loss varies approximately with the cube of the speed.**

The various copper, or electrical, losses can be readily separated and each of them can be accurately measured or calculated.

It is difficult to separate or to measure accurately either of the iron losses or the friction losses. And since these four losses

(hysteresis, eddy-current, windage and friction) vary as some function of the flux or the speed, or both, and are independent of the load, they are grouped together and called the Stray Losses. The power consumed by these combined losses is called the **Stray Power.** This is also the rotational loss.

For convenience, the losses in a constant-speed motor or generator are often divided into two classes: (1) the **variable** losses or those which vary with the load on the machine; (2) the **constant,** or nearly constant losses which are practically independent of the load.

Variable Losses — $I^2R$ Losses in Armature, Series and Commutating Fields.

Constant Losses — $I^2R$ Losses in Shunt-Field and Stray-Power.

**6. Efficiency by Determination of Losses.** In any machine:

$$\text{The output} = \text{input} - \text{losses}$$

and

$$\text{the input} = \text{output} + \text{losses}.$$

Thus efficiency, which is always $\dfrac{\text{output}}{\text{input}}$, may be written,

$$\text{efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} \tag{1}$$

or

$$\text{efficiency} = \frac{\text{output}}{\text{output} + \text{losses}}. \tag{2}$$

Therefore, if the losses in a machine can be determined for any given input or output, the efficiency can be readily computed.

In motors, it is easier, and generally more precise, to measure (or calculate) the electrical input and the losses, and to compute the efficiency from equation (1) above.

In generators, it is easier to measure (or calculate) the electrical output and to compute the efficiency from equation (2) above.

Furthermore, if the losses can be determined for any **assumed** input or output, it is possible to **calculate the efficiency** of a machine without loading it. This is a very great advantage. Large motors today are built of several thousand horsepower capacity. While the electrical input can be readily measured, it is manifestly impossible to load, for instance, a 1000-hp motor by

means of a prony brake; for this amount of energy cannot be absorbed by such a device.

It is also more convenient to calculate the efficiency of even small motors by a determination of the losses.

**7. Determination of Stray-Power Loss in Shunt Motors.** Stray-power loss is determined by running the motor unloaded with normal impressed voltage and at normal speed. The motor is connected, as in Fig. 7–12, with the shunt field connected across the line in series with a field rheostat. The impressed voltage $E_t$, the line current $I_l$, and the field current $I_f$ are measured. The machine is now shut down and armature resistance, including that of the brushes and the brush contact, is measured. These readings for a commercial test should be taken when the machine is "hot," or at normal temperature.

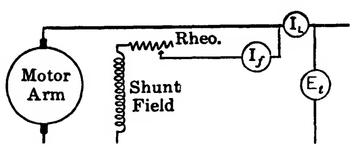



FIG. 7–12. Connections for determining stray power, and field I²R loss in a shunt motor. Motor is run at normal speed without load.

The total power input, $E_tI_l$, is all power expended in losses, since the motor is running without load. This power is distributed as follows: some power ($E_tI_f$) is used in $I^2R$ losses in the field; some is expended in a small $I^2R$ loss in the armature; and the remainder is the stray-power loss — loss due to the rotation of the armature.

**Example 3.** A 230-volt shunt motor, Fig. 7–12, running without load, takes a current of 5.7 amperes from the line at 1250 rpm. The field current is 1.7 amperes. Armature resistance is 0.185 ohm. What is the stray-power loss?

**Solution.**

Armature current = 5.7 − 1.7 = 4 amperes.
Power input to armature = 230 × 4 = 920 watts.
$I^2R$ loss in armature = $4^2 \times 0.185 = 3$ watts.
Stray power = 920 − 3 = 917 watts.

Note that

$I^2R$ loss in shunt field = 230 × 1.7 = 391 watts.

And the total loss = 391 + 3 + 917 = 1311 watts.
This equals the total input of 230 × 5.7, or 1311 watts.

It is important to note that the **stray-power loss is equal to the total power input to the armature at no load, minus the armature I²R loss.**

**8. Calculation of Efficiency by Determination of the Losses. Stray-Power Method. Constant-Speed Machines.** If it be **assumed** that the **speed** and the flux density of the motor in Example 3 are **constant at all loads,** the stray-power loss of 917 watts as figured is constant. The shunt-field $I^2R$ loss is also constant. Then, since the armature resistance is known, the $I^2R$ loss in the armature can be determined for **any assumed current** without putting any load on the machine.

**Example 4.** The full-load current of the motor in Example 3 is 57.7 amperes. What is the efficiency at full load? At half load?

**Solution.**

At full load,

Armature current $= 57.7 - 1.7 = 56$ amperes.

$$\begin{aligned}
&\text{Armature } I^2R \text{ loss} = 56^2 \times 0.185 &&= \phantom{1}580 \text{ watts.}\\
&\text{Stray-power loss (Example 3)} &&= \phantom{1}917 \text{ watts.}\\
&\text{Field } I^2R \text{ loss (Example 3)} &&= \underline{\phantom{1}391} \text{ watts.}\\
&\text{Total loss at full load} &&= 1888 \text{ watts.}
\end{aligned}$$

Input $= 230 \times 57.7 = 13{,}280$ watts.

Output (input − losses) $= 13{,}280 - 1888 = 11{,}392$ watts.

$$\text{Efficiency} = \frac{11{,}392}{13{,}280} \times 100 = 86 \text{ per cent.}$$

At half load,

$$\begin{aligned}
&\text{Armature current} &&= \frac{56}{2} = 28 \text{ amperes.}\\
&\text{Armature } I^2R \text{ loss} &&= 28^2 \times 0.185 = 145 \text{ watts.}\\
&\text{Total loss} &&= 391 + 917 + 145 = 1453 \text{ watts.}\\
&\text{Input} &&= 230 \times (28 + 1.7) = 6831 \text{ watts.}\\
&\text{Output} &&= 6831 - 1453 \qquad = 5378 \text{ watts.}\\
&\text{Efficiency} &&= \frac{5378}{6831} \times 100 \qquad = 78.7 \text{ per cent.}
\end{aligned}$$

**Prob. 7–12.** A 115-volt shunt motor has an armature resistance of 0.13 ohm and a field resistance of 84 ohms. The motor takes 5.2 amperes when running idle at 1500 rpm on a 115-volt line. Full-load line current is 60 amperes. What is the efficiency at full load?

**Prob. 8–12.** Compute the efficiency and plot the efficiency curve of the motor in Prob. 7–12 (using per cent efficiency as ordinates and armature current as abscissae) for $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full and $1\frac{1}{4}$ load.

**Prob. 9–12.** It can be shown that any machine operates at maximum efficiency under that load at which the variable losses are equal to the constant losses. At what armature current does the motor of Prob. 7–12 operate at maximum efficiency?

**9. Stray-Power Loss Corrected for Change in Speed.** In the previous article, Example 4, the stray power was assumed constant at all loads. This is strictly true only if the flux and speed are

constant, for stray power is a function of flux and speed. But the speed of a shunt motor falls off somewhat as the load increases. This changes to some extent the stray power.

It has been shown that, if the flux is constant, the counter voltage varies with speed: $E_c = K'\Phi N$ (Equation 8, Chapter XI).

It has also been shown that:

| | |
|---|---|
| Stray Power | Hysteresis loss varies with speed and $B$ (usually considered to the 1.6 power),<br>Eddy-current loss varies with speed$^2$ and $B^2$,<br>Friction loss varies with speed,<br>Windage loss varies with speed$^3$. |

Hysteresis is probably the greatest of the above losses in most motors. Therefore, for small changes in speed, the error will be slight, if it be assumed that the stray-power loss varies with the speed.

Since both $E$ (counter) and stray power can be considered to vary with speed (flux constant), the stray power can be said to vary with $E$ (counter).

Thus the efficiency of the motor in Example 4 can be more accurately determined if the stray-power loss is corrected for change in speed, as shown in the example below.

**Example 5.** In Example 3, at no load, the stray-power loss is 917 watts and the counter voltage $= 230 - 4 \times 0.185 = 229.26$ volts. The counter voltage at full load, Example 4, $= 230 - 56 \times 0.185 = 219.64$ volts.

Now,

$$\frac{\text{Stray power (full load)}}{\text{Stray power (no load)}} = \frac{219.64}{229.26} = \frac{x}{917}.$$

$$\text{Stray-power loss at full load} = \frac{219.64}{229.26} \times 917 = 880 \text{ watts.}$$

Total losses $= 391 + 880 + 580 = 1851$ watts.
Input, as before, $= 13{,}280$ watts.
Output (input $-$ losses) $= 13{,}280 - 1851 = 11{,}429$ watts.

$$\text{Efficiency} = \frac{11{,}429}{13{,}280} \times 100 = 86.3 \text{ per cent.}$$

At half load,

$E$ (counter) $= 230 - 28 \times 0.185 = 224.82$ volts.

$$\frac{\text{Stray-power half load}}{917} = \frac{224.82}{229.26}.$$

$$\text{Stray power at half load} = \frac{224.82}{229.26} \times 917 = 900 \text{ watts.}$$

Total losses = 391 + 900 + 145 = 1436 watts.

Input, as before, = 6831 watts.

Output (input − losses) = 6831 − 1436 = 5395 watts.

$$\text{Efficiency} = \frac{5395}{6831} \times 100 = 79 \text{ per cent.}$$

Note that there is a difference of about 0.3 per cent in the resulting efficiencies in Examples 4 and 5. This is negligible in small machines, but amounts to a considerable item in the cost of energy for a large motor.

**Prob. 10–12.** Repeat the calculations called for in Prob. 8–12, correcting the stray-power loss for change in speed.

**10. Measured Efficiency vs Calculated Efficiency. Precision of the Two Methods.** The efficiency of most electrical machines is high, and therefore the losses are small compared with either the input or the output. For this reason, the method of calculating the efficiency by determination of the losses is generally more precise than that of making actual measurements of input and brake-horsepower output. The more efficient the machine, the more precise is the former method.

For instance, consider a machine having an actual efficiency of 90 per cent. If the output is measured at the brake and the reading is in error and 1 per cent low, the efficiency as found will be 1 per cent low. On the other hand, the **losses** in this machine are only 10 per cent; and if an error of 1 per cent is made in determining the losses, this error amounts to only 1 per cent of 10 per cent or 0.1 of one per cent of the actual efficiency.

It is probably true that the **exact** losses in a machine under load cannot be calculated. Some are determined by resistance readings, others are approximated; and the effect of armature reaction is not considered, as in the previous article. On the other hand, it is difficult to get accurate readings of speed and scales in brake-horsepower tests, and this method is open to error.

Therefore, allowing for errors in both methods, the calculated efficiency probably gives results as near the actual efficiency as it is possible to obtain.

**11. Efficiency of Machines in Which Speed or Flux, or Both, Vary with Load.** Stray-power and field $I^2R$ losses which are

constant, or nearly constant, in the shunt motor, vary with load in other types of motors, and in generators.

Thus, in these machines the stray-power loss must be determined separately for each particular load; and it is therefore necessary to know the speed and field currents when the machine operates normally under the precise load for which the efficiency is desired. To duplicate the stray-power loss at this load, it is merely necessary to run the machine unloaded as a motor with the same field current (same flux) and at the same speed. If equation (8), Chapter XI, be transposed, $N$ (speed) $= \frac{E_c}{K'\Phi}$. Thus if field current (flux) is adjusted to the same value as under load, the voltage impressed upon the armature, and therefore $E$ (counter), can be adjusted until the machine runs at the same speed. If the flux and speed are the same in the two cases, the induced voltage or $E_c$ must be the same.

For instance, a series motor may be run at full load on normal voltage, and the speed and current noted. It can then be run unloaded as a separately excited motor with the field excited to full-load value. Then enough voltage is applied to the armature to drive it at the full-load speed noted above. Since the field is excited to full-load value, the flux is the same as at full load (neglecting armature reaction); and since both the speed and flux are the same as at full load, the stray-power loss at full load is duplicated. Then the input to the armature minus the no-load $I^2R$ loss is the stray power at full load. The full-load $I^2R$ loss in both field and armature can then be determined from the resistance measurements, and the efficiency calculated.

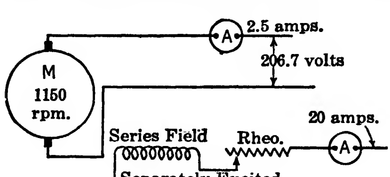



FIG. 8–12. A series motor operated idle at full load speed with field separately excited to full load current, to determine the stray power loss at full load.

**Example 6.** A 220-volt series motor at full load draws 20 amperes and runs at 1150 rpm when operated from a 220-volt line; field resistance = 0.14 ohm; armature resistance = 0.6 ohm. When run light with the fields separately excited to 20 amperes (Fig. 8–12), and sufficient voltage applied to the armature to cause the motor to run at 1150 rpm, the armature now draws 2.5 amperes and the voltage necessary is 206.7 volts.

(The counter emf at full load = 220 − 20 (0.14 + 0.6) = 205.2 volts. If the machine operates at no load at the **same** flux and speed, the

counter emf must be the same, or 205.2 volts. And if the armature draws 2.5 amperes at no load, the necessary impressed voltage must be $205.2 + (2.5 \times 0.6)$ or 206.7 volts at no load).

What is the full-load efficiency of the motor?

**Solution.**

Armature input at no load $= 206.7 \times 2.5 = 516.75$ watts.
Armature $I^2R$ loss at no load $= 2.5^2 \times 0.6 = 3.75$ watts.
Stray-power loss (at full load) $= 516.75 - 3.75 = 513$ watts.
Field $I^2R$ loss at full load $= 20^2 \times 0.14 = 56$ watts.
Armature $I^2R$ loss at full load $= 20^2 \times 0.6 = 240$ watts.
Total loss at full load $= 513 + 56 + 240 = 809$ watts.
Input at full load $= 220 \times 20 = 4400$ watts.
Output at full load $= 4400 - 809 = 3591$ watts.

$$\text{Efficiency at full load} = \frac{3591}{4400} \times 100 = 81.5 \text{ per cent.}$$

**Example 7.** A 115-volt 10-kw shunt **generator** when delivering full load at 115 volts and 900 rpm requires a field current of 2.4 amperes. Resistance of armature is 0.09 ohm. When the machine is run as a motor at no load with the same field current, 2.4 amperes, and at the same speed, the armature draws 4.2 amperes. (*a*) What voltage must necessarily be impressed on the armature when run as a motor at no load? (*b*) What are the stray-power loss and the efficiency of the generator at full load?

**Solution.**

$$(a)\ \text{Line current at full load} = \frac{10{,}000}{115} = 87 \text{ amperes.}$$

Armature current at full load $= 87 + 2.4 = 89.4$ amperes.
Generated voltage at full load $= 115 + (89.4 \times 0.09) =$ 123 volts.

To run the machine as a motor at no load, under full-load conditions of flux and speed, necessitates the same generated (or counter) emf as at full load; therefore $E$ (counter) at no load must be 123 volts. Impressed voltage at no load must then be $123 + (4.2 \times 0.09)$ or 123.4 volts.

(*b*) Stray-power loss (full-load conditions) $= 123.4 \times 4.2 - 4.2^2 \times 0.09 = 517$ watts.

Field $I^2R$ loss $= 115 \times 2.4 = 276$ watts.
Armature $I^2R$ loss at full load $= 89.4^2 \times 0.09 = 720$ watts.
Total loss at full load $= 517 + 276 + 720 = 1513$ watts.
Output at full load $= 115 \times 87 = 10{,}000$ watts.

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{10{,}000}{10{,}000 + 1513} = 0.868 \text{ or } 86.8 \text{ per cent.}$$

**Prob. 11–12.** When the load on the series motor of Example 6 is such that it draws 12 amperes from a 220-volt line, it runs at 2000 rpm. When run light with the field separately excited to 12 amperes and the

necessary impressed voltage to drive it at 2000 rpm, the armature draws 2.3 amperes. What is the efficiency?

**Prob. 12–12.** If the efficiency of the motor of Example 6 is 75 per cent, when running on a 220-volt line under a load which requires 10 amperes, what is the stray-power loss?

**Prob. 13–12.** The shunt generator of Example 7, when delivering half load at 115 volts and 900 rpm, requires a field current of 2.1 amperes. When run light as a motor at 900 rpm with the field separately excited with 2.1 amperes, the armature takes 4.1 amperes. What is the efficiency?

**Prob. 14–12.** A long-shunt compound generator is rated as 15 kw at 230 volts and 1200 rpm. Shunt-field current at full load = 1.8 amperes. Series-field resistance = 0.04 ohm. Armature resistance = 0.16 ohm. When the machine is run as a motor unloaded, with both fields excited to their full-load value and the necessary voltage impressed upon the armature to drive it at 1200 rpm, the armature current is 4 amperes. What is the efficiency at full load?

**12. Relation of Voltage to Efficiency of Transmission.** It has been shown in Chapter III that the losses in a direct-current transmission system are the $I^2R$, or copper losses. By increasing the voltage, the current necessary to transmit a certain amount of energy over a given line is decreased. Thus the $I^2R$ losses are reduced and the efficiency of transmission is increased. This accounts for the high-voltage alternating-current systems by which electrical energy today is transmitted long distances with little loss and at slight maintenance expense. However, the high voltages employed in alternating-current transmission are not today generally available for direct-current transmission, due in part to the fact that these voltages cannot be obtained by a machine equipped with a commutator. The limiting voltage for a d-c generator is around 1500 volts.

Since the conditions which must be complied with to obtain high efficiency are essentially the same in either an a-c or a d-c system, let us consider, in the example below, a d-c transmission system without regard to the limiting d-c voltages in use today.

**Example 8.** Suppose 30 kw is to be generated, and as much power as is practicable is to be transmitted over a 3-ohm line to drive a motor, say, 2 miles distant from the generator, as indicated in Fig. 9–12. The power is to be transmitted with as small a loss as is practicable. Let us try the effect of using various voltages ranging from 100 to 10,000 volts at which we might generate the 30 kw, and compute the corresponding efficiencies.

**First:**

Assume we generate at 100 volts. To supply 30 kw at this voltage, the generator must deliver $\frac{30{,}000}{100}$ or 300 amperes to the line. To force 300 amperes through a 3-ohm line requires $3 \times 300$ or 900 volts; which is 800 more volts than we are even generating. It is therefore clearly impossible to use 100 volts for the transmission.

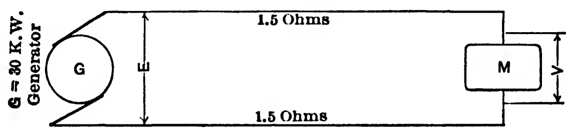



FIG. 9–12. A generator feeding a motor over a 3-ohm line.

**Second:**

Assume we generate at 200 volts. To supply 30 kw at this voltage, the generator must deliver $\frac{30{,}000}{200}$ or 150 amperes to the line. Here, again, to force 150 amperes through a 3-ohm line requires $3 \times 150$ or 450 volts; which is more than we assumed generated.

**Third:**

If we generate at 300 volts, only $\frac{30{,}000}{300}$ or 100 amperes must be transmitted. This requires $3 \times 100$ or 300 volts to force the current through the line and leaves no voltage to force current through the motor.

**Fourth:**

If we generate at 400 volts, it is necessary to force but $\frac{30{,}000}{400}$ or 75 amperes through the line. This requires $3 \times 75$ or 225 volts, leaving $400 - 225$ or 175 volts to force current through the motor.

Watts lost in the line $= 75^2 \times 3 = 16{,}875$ watts.

Watts delivered to the motor $= 30{,}000 - 16{,}875 = 13{,}125$ watts.

Efficiency of transmission $= \frac{13{,}125}{30{,}000} \times 100 = 43.8$ per cent.

The results of similar computations for a range of voltages are tabulated as follows:

30 KW GENERATED AT $E$ VOLTS

| Volts $E$ | Amperes $I$ | Line Loss in Volts. $R=3$ ohms $IR$ | Line Loss in Watts $I^2R$ | Volts Left for Motor $V$ | Watts Transmitted to Motor $W$ | Efficiency Per cent |
|---|---|---|---|---|---|---|
| 100 | 300 | 900 | Impossible | | | |
| 200 | 150 | 450 | " | | | |
| 300 | 100 | 300 | 30,000 | 0 | 0 | 0 |
| 400 | 75 | 225 | 16,875 | 175 | 13,125 | 43.8 |
| 500 | 60 | 180 | 10,800 | 320 | 19,200 | 63.3 |
| 600 | 50 | 150 | 7,500 | 450 | 22,500 | 75 |
| 800 | 37.5 | 112.5 | 4,220 | 687.5 | 25,780 | 86 |
| 1,000 | 30 | 90 | 2,700 | 910 | 27,300 | 91 |
| 1,200 | 25 | 75 | 1,875 | 1,125 | 28,125 | 93.8 |
| 1,500 | 20 | 60 | 1,200 | 1,440 | 28,800 | 96 |
| 2,000 | 15 | 45 | 675 | 1,955 | 29,325 | 97.8 |
| 3,000 | 10 | 30 | 300 | 2,790 | 29,700 | 99 |
| 5,000 | 6 | 18 | 108 | 4,994 | 29,964 | 99.8 |
| 10,000 | 3 | 9 | 27 | 9,991 | 29,973 | 99.9 |

The curve plotted in Fig. 10–12 shows the relation of voltage to efficiency, as brought out in Example 8 above. Note that, in this case, the efficiency increases rapidly with the voltage until about 1200 volts are reached. From there on, the increase is much slower, though the efficiency continues to rise a little with each increase of voltage. If the voltage were the only condition affecting the efficiency, it would be advisable to use indefinitely higher voltages. The increased cost of the equipment, and difficulties in insulating the line, limit the voltage at the present time to some more moderate value. A rough rule for the most efficient voltage is "1000 volts per mile." Yet the fact is evident that as far as the line loss ($I^2R$) is concerned, the higher the voltage used, the higher the efficiency of transmission.

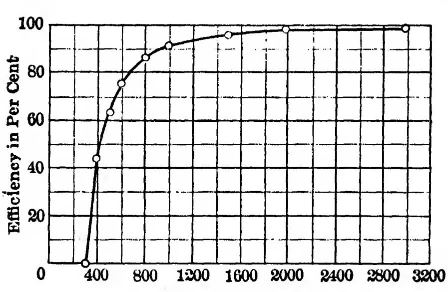



Fig. 10–12. The relation between voltage and efficiency of transmission.

**Prob. 15–12.** Construct a table as above for a 60-kw generating station and a 2-ohm line, starting at 100 volts and gradually increasing it to 10,000 volts. Plot a curve between volts and efficiency of transmission.

**Prob. 16–12.** Repeat Prob. 15–12 using a 6-kw generator.

**13. Relation of Line Loss to Voltage. Generator Output Constant.** On inspection of the table in Example 8, it is seen that the line loss in watts varies inversely as the square of the voltage of the generator, or the voltage of transmission (assuming constant generated power).

For instance, the line loss at 500 volts is 10,800 watts, while at 1000 volts it is only $\frac{10{,}800}{4}$ or 2700 watts. By doubling the voltage, the line loss is quartered.

**Prob. 17–12.** (*a*) If the line loss on a 115-volt system is 4 kw, what will it become if the voltage of the system is changed to 230 volts? (*b*) To 550 volts? Assume the same power delivered to the line.

**14. Relation of Size of Conductor to Voltage. Line Loss Constant.** In the above paragraphs, we have been considering a means of decreasing the operating expenses by decreasing the power lost in the line. We have seen that the power lost decreases as the square of the voltage of transmission. It is also important to consider the effect on the original cost of transmission if we raise the voltage. In analyzing this point, we will assume that we have fixed upon the amount of power which we are willing shall be lost in the line, and that we keep this constant.

By referring again to the table in Example 8, we find the line loss is 1.2 kw, when the transmission voltage is 1500 volts. Let us assume this to be the line loss we are willing shall be kept constant for economical reasons. By doubling the voltage to 3000 volts, we of course decreased the line loss to $\frac{1.2}{4}$ or 0.3 kw. But since we are allowed a line loss of 1.2 kw, we may now increase our line resistance by using a wire of a diameter enough smaller to make up this line loss.

The resistance of the line has been kept at 3 ohms. At 1500 volts, the line current was 20 amperes; line loss $= 20^2 \times 3 = 1.200$ kw.

At 3000 volts, line current was 10 amperes; line loss $= 10^2 \times 3 = 0.300$ kw.

But being allowed 1.2 kw line loss, if we use the 3000 volts, and 10 amperes line current, we may use a wire of 12 ohms, instead of 3 ohms. Then the line loss would become $10^2 \times 12 = 1.2$ kw, the same value it had at 1500 volts with a 3-ohm line. That is, by doubling the voltage, we are able to use a line wire of four times the resistance, and not increase the line loss.

This may be stated in more general terms as follows:

The line resistance for a constant line loss varies directly as the square of the voltage of transmission.

Since the resistance of a wire varies directly as the length, and inversely as the cross-section area, the above law may be applied as meaning that with the same line loss we may either:

(*a*) Use the same size wire and transmit four times as far at double voltage, nine times as far at triple voltage, etc., or

(*b*) Transmit the same distance and use wire of one-quarter cross section or of one-quarter weight if we double the voltage; one-ninth weight, if we triple the voltage, etc.

It can be seen from (*b*) that in transmitting electric power between any two points at given line loss, a great saving in the cost of copper can be made by transmitting at a high voltage. In fact, since the cost of installed moderate-size copper wire is almost directly proportional to its weight, we may say that as far as the cost of the line wire is concerned, the initial expense varies inversely with the square of the voltage of transmission.

In choosing the line wire to be used in the installation of a system, such a size must be used as will maintain a fair balance between the initial cost and the cost of running, i.e., the line loss. The initial cost **increases** as the size of the wire increases, while the cost of running (line loss) **decreases** as the size increases.*

**Prob. 18–12.** A 115-volt line is to transmit the same power at the same line loss as a 230-volt line of equal length. How will the size of the line wires of the two systems compare?

**Prob. 19–12.** If No. 0 (B & S) wire is used for the 115-volt line of Prob. 18–12, what size wire would be necessary for the 230-volt line?

* Lord Kelvin deduced the following rule with regard to the **most economical area of conductor** for transmitting a given amount of power at a given voltage:

The conductor should be of such an area that the value of power lost per year in the line equals the interest per year on the money invested in the line.

## Parallel Operation of D-C Generators

**15. Advantage of Operating Generators in Parallel.** In most power stations it is common practice to supply the power from several smaller units, rather than from a single large unit. Thus all the generators, or only one, may be operated to supply the station output, depending on the total load. The reasons for this arrangement are **efficiency** and **reliability.**

Consider Fig. 11–12, which is a curve representing the daily load on an average small station supplying power for lighting, for factories and for railway traction. It is plotted between kilowatt output and time. The "peak" load occurs about 7 o'clock P.M., and is about 2500 kw.

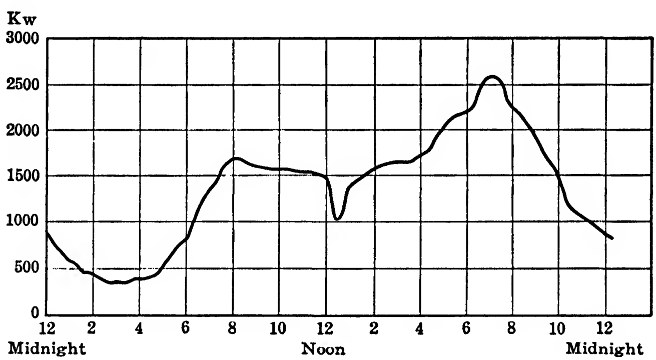



FIG. 11–12. Daily load on an average small station.

If this station were equipped with one 2000-kw generator, it could supply the load satisfactorily. The generator would be operated at an over-load for a short period each day during the "peak," but generators are usually able to carry 25 per cent overload for short periods. However, this arrangement would be neither **efficient** nor **reliable,** for the generator would be operating at one-quarter to one-half load the greater part of the time, and consequently at low efficiency. Most machines operate at greatest efficiency at, or near, full load. Furthermore, in case of a breakdown or an accident, the plant would be without power.

A better arrangement would be to install, for example, two 1250-kw generators and one 750-kw generator. Then the 750-kw generator could handle the load from midnight to about 6 o'clock in the morning. During most of the day, one 1250-kw generator

could handle the load; and during the "peak" either the two 1250-kw machines, or one 1250 and the 750-kw machine, could be operated in parallel to carry the maximum load. By this arrangement, the machines would be operating at nearly full load and at their best efficiency. Furthermore, in case of accident to any machine, the other two could carry the load, and the reliability of the plant is assured. The first cost of the three-generator equipment would be considerably greater than if one large generator were used; but the operating cost would be decreased, and continuity of service would be amply insured.

**16. Conditions for Paralleling.** Generators, to operate in parallel, must be designed to produce the same terminal voltage. They must also be connected in **opposition** to each other. In a d-c power station, this is accomplished by connecting all the generators to a pair of "bus bars." The bus bars are the heavy copper conductors, installed behind the switchboard, to which all the generators and feeder lines are connected through circuit breakers and switches. The positive terminals of all the generators in operation are connected to the **positive bus**; and all the negative terminals are connected to the **negative bus,** as shown in the simple diagram of Fig. 12–12. This is called connecting the generators in parallel.

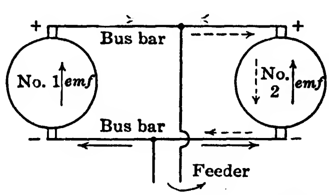



Fig. 12–12. A simple arrangement of two generators in parallel.

Assume no external load and a terminal voltage of 230 volts for each generator in Fig. 12–12. Generator No. 1 would tend to send a current through generator No. 2 against its emf; and generator No. 2 would tend to send a current through No. 1 against its emf. Since the terminal voltages of the two generators are the same, they will exactly counteract each other and no current will circulate between the two machines. If, however, the emf of generator No. 1 were raised (by increasing the field current) to, say, 235 volts, it would now force a circulating current through machine No. 2 against the emf of that machine as shown by the dotted arrows. The **terminal** voltage, however, would be the same for both machines. Again, if the emf of generator No. 1 were lowered, generator No. 2 would send a circulating current in the reverse direction through generator No. 1 against its emf. Since armature and bus-bar resistances are very small, a slight

change in the emf of either machine will send a large circulating current through the circuit.

While the emf's of the two generators in Fig. 12–12 are in opposition to each other in the local circuit between the machines, yet with an external load, both emf's act in a direction to supply current to the feeder; therefore they act in the same direction in the external circuit.

**17. Parallel Operation of Shunt Generators.** Shunt generators, when operated in parallel, are stable in their performance, due to the fact that they have a drooping external characteristic. When two or more shunt generators are once adjusted by field rheostat to share a constant external load, the division of the load by the machines does not change. Any tendency of one machine to take more than its share of the load results in a drop in terminal voltage, which prevents this action. Thus each machine tends to "shirk" the load, as it is called, and the total load divides in inverse ratio to the resistance of the individual generator circuits. Even when the external load changes, each machine is **compelled to take some part** of the increase or decrease in total load.

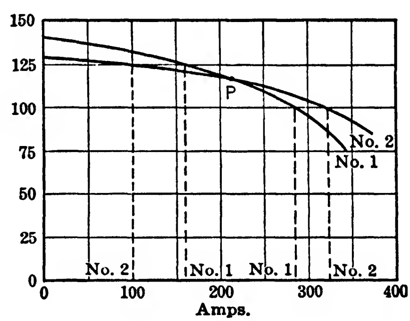



Fig. 13–12. External characteristics of two shunt generators in parallel.

In order that shunt generators operating in parallel shall divide the load proportionately, as the total load on the station changes, it is very important that the external characteristics of all the machines be the same; that is, have the same voltage at no load and at full load.

Consider two shunt generators with different external characteristics, Fig. 13–12, operating in parallel at 115 volts terminal pressure (point $P$), and adjusted so that each machine is carrying half of a total load of 450 amperes, or 225 amperes each.

When the total load is increased to 610 amperes, machine No. 1, due to the fact that its external characteristic droops or falls off more than that of machine No. 2, shirks its proportional share of the load and is carrying only 285 amperes; while machine No. 2 is forced to carry 325 amperes.

When the total load decreases to 260 amperes, machine No. 2,

due to the fact that its voltage does not rise as fast with decrease of load as does machine No. 1, now shirks its proportional share of the load and carries only 100 amperes; while machine No. 1 is forced to carry 160 amperes. Thus, due to the fact that the characteristics of the two generators are not the same, the machines, while stable in operation, do not divide the load proportionately as the load on the system changes. It is evident that if the generators are to proportionately divide the load at all load values, the external characteristics should be similar. This fact emphasizes the importance of determining the shape of the load curve of a generator as discussed in Chapter X.

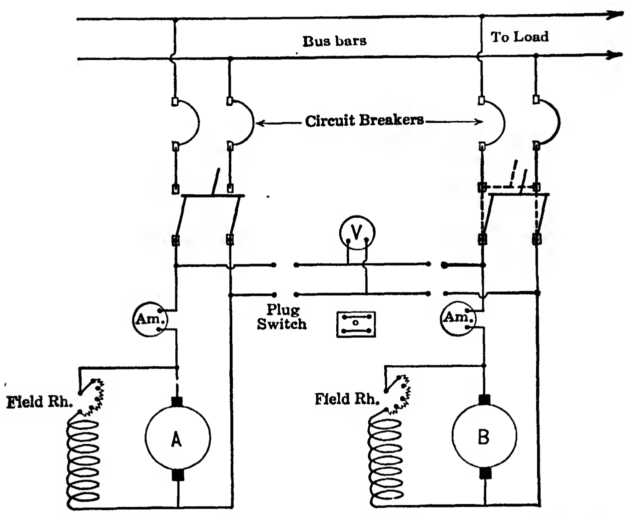



FIG. 14–12. Diagram of connections for operating two shunt generators in parallel.

Figure 14–12 shows the connections for operating two shunt generators in parallel. Each machine has its own ammeter and, by means of a plug connection, the voltmeter can be connected across the terminals of either machine. Machine *B* is shown connected to the bus bars and supplying power to the load. Assume the load has increased so that it is necessary to bring in machine *A*. This machine is brought up to speed and its emf adjusted by the field rheostat until it is just equal to the terminal voltage of *B*, or the voltage of the bus bars. The switch is now closed connecting *A* to the bus bars; but *A* furnishes no current to the system until its emf is raised slightly above the terminal volt-

age of $B$. Now, since the terminal voltage of both machines must be the same, machine $A$ supplies current to the load, the amount depending upon how much the emf of $A$ is raised. By adjusting the resistance in the field rheostat of $A$, this machine can be made to take any desired part, or all, of the total load. Because of the drooping characteristic, the terminal voltage of generator $B$ rises as the load on it is decreased; and, therefore, its emf must be decreased (by field-rheostat control) as that of machine $A$ is increased, in order that the bus-bar voltage may be kept constant while the load is adjusted.

When the emf of generator $B$ is decreased until it is equal to the terminal voltage of $A$, or of the bus bars, it will supply no current and its switch may be opened and the machine taken out of service, the total load having been shifted to $A$. If the emf of generator $B$ is decreased below the terminal voltage of $A$, or the bus-bar voltage, the armature current in $B$ is reversed and it operates as a motor running in the same direction as before.

**Example 9.** Two shunt generators are connected in parallel. The full-load capacity of generator $A$ is 40 amperes, and that of $B$ is 100 amperes. The full-load voltage of $A$ is 110 volts and the no-load voltage is 125 volts. Generator $B$ has a no-load voltage of 120 volts and a full-load voltage of 112.5 volts. Consider the external characteristic of each machine to be a straight line.

Find the terminal voltage and current delivered by each machine when the total current taken by the load is 140 amperes.

**Solution.**

Let $a$ be the current delivered by generator $A$;
Let $b$ be the current delivered by generator $B$;
Generator $A$ drops 15 volts when delivering 40 amperes.

Therefore, it drops $\frac{a}{40}$ of 15 volts, or $\frac{3a}{8}$ volts when delivering $a$ amperes.

Generator $B$ drops 7.5 volts when delivering 100 amperes.

Therefore, it drops $\frac{b}{100}$ of 7.5 volts or $\frac{3b}{40}$ when delivering $b$ amperes.

Terminal voltage of $B$ when delivering $b$ amperes $= 120 - \frac{3b}{40}$, and that of $A = 125 - \frac{3a}{8}$ volts.

Since the two generators are in parallel, their terminal voltages must be the same. Therefore,

$$125 - \frac{3a}{8} = 120 - \frac{3b}{40}.$$

But $\quad a + b = 140,$

so $\quad b = 140 - a,$

and
$$125 - \frac{3a}{8} = 120 - \frac{3(140 - a)}{40}.$$
$$5 = \frac{3a}{8} - \frac{420 - 3a}{40},$$
$$200 = 15a - 420 + 3a,$$
$$18a = 620 \text{ and } a = 34.4 \text{ amperes,}$$
$$b = 140 - 34.4 = 105.6 \text{ amperes.}$$

Therefore machine $A$ delivers 34.4 amperes and $B$, 105.6 amperes.

Terminal voltage of $A$ =
$$125 - \frac{3 \times 34.4}{8} = 112.1 \text{ volts.} \quad \textit{Answer.}$$

Terminal voltage of $B$ =
$$120 - \frac{3 \times 105.6}{40} = 112.1 \text{ volts.} \quad \textit{Check.}$$

The problem also can be solved graphically by plotting the characteristic curves of the two machines.

**Prob. 20–12.** Two shunt generators are operated in parallel. $A$ has a full-load current of 50 amperes at 115 volts, with a 10 per cent voltage regulation. $B$ has a full-load current of 80 amperes at 113 volts and an 8 per cent voltage regulation. (*a*) What current will each deliver to a load demanding a total of 120 amperes? (*b*) What will be the terminal voltage of the machines at this load?

**Prob. 21–12.** Shunt generator $A$ has a no-load voltage of 228 volts and a full-load voltage of 220 at 50 amperes. Shunt generator $B$ has a no-load voltage of 228 volts and a full-load current of 100 amperes. What must be the full-load voltage of generator $B$ in order that the two generators may be operated in parallel and each take its proportional share of the load?

**Prob. 22–12.** The full-load voltage of each of two shunt generators is 112 volts. $A$ has a voltage regulation of 5 per cent; $B$, 10 per cent. The full-load current of $A$ is 100 amperes; of $B$, 60 amperes.

(*a*) What current will each deliver when a load of 160 amperes is demanded, if they are connected in parallel?

(*b*) When a load of 30 amperes is demanded?

(*c*) When a load of 10 amperes is demanded?

**18. Parallel Operation of Compound Generators.** Compound generators are usually either flat or over-compounded, and therefore, have rising characteristics, as shown in Fig. 15–12. If two such generators are connected in parallel at their terminals only, they will be **unstable** in their operation.

Consider two over-compounded generators, connected as in Fig. 16–12. Assume they are dividing the load on the feeder, and that due to a momentary change in the speed of the driving engine, the voltage of generator No. 2 rises slightly. It at once takes more of the load and generator No. 1 takes less. This shift in the load raises the voltage of No. 2 still more, due to its rising charac-

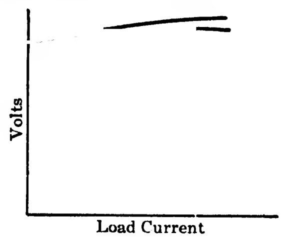



FIG. 15–12. Rising characteristics of two over-compounded generators.

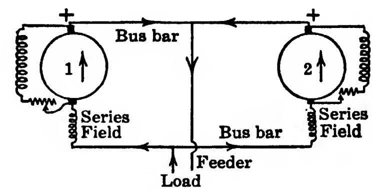



FIG. 16–12. Unstable connections of compound generators in parallel.

teristic, and further reduces the voltage of machine No. 1. Consequently, machine No. 2 takes more and more load and machine No. 1, less. The action continues until one machine takes all the load with constantly rising terminal voltage, and drives the other as a motor. When generator No. 1 runs as a motor, the current in the series field is reversed and the machine operates as a differential compound motor with, consequently, an additional decrease in emf and increase in current until the series ampere-turns reverse the polarity of the machine. The result is therefore practically a short-circuit on generator No. 2 and the circuit breakers will open.

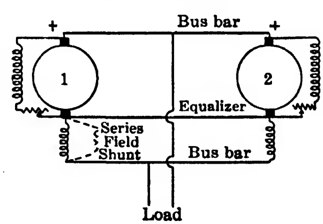



FIG. 17–12. Stable connections of compound generators in parallel. This condition is obtained by the use of an "equalizer" connecting the series fields in parallel.

Compound generators **can** be made to operate in parallel under stable conditions, if the series fields are all connected in parallel by means of a low resistance connection called an "**equalizer,**" as shown in Fig. 17–12. The load current now divides between the several field windings inversely as their resistances; and any tendency of one machine to grab the load results in a division of current in the various series fields, and prevents the emf of one machine from rising, without also increasing that of the others.

To maintain the proper division of load among the several machines as the load on the station varies, it is essential that the external characteristics of all the machines be the same, and that the **resistances of the series-field windings** be **inversely proportional** to the **current ratings** of the several machines.

It was shown in Chapter X that the shape of the characteristic and the over-compounding of a generator can be changed by placing a shunt across the series field. However, when one machine, equipped with a shunt, is placed in parallel with another, this shunt also affects the compounding of the other; for since the series windings are in parallel, the shunt (shown dotted in Fig. 17–12), because of the negligible resistance of the bus bar and equalizer, shunts both series-field windings. The compounding of both machines can be made the same, either by placing a low resistance in series with the series field of the higher-compounded machine, or by driving it at a somewhat lower speed.

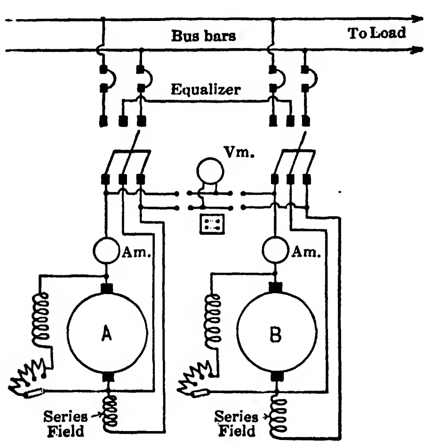



Fig. 18–12. Diagram of connections for operating two compound generators in parallel.

A typical arrangement of connections for the parallel operation of two compound generators is shown in Fig. 18–12. In the figure, the equalizer is connected through one pole of the three-poles switches. Compound generators operated in parallel are connected short shunt, generally with the series field in the negative line. Ammeters to measure the load current of the several machines must be placed in the armature circuit of each machine, as indicated in the figure. When placed in the series-field circuit, the ammeter does not measure the load current, if the equalizer is carrying any current.

To parallel a compound generator, it is brought up to speed and to the same voltage as the bus bars; and the circuit breaker and main switch, including the equalizer, are then closed. The desired division of the load is obtained by adjustment of the shunt-field rheostat, exactly as in the case of shunt generators.

When three-wire generators having two series fields are operated in parallel, Fig. 19–12, they must have two equalizers, and therefore are connected to the bus bars, either with a four-pole switch,

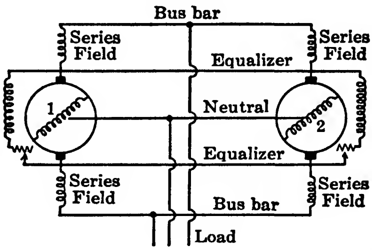



FIG. 19–12. Simple arrangement of two three-wire generators connected in parallel.

or with two two-pole switches. In the latter case, one side of each two-pole switch is connected in an equalizer circuit. Otherwise, the operation of paralleling is the same as for the standard compound generator.

## SUMMARY OF CHAPTER XII

**COMMERCIAL EFFICIENCY of a motor or a generator is the ratio of the power output to the power input.**

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

**Both output and input must be expressed in the same units. Input to a motor is measured in watts and output in horsepower. To compute the efficiency, it is necessary either to reduce watts input to horsepower, or horsepower output to watts.**

**Low-efficiency generators and motors require more power for a given output than do high-efficiency machines. The cost of this additional power for large machines is high. Therefore, it is IMPORTANT THAT LARGE MACHINES HAVE HIGH EFFICIENCY.**

**For very small motors, high efficiency is not important because the cost of additional power is not a large item.**

**BRAKE-HORSEPOWER (MEASURED EFFICIENCY) of a motor can be obtained by equipping it with some form of PRONY BRAKE. The output is measured by obtaining the NET pull (or weight) transmitted by the rotating pulley and the speed, and substituting in the equation for mechanical horsepower. The product of the current input and the terminal voltage, as read from a voltmeter and an ammeter, gives the input.**

**It is difficult to measure the mechanical input to a generator; so**

efficiency of generators is usually obtained by the method of determining the losses.

Approximate Efficiency of a motor may be determined from the NAME-PLATE RATING, which gives the rated voltage, current and horsepower output

$$\frac{\text{HP} \times 746}{E_l I_l} = \text{efficiency.}$$

LOSSES IN A MOTOR (OR GENERATOR) are — (1) The electrical or copper ($I^2R$) losses in the armature, shunt and series fields, etc., which can readily be separated and measured, or calculated. (2) Mechanical losses, or losses due to the rotation of the armature, which consist of:

Hysteresis..................varies with speed and approximately with $B^{1.6}$;
Eddy currents..............vary with speed$^2$ and $B^2$;
Bearing and brush friction...vary with speed;
Windage (air friction)......varies with speed$^3$.

These losses cannot easily be separated or measured, but the combination or total of these losses can be measured and is called the Stray-Power Loss. Stray-power losses are a function of the speed and the flux, and are independent of the load.

In a constant-speed motor, stray-power and shunt-field copper ($I^2R$) losses are known as the constant losses. Copper ($I^2R$) losses in the armature and series field are known as the variable losses.

DETERMINATION OF STRAY-POWER LOSS. Stray power is the input to the armature required to run the machine idle as a motor, minus the armature copper loss.

$$\text{Stray power} = E_a I_a - I_a^2 R_a \text{ at no load.}$$

EFFICIENCY BY DETERMINATION OF THE LOSSES. If the losses in a motor or a generator can be determined for any given load, the efficiency of the machine can be calculated. For,

$$\text{output} = \text{input} - \text{losses},$$
$$\text{input} = \text{output} + \text{losses}.$$

For a motor,

$$\text{efficiency} = \frac{\text{input} - \text{losses}}{\text{input}}.$$

For a generator,

$$\text{efficiency} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

Copper losses in the circuits of a machine can be calculated for any load provided the currents in the various windings and the resistances are known.

Stray-power loss for any particular load can be duplicated, if the

**machine is driven idle as a motor at the same speed and flux density at which it operates under that load.**

**Efficiency, calculated from the losses, can be determined more accurately than from the measured efficiency of a brake-horsepower test.**

**ELECTRICAL TRANSMISSION. Is convenient and efficient over long distances.**

**LINE LOSS. Varies inversely as the square of the voltage of transmission. For a given line loss, the size of the wire, either by cross section or by weight, varies inversely as the square of the voltage.**

**PARALLEL OPERATION. Generators, to operate in parallel, must be connected in OPPOSITION to each other — positive terminals connected to positive terminals, and negative to negative.**

**SHUNT GENERATORS. When connected in parallel, attempt to "shirk" the load due to their drooping characteristics and hence are in stable relations with respect to division of the load.**

**COMPOUND GENERATORS, either flat- or over-compounded, when connected in parallel attempt to take the load from each other due to their rising characteristics; and therefore are unstable in their relation to each other. It is necessary to connect the series fields of all machines in parallel by means of a low-resistance "equalizer."**

**SATISFACTORY PARALLEL OPERATION of generators, either shunt or compound, demands that their external characteristics be the same; and that the resistances of the series fields of compound generators be inversely proportional to their capacities.**

## PROBLEMS ON CHAPTER XII

**Prob. 23–12.** A brake-horsepower test of a 115-volt shunt motor is made with a prony brake similar to that in Fig. 3–12. The rope is $\frac{3}{4}$ inch and the brake pulley is 9 inches in diameter. The brake frame with rope weighs 20 pounds. When the motor is loaded to full-load current of 55 amperes, it runs at 1480 rpm, and the scales are balanced at 82 pounds. (*a*) What torque in pound-feet is developed? (*b*) What is the horsepower output? (*c*) What is the efficiency?

**Prob. 24–12.** A motor is rigged with a prony brake similar to that in Fig. 1–12. The reading of balance $B$ is 3.5 pounds and that of balance $A$ is 35.1 pounds when the motor draws 30 amperes at 230 volts. The distance $R$ is 6.5 inches. If the efficiency of the motor at this load is 78 per cent, at what speed does it run?

**Prob. 25–12.** A motor is rigged with a prony brake similar to that in Fig. 2–12, and the distance $R$ is 2 feet, 9 inches. When the motor is at standstill, the balance reads 2.6 pounds. When the motor draws 100 amperes at 120 volts, the speed is 1190 rpm. If the efficiency of the motor at this load and speed is 89 per cent, what is the reading of the balance?

**Prob. 26–12.** The data on the name plate of a motor are as follows: 75 hp, 230 volts, 280 amperes, 1200 rpm. What is the efficiency?

**Prob. 27–12.** A shunt motor at no load takes 5.5 amperes from a 230-volt line, and runs at 1300 rpm. The resistance of the field circuit is 164 ohms. Resistance of armature including brushes is 0.15 ohm. (*a*) What is the stray-power loss? (*b*) What is the counter emf?

**Prob. 28–12.** If the full-load line current of the motor in Prob. 27–12 is 80 amperes, and the flux is assumed to be constant: (*a*) What is the stray-power loss at this load? (*b*) What is the speed?

**Prob. 29–12.** What is the efficiency of the motor of Prob. 28–12 at full load?

**Prob. 30–12.** At what load (line current) will the motor of Prob. 28–12 operate at its maximum efficiency?

**Prob. 31–12.** The data for a certain shunt motor are as follows:

| | |
|---|---|
| Flux per pole = 1,500,000 lines | No. conductors = 400 |
| No. of paths = 2 | Resist. of armature and brushes = 0.1 ohm |
| No. of poles = 6 | Resist. of field circuit = 115 ohms. |

(*a*) If the counter emf at no load is 229.5 volts, at what speed does it run?

(*b*) If the armature current at no load is 5 amperes, what is the impressed voltage and the total no-load current?

(*c*) If the motor takes 80 amperes from the line at full load and the flux is assumed constant, what is the full-load speed?

(*d*) What is the percentage regulation?

(*e*) What is the stray-power loss at full load?

(*f*) What is the efficiency at full load?

(*g*) What is the horsepower output at full load?

(*h*) What is the torque in pound-feet at the pulley?

(*i*) What must be the resistance in a starting box to start this motor with 150 per cent of normal armature current?

**Prob. 32–12.** A series motor operating under a certain load draws 54 amperes from 500-volt mains and runs at 900 rpm. The armature resistance is 0.4 ohm and that of the field is 0.3 ohm. When the motor is run at no load with the field separately excited to 54 amperes, and sufficient voltage impressed on the armature to drive it at 900 rpm, the armature current is 4 amperes. (*a*) What is the stray-power loss? (*b*) What is the efficiency?

**Prob. 33–12.** A 240-volt long-shunt compound motor at full load takes 75 amperes from the line and runs at 1050 rpm. Resistance of shunt-field circuit is 60 ohms. Resistance of armature is 0.04 ohm and that of the series field is 0.02 ohm. When the motor is run idle with both shunt and series fields separately excited by the same current as at full load, and the necessary voltage impressed on the armature to run it at 1050 rpm, the armature current is 8.3 amperes: (*a*) What is the stray-power loss? (*b*) What is the efficiency? (*c*) The horsepower output at full load?

**Prob. 34–12.** A 25-kw, 1400 rpm shunt generator, when delivering full load at 125 volts, requires a field current of 3.5 amperes. Resistance of armature is 0.04 ohm. When the generator is run idle as a motor with the field separately excited to 3.5 amperes and sufficient voltage impressed to drive it at 1400 rpm, the armature takes 11 amperes. (*a*) What is the full-load efficiency? (*b*) The horsepower input?

**Prob. 35–12.** The generator of Prob. 34–12 operating at half load and at 125 volts terminal pressure requires a field current of 2.9 amperes. When run idle as a motor at the same speed and flux, the armature current is 10.5 amperes. What is the efficiency and the horsepower input at half load?

**Prob. 36–12.** A 500-kw generator is to be selected to transmit power 6 miles over a 2-wire line, No. 00 copper wire. Plot a curve showing relation of voltage to efficiency of transmission for above power, ranging from 0 per cent efficiency to 99 per cent. Have at least 12 points about equally spaced along the curve.

**Prob. 37–12.** State what voltage you would select to transmit power in Prob. 36–12. Using three times this voltage, what size wire could be used with the same line loss?

**Prob. 38–12.** Allowing 5 per cent of the power generated to be lost in the line, plot a curve between size of the copper wire (in circular mils) and voltage of transmission in Prob. 36–12. Plot at least 8 points.

**Prob. 39–12.** What size aluminum wire could be used in Prob. 36–12 with the same line loss?

**Prob. 40–12.** According to Kelvin's rule for most economical size of conductor, what size copper wire (B & S) should be used under the following conditions?

Cost of installed copper wire per pound, $0.20;
Cost of generating power, $0.012 per kwhr;
Probable rate of interest, 5 per cent;
Power to be transmitted, 100 kw;
Voltage of transmission, 600 volts;
Copper weighs 555 pounds per cubic foot.

**Prob. 41–12.** Compute size of aluminum wire that should be used under conditions of Prob. 40–12, assuming that aluminum costs twice as much as copper per pound installed. Aluminum weighs 166 pounds per cubic foot.

**Prob. 42–12.** If double the voltage of Prob. 40–12 is used, what size copper conductor is the most economical?

**Prob. 43–12.** Two 100-kw, 240-volt shunt generators, having straight-line characteristics, are to be operated in parallel. The voltage of generator No. 1 drops from 240 volts at no load to 220 volts at full load, and that of generator No. 2, from 240 volts at no

load to 216 volts at full load. The voltage of each machine is adjusted to 240 volts at no load and the machines are paralleled. (*a*) When the total load on the system is 600 amperes, what current does each supply? (*b*) What will be the terminal voltage of each machine? (*c*) How many kilowatts does each supply?

**Prob. 44–12.** Answer the questions of Prob. 43–12 when the total load on the system is 700 amperes.

**Prob. 45–12.** Two 240-volt compound generators, one of 100 kw and the other of 150 kw, are to be operated in parallel. Each has a straight-line characteristic and are over-compounded 8 per cent. (*a*) When the total load on the system amounts to 800 amperes, what current does each deliver? (*b*) What is the terminal voltage of the generators? (*c*) What load in kilowatts does each supply?

**Prob. 46–12.** If the resistance of the series-field winding of the 100-kw generator in Prob. 45–12 is 0.006 ohm what should be the resistance of the series-field winding of the other in order that they may divide the load in proportion to their ratings?

**Prob. 47–12.** Answer the questions of Prob. 45–12, if the total load on the system is 1000 amperes..

**Prob. 48–12.** Describe the operations in the order in which they would be performed when one of a number of parallel shunt generators is to be taken off the line.

# CHAPTER XIII

## ARMATURE WINDINGS

The fundamental principles underlying the standard forms of armature windings are few and simple. There are, also, special variations of these standard forms which call for special treatment. These fundamentals will be taken up in this chapter and their application to present-day usage shown by means of practical examples.

**1. Total Number of Armature Conductors Required.** It has already been shown that the voltage developed in a generator (or motor) depends upon the flux cut per second and the number of conductors connected in series on the armature. This relation has been expressed by Equation (1), Chapter X, as written below:

$$E_{\text{gen.}} = \frac{\Phi P N}{10^8 \times 60} \times \frac{Z}{a} \tag{1}$$

where
$\Phi$ = flux per pole;
$P$ = number of poles;
$N$ = speed in rpm;
$Z$ = number of conductors on the armature;
$a$ = number of paths in the armature.

Thus, for any machine, after the number of poles, their field strength and the speed of the armature have been tentatively decided upon, the desired voltage depends not only upon the total number of conductors on the armature, but also upon the number of parallel paths in the winding. This has a direct bearing on the type of armature winding which may be chosen.

**Example 1.** How many conductors are necessary on a 4-pole generator, having 3,000,000 lines per pole, running at 1700 rpm, and generating 115 volts?

**Solution.**

Before we can determine definitely the number of conductors needed, we must tentatively decide upon how many paths (or what value of $a$) we will use.

Let us assume 4 paths:

Then
$$E = 115,$$
$$\Phi = 3{,}000{,}000,$$
$$N = 1700,$$

and
$$a = 4.$$

$$E = \frac{\Phi PNZ}{10^8 \times 60 \times a}$$

and transposing,
$$Z = \frac{E \times 10^8 \times 60 \times a}{\Phi PN}$$
$$= \frac{115 \times 10^8 \times 60 \times 4}{3{,}000{,}000 \times 1700 \times 4}$$
$$= 135.5 \text{ or } 136 \text{ conductors.}$$

If we decide to use only two paths through the armature, we shall need only

$$Z = \frac{115 \times 10^8 \times 60 \times 2}{3{,}000{,}000 \times 1700 \times 4}$$
$$= 67.75 \text{ or } 68 \text{ conductors.}$$

**Prob. 1–13.** An armature is to be wound for a 125-volt generator having 8 poles and a speed of 800 rpm. The flux per pole is $3.5 \times 10^6$ lines. How many conductors are needed, if it is decided to have 8 parallel paths in the winding?

**Prob. 2–13.** How many armature conductors would be needed in Prob. 1–13, if it is decided to wind the armature with only two paths?

## 2. Types of Armature Windings.

**Open or Closed.** An open winding is one that can be traced through from one open end to another open end. Thus a circuit can be traced in Fig. 1–13 from commutator segment *a* through coils 1 and 3, and end at segment *c*. The two segments *a* and *c* are not connected to any other coils.

In a closed winding, a circuit can be traced from one commutator segment through the complete winding back to the starting point. Thus a circuit can be traced in Fig. 2–13 from commutator segment *a*, through coils 1, 2, 3 and 4, back to segment *a* again. Note that the brushes are set on segments *a* and *c* and divide the winding into two parallel paths.

Closed windings are the only ones used in direct-current practice and will be the only type considered here. Open windings are in common use in alternating-current machines.

**Drum or Ring.** The principal difference between drum and ring windings was pointed out in Chapter X. The ring winding is no longer used because it is expensive to wind, hard to insulate,

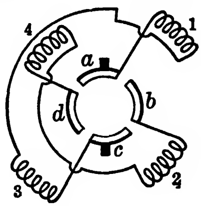



Fig. 1–13. An open armature winding.

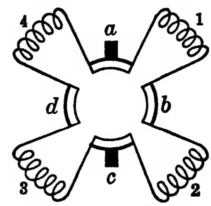



Fig. 2–13. A closed armature winding.

and only a small part of the actual wire wound on the armature is active; that is, **only one side of the coils** act as conductors and cut the magnetic flux.

The drum winding, on the other hand, is wound with coils, both sides of which are placed in slots at the surface of the armature core; and **two sides of the coils are active.** Thus in Fig. 3–13 the two sides *mn* and *op* of the coil are active, cut the flux and are the "conductors." The parts *no*, *mr* and *ps* are called the **end connections.** The sides or edges of each coil should span approximately the distance between the centers of two adjacent poles, so that when a voltage is induced in a direction from *m* to *n* in one conductor, it will be induced in the other from *o* to *p* as shown by the arrows. The various coils on the armature are so connected that their voltages are additive in all the coils in any one path.

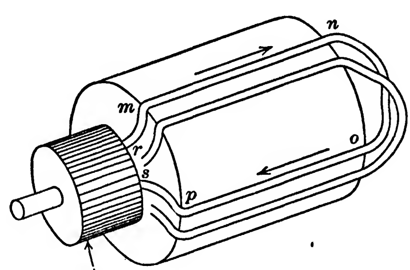



Fig. 3–13. Coils on the surface of a drum armature.

Drum armatures, especially for small machines, may be hand wound or machine wound. Most armatures, however, are usually "form wound." The separate coils are wound by machine with the required number of turns, insulated from each other. They are then wound with cotton, oiled linen, or mica tape and bent into proper shape on a former. The ends of the coils are left bare and tinned so that they may be soldered to the commutator seg-

ments. Figure 4–13 shows a partially wound drum armature and the general appearance of the coils. Coils are wound of copper wire, rectangular strap, or bars, depending somewhat upon the current the machine is to carry.

Armatures may be wound as single-layer or double-layer windings. Most armatures today have double-layer windings.

**Lap and Wave.** A closed winding on a drum armature may be either **lap** wound or **wave** wound. Both lap and wave are the standard types of windings in common use.

The **lap** winding is called a **multiple-circuit** or **parallel** winding, because it has more parallel paths than the wave winding. Large

FIG. 4–13. A partially wound drum armature showing the general shape of the coils; how they are placed in the slots and connected to the commutator. *General Electric Company.*

machines which carry high values of current almost universally have lap-wound armatures.

The **wave** winding is called a **series** or **two-circuit** winding, and does not require as many total conductors to develop the same voltage as does the lap winding. Consequently, it is cheaper to manufacture. Small generators and motors which are not required to carry high values of current are generally wave wound.

**3. Lap Windings.** A lap winding is so called because the sides of successive coils **lap back** over one another. Figure 5*a*–13 shows the typical shape of a lap-wound coil of a single turn. Note that side *a* represents a conductor and corresponds to the coil side *mn* in Fig. 3–13, and the side *a′* corresponds to the coil side *op*. When these coils are put in place on an armature core, they are arranged

as in Fig. 5b–13 with one coil side lapping back over the other. Note that coil $a'$ of coil 1 is connected through the commutator bar to side $b$ of coil 2; and side $b'$ of coil 2 is connected to side $c$ of coil 3. A circuit can thus be traced up through $a$, down $a'$, up $b$, down $b'$, up $c$, etc., through all the coil edges in the winding back to coil edge $a$; and the winding closes on itself. In an armature wound with coils of one turn, the number of conductors is equal to the number of coil edges. Figure 6a–13 shows a lap-wound coil with two turns per coil. It is not different from the coil of Fig. 5a–13 except that each **coil edge** consists of **two** conductors. It can be joined to other lap-wound coils of two turns in exactly the same way that one-turn coils are joined, as shown in Fig. 6b–13. The connections are the same as in Fig. 5b–13. Note that in the two-turn coil there are two conductors per coil edge; or the number of conductors on the armature is twice the number of coil edges.

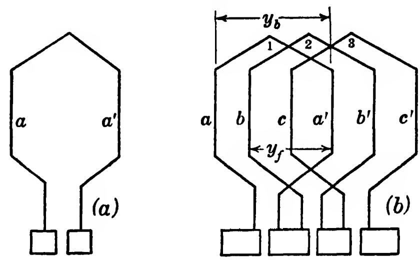



FIG. 5–13. (a) Lap-wound coil of a single turn. (b) Lap-wound coils connected together.

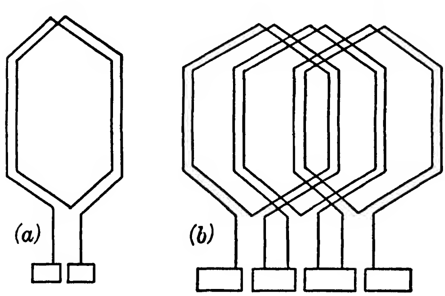



FIG. 6–13. (a) A lap-wound coil of two turns. (b) Lap-wound coils of two turns connected together.

Coils may be wound with several turns, depending upon the number of conductors and commutator segments desired on the armature.

The single coil, whether of one or more turns, is generally called a "winding element." It is that part of a winding connected between two commutator bars. Figures 5a–13 and 6a–13 are winding elements.

Note that, regardless of the number of turns per coil, all winding diagrams are generally shown as one-turn coils for the sake of clearness.

**4. Coil Pitch or Winding Pitch.** Coil pitch is the coil "span." It may be measured as a fraction of the distance between the cen-

ters of adjacent poles, in slots, or in **coil edges.** In this chapter it will be measured in **coil edges** (or **winding spaces**). This method of measurement is applicable to both single-layer and double-layer windings.

The **front pitch** is the distance between any two coil edges connected to the same commutator segment, measured in coil edges at the **front or commutator end** of the armature; and is shown as $y_f$ in Fig. 5b–13.

The **back pitch** is the distance between the two sides of any coil, measured in coil edges. It is also called the "spread" or "throw" of the coil, shown as $y_b$ in Fig. 5b–13. A coil, with a back pitch of 9, spans nine coil edges.

**Example 2.** Consider carefully Fig. 7–13, which is a diagram of part of the development of a lap-wound armature. In such a diagram, the winding is shown as though it had been split axially and peeled off the armature core and laid flat. The coil edges are numbered for convenience, and, of course, half the numbers are even and half odd. In tracing through the circuit, note that the coil shown in heavy lines is attached to commutator segment *a*, and we go out on coil edge numbered 1 and call it an "outgoing" edge. Now if the coil spans 9 coil edges, the other side of the coil, or "incoming" edge, must be that one numbered 10, and it is attached to commutator segment *b*. Note that we went **out** on coil edge 1, and came back on coil edge 10. The **back pitch** is thus 10 − 1 or 9 coil edges, or winding spaces.

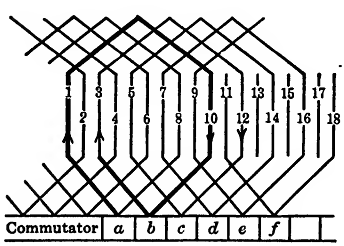



FIG. 7–13. Lap-wound coils connected to a commutator.

To the same segment *b* is also attached the "outgoing" side or edge of the next lapped coil. Note that the edge is numbered 3 and not 2. Coil edge 2 is an **incoming** edge and is part of another coil to the left. Since the incoming edge 10 and the outgoing edge 3 are attached to the same commutator segment, the **front pitch** is 10 − 3 or 7 coil edges.

Tracing on through the next coil, we go out on 3 and come back on 12; so this coil also has a back pitch of 9, and comes back to the next commutator segment *c*. From there, we go out on coil edge 5, the front pitch again being 12 − 5 or 7.

Thus we continue through the winding, the **back pitch** of every coil being 9 and the **front pitch** 7.

A partial diagram of a two-layer winding, with the same pitches as in Fig. 7–13, is shown in Fig. 8–13. Note that there are **two** coil edges or winding spaces per slot, one above the other; and

that those at the top of the slots are given odd numbers and are **outgoing** edges, while those at the bottom of the slots are given even numbers and are **incoming** edges. As in Fig. 7–13, from commutator segment *a*, we go out on edge 1 (at the top of the slot), back on edge 10 (at the bottom of a slot), to commutator segment *b* — a back pitch of 9.

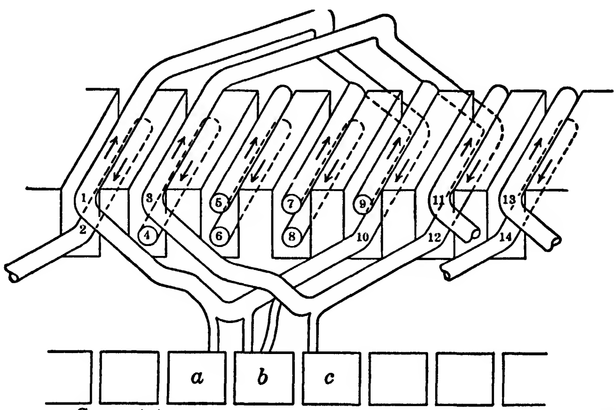



FIG. 8–13. A two-layer diagram similar to that of Fig. 7–13, showing the coil edges in slots.

To the same segment *b* is also attached the outgoing edge of the next coil in the **top** of the next slot, numbered 3—a front pitch of 7. Note that coil edges 2 and 4 are the incoming edges of two other coils to the left.

Again, tracing on through the next coil we go out on 3, and back on 12 at the bottom of the slot to commutator segment *c*, and so on through the winding.

Figure 9–13 shows the complete development of the winding, of which Figs. 7–13 and 8–13 are only a part.

**Note:** (1) That the back pitch of all the coils is the same and the front pitch of all the coils is the same.

(2) That starting anywhere in the winding and tracing through it, we come back to the starting point or the winding closes on itself.

(3) That in a single-layer winding, there is one coil edge per slot; while in a two-layer winding, there are always at least two coil edges per slot, and these edges are parts of different coils.

(4) That all the outgoing coil edges are **odd** numbered and all the incoming edges are **even** numbered, as shown by the arrows in Figs. 7–13 and 8–13. All the outgoing edges might have been

even numbered and all the incoming odd; but part of the outgoing cannot be even and part odd, because **one set** of numbers must be used to **go out** on, and the others saved to **come in** on. Thus if we had tried to go out on 1 and back on 9, we could not make a symmetrical winding that closed; because some of the armature slots in which we wished to put coil edges would already be filled. It is usually customary to number all outgoing coil edges **odd.** This leaves the incoming **even.**

(5) That since the pitch is always the difference between the number of an outgoing and an incoming edge, it is always the

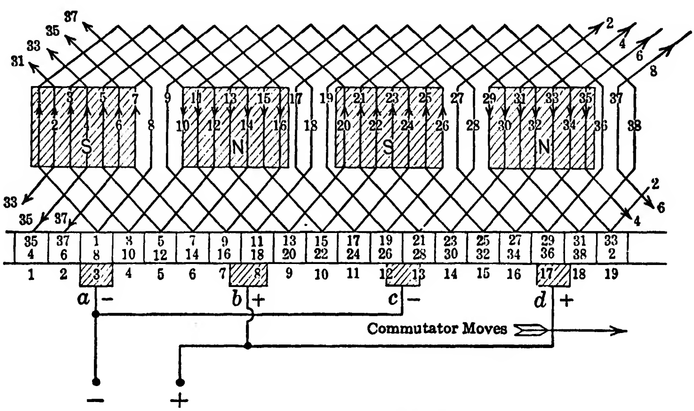


FIG. 9–13. A lap-wound armature showing the location of the poles and brushes.

**difference** between an **odd** and an **even** number. The difference between an odd and an even number is always odd. Therefore, **the coil pitch (either front or back) must always be odd.** There is nothing mysterious about this. In our system of numbers, the difference between any odd and any even number is always odd. So in determining the winding pitch, the first thing to remember is that **it must always be odd.**

**5. Determination of Coil Pitch. Simplex Lap Winding.** Consider Fig. 10–13, which is a circular diagram of an elementary single-layer four-pole lap winding with 20 coil edges (showing one turn per coil) viewed from the front or commutator end. The ends of the coil edges are shown as circles. The slots in the armature core are omitted for the sake of clearness. The dotted lines represent the end connections at the back, or pulley end, of the armature. Also, for the sake of clearness, although the brushes

actually bear on the outside surface of the commutator, they are here shown on the inside.

From Fig. 3–13, we have seen that one side of a coil must be under a north pole, while the other is under a south pole, in order that the voltage induced in the two sides of a single coil may be in the same direction in the coil. Note in Fig. 10–13, that outgoing edge number 1 is under the center of a north pole, while the incoming edge 6 is under the center of a south pole; and that the induced voltage in 1 is **in** (clockwise rotation), while that in 6 is **out**. Thus the voltage between commutator segments $a$ and $b$ is the sum of

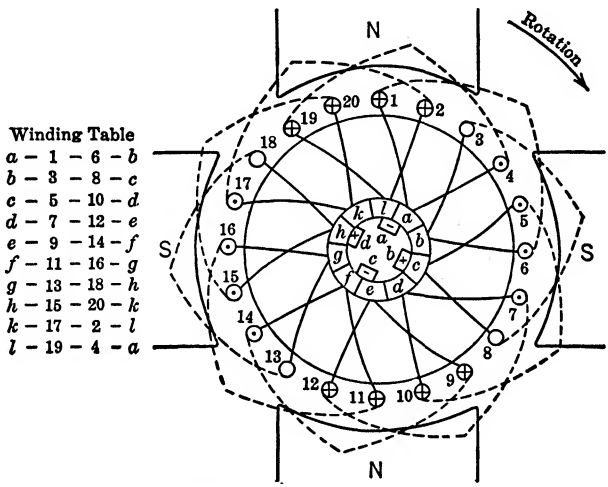



FIG. 10–13. A circular diagram of an elementary four-pole lap-winding.

the induced voltages in coil sides 1 and 6. It is seen that the back pitch is 5. If the coil were made with too small a back pitch, at certain instants both sides of the coil might be under the same pole, and the voltages induced at this instant would tend to neutralize each other. Note, also, that coil edge 6 is connected through commutator segment $b$ to edge 3 (not 2), and that the front pitch is 3.

It is readily seen that in order for the two sides of a coil to be under the center of adjacent poles in this four-pole machine, it must span one-quarter of the coil edges on the armature. Therefore, to determine the back pitch, $y_b$, we divide the total number of coil edges by four, or by the number of poles, and have $\frac{20}{4}$ or 5. Since this is an odd number, we take 5 as the back pitch.

Now the front pitch, $y_f$, as we have seen, is 3 and it differs from

the back pitch by 2. In tracing through this winding it is readily seen that it closes on itself.

Since the back pitch in this winding is just equal to one-quarter of the winding spaces, or to the "pole pitch," it is called **a full-pitch** winding. In most windings the number of coil edges, or slots, is such that the two sides of a coil cannot both be under the exact center of a pole at the same time.

Consider now the developed winding of Fig. 9–13. The same winding is also shown, Fig. 11–13, in a circular diagram, as a two-

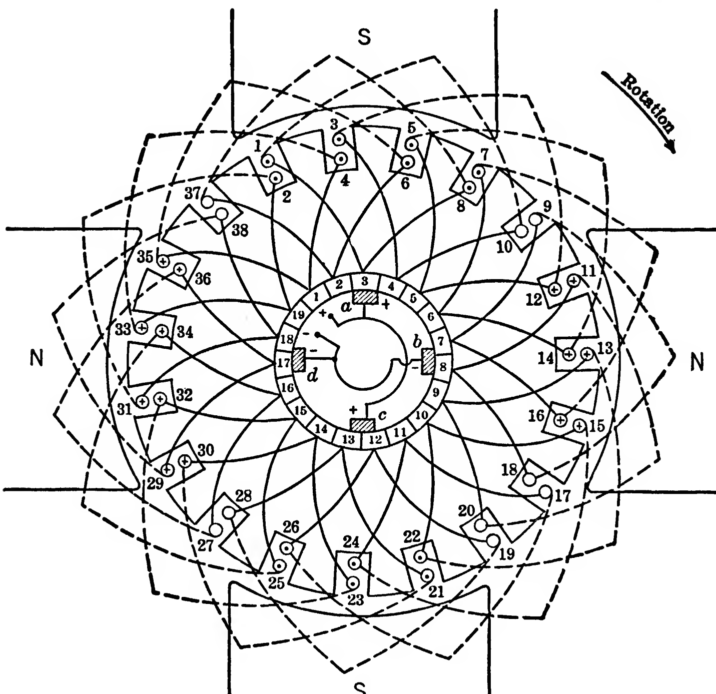



FIG. 11–13. A circular diagram of the same lap winding as that of Fig. 9–11, with opposite rotation. Shown as a two-layer winding.

layer winding. This is a four-pole winding of 38 coil edges. For the two sides of a coil at all times to be in exactly similar positions with regard to adjacent poles, it would require a back pitch of $\frac{38}{4}$ or $9\frac{1}{2}$. We cannot use a pitch of $9\frac{1}{2}$, but must use some odd number near $9\frac{1}{2}$. The nearer the pitch is to $9\frac{1}{2}$ the better the induced voltages will add up in all the coils. We could choose either 9 or 11 as a back pitch; but should select 9 because it would

save copper, and would produce better commutation. This winding would be called a "short-chord" or "fractional-pitch" winding. Most armatures have fractional-pitch windings.

The rule for back pitch then may be expressed by the equation:

$$y_b = \frac{C}{p} \text{ **approximately** (smaller, rather than greater)} \qquad (2)$$

where $y_b$ = back pitch or span of the coil,
$C$ = number of coil edges,
$p$ = number of poles.

Note that in Figs. 9–13 and 11–13 the front pitch is 7 or two fewer than the back pitch of 9. Starting with any "outgoing" coil edge the winding proceeds to the right, or in a clockwise direction around the armature and is therefore called a **progressive** winding. If 11 had been selected for the front pitch, the winding would have proceeded to the left, or in an anti-clockwise direction and would thus be called a **retrogressive** winding.

Also, if the front pitch in Fig. 10–13 had been taken as 7, instead of 3, this winding could have been shown as a retrogressive winding.

In any **simplex** lap winding (e.g., in a single winding), the **front pitch** is always **two fewer** or **two more** than the back pitch. That is

$$y_f = y_b \pm 2 \qquad (3)$$

The (−) sign indicates a **progressive** winding; the (+) sign indicates a **retrogressive** winding.

The rules, then, for a **simplex lap-wound armature** are

First. **Both front and back pitches must be odd and differ by 2.**

Second. **The back pitch must be approximately equal to** $\frac{C}{p}$.

Note that in the above diagrams there are half as many commutator segments as coil edges, or the same number as there are coils. This must be so because each segment has two coil edges attached to it. The winding of Fig. 10–13 has 20 coil edges and therefore 10 coils and 10 commutator segments — both even numbers. The winding of Figs. 9–13 and 11–13 has 38 coil edges, and therefore 19 coils and 19 commutator segments — both odd numbers.

Therefore, in a simplex lap winding, the number of **coils** and **commutator segments** may be either **odd** or **even.**

The **poles** of a machine usually cover from 60 to 80 per cent of the surface of the armature and are symmetrically placed with respect to the winding. In Fig. 9–13, each pole face covers the equivalent of $7\frac{1}{2}$ coil edges. The four poles cover $4 \times 7\frac{1}{2}$ or 30 coil edges. The pole faces then cover $\frac{30}{38}$ or 79 per cent of the coil edges. In the circular diagram of Fig. 11–13, it is simpler to place the poles symmetrically with respect to the slots.

The **brushes** must be equally spaced around the commutator. In Fig. 9–13, since there are 19 commutator segments, the brushes must be placed $\frac{19}{4}$ or $4\frac{3}{4}$ segments apart. They must also be placed so that when they are short-circuiting coils, these coils are as nearly as possible in a neutral position; that is, where both sides of the coils are cutting as little flux as possible. Note in Figs. 9–13 and 10–13 that the induced voltages in the windings are from brushes *a* to *b* and from *c* to *d*; therefore, brushes *b* and *d* are positive, while *a* and *c* are negative.

**Note.** In practice, a four-pole winding would have a considerably larger number of coil edges than have been shown. In the diagram for a commercial armature, the exact position of the brushes would be more readily apparent. Diagrams of elementary windings are shown for the sake of simplicity.

**Prob. 3–13.** Draw the diagram of a 4-pole lap winding similar to that of Fig. 9–13 or Fig. 11–13 for an armature having 38 coil edges. **Back** pitch in this case is to be **9** and **front** pitch **11.** The poles cover the same amount of the armature as in Fig. 9–11. Place the brushes in the proper position.

**Prob. 4–13.** An armature for a 2-pole machine is to have 28 coil edges and be a simplex lap winding. The poles cover 75 per cent of the surface of the armature. Draw a diagram of the winding and place the brushes.

**Prob. 5–13.** The armature of a certain 6-pole generator has 62 coil edges. The poles cover 70 per cent of the armature surface. Draw a diagram of the winding which is to be a simplex lap. Place the brushes.

**Prob. 6–13.** Specify the front and back pitches and the number of commutator segments in a 6-pole simplex lap winding having 162 coil edges. How many coils would be used? If the winding were two layer, how many slots would the armature core have?

**6. Number of Paths through a Lap-Wound Armature.** In a simplex lap-wound armature, there are as many armature paths as there are brushes. Since for all lap windings there are always as many brushes as there are poles, we can say that there are as

many paths through a simplex lap winding as there are poles on the machine.

Note in Fig. 9–13, which is the development of a simplex lap-wound armature for a four-pole machine, that, starting with negative brush *a*, we have two paths through the winding to the positive brushes. Starting with negative brush *c*, we have two parallel paths from this brush to the positive brushes. Since the two negative brushes are in parallel, the two paths from brush *a* and the two from brush *c* must be in parallel. Thus we have four parallel paths through the armature. The four parallel paths are as follows:

| From Negative Brush *a* | | From Negative Brush *c* | |
|---|---|---|---|
| Path 1 | Path 2 | Path 3 | Path 4 |
| Edge 1 | Edge 8 | Edge 26 | Edge 21 |
| 10 | 37 | 17 | 30 |
| 3 | 6 | 24 | 23 |
| 12 | 35 | 15 | 32 |
| 5 | 4 | 22 | 25 |
| 14 | 33 | 13 | 34 |
| 7 | 2 | 20 | 27 |
| 16 | 31 | 11 | 36 |
| + Brush *b* | + Brush *d* | + Brush *b* | + Brush *d* |

Note that in addition to these four paths running from negative to positive brushes, there are some short-circuit paths through the armature; that is, paths which start and end at the same brush. Thus we have from brush *c* a path through edges 19 and 28 back to brush *c* again. Of course, these short-circuit paths, out from and back to the same brush, do not contribute current to the outside and are not counted as paths through the armature.

It is also seen that in the lap winding, the induced voltages add up in any one path in very much the same manner as in the ring armature. A comparison of the two windings in Fig. 12–13 makes this plain. The coil edges in Fig. 12*b*–13 are shown as radial lines.

**Prob. 7–13.** A 4-pole simplex lap-wound 115-volt generator has a flux per pole of $8.35 \times 10^6$ lines. The speed is 1800 rpm. Draw a winding diagram, using coils of a single turn, as in Fig. 9–13 or Fig. 11–13. Place the brushes.

**Prob. 8–13.** A machine is to generate 240 volts. It has 4 poles with a flux of $1.2 \times 10^6$ lines per pole. The speed is 1500 rpm. Specify front and back pitches and number of commutator segments, if the armature is to be simplex lap wound, one turn per coil.

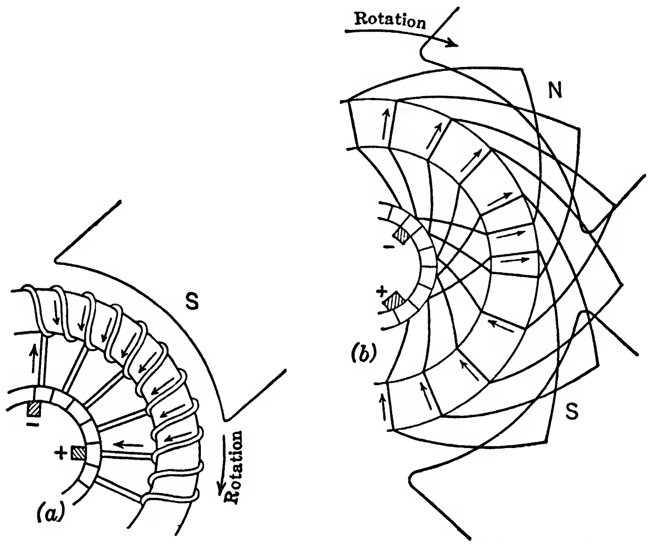



FIG. 12–13. The induced voltages in the conductors of a single armature path are in the same direction in the circuit; (a) Ring winding; (b) Lap-wound armature.

**Prob. 9–13.** If the machine of Prob. 8–13 is to run at 1800 rpm, specify what front and back pitches would be used in a simplex lap winding, one turn per coil.

**7. Number of Turns per Coil.** In the previous articles on armature winding, we have considered but one turn per coil. It is always desirable to use but one turn per coil wherever possible and rarely more than two turns. But if the number of coils (and consequently the number of commutator segments) will be excessive if only one turn is used, two or more may be employed.

**Example 3.** An armature is to have 864 conductors, and be used in a 4-pole frame. What front and back pitches would be used for a simplex lap winding?

**Solution.** If one turn per coil is to be used, there would be the same number of coil edges as conductors and the back pitch must approximately equal $\frac{864}{4}$ or 216.

That is, we could use 215 for **back** pitch and 213 for **front** pitch and have $\frac{864}{2} = 432$ commutator segments.

These long pitches would result in a great deal of wasted wire and a commutator of 432 segments would be very expensive. If we use 2 turns per coil, each coil edge would contain 2 conductors. The coil edges would then be only $\frac{864}{2} = 432$, and the pitches approximately $\frac{432}{4} = 108$. This would mean back pitch of 107 and front pitch of 105, and $\frac{432}{2} = 216$ commutator segments.

If we use 3 turns per coil, there would be $\frac{864}{3} = 288$ coil edges, and the pitches would be approximately $\frac{288}{4} = 72$. This would mean a back pitch of 71 and a front pitch of 69, with $\frac{288}{2} = 144$ commutator segments. This would be a reasonable winding, though we might use 4 turns per coil, and get still smaller pitches, and a smaller number of commutator segments. We should probably have difficulties in commutation which would offset the advantages gained by the use of a smaller pitch and commutator.

The winding of an armature with coils of more than one turn is shown in Fig. 6*b*–13. The winding would look exactly like that of Figs. 9–13 and 11–13, except that each coil edge will represent two conductors. Of course, the pitches would be 71 and 69 instead of 9 and 7.

**Prob. 10–13.** Make a new specification of the back and front pitches in the armature winding in Prob. 8, using more than one turn per coil.

**Prob. 11–13.** What will be the number of commutator segments and front and back pitches in the winding of Prob. 9–13, if 3 turns per coil are used?

**8. More than Two Coil Sides per Slot.** In large machines, the armature is often wound with more than two coil sides per slot such as 4, 6 or 8, etc. This is shown diagrammatically in Fig. 13–13. By this arrangement, the number of slots is reduced and the cross section at the base of the teeth on the core is larger. In such a winding, in order that all the coils may have the same shape, care must be taken to select such a pitch that all the coils span the same number of slots. For instance, if the corresponding edges of two coils lie in the top of slot No. 1, Fig. 13*b*–13, the other edges of these two coils must both lie in the bottom of another slot — slot No. 4 in the figure.

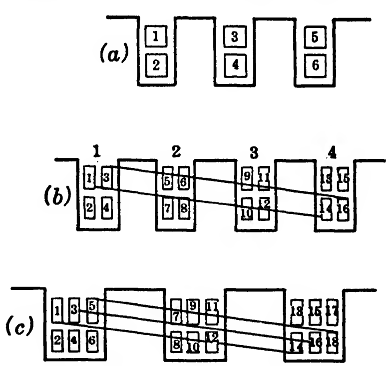



Fig. 13–13. The method of placing and numbering coil edges in the slots of an armature core.

The coils will then all be of the same shape and size. Each

group of coils for the same slot can be taped together and the whole has the appearance of one coil. Such a coil is called a double or "poly" coil, and is connected to the commutator, as indicated in Fig. 14–13.

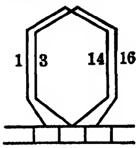


FIG. 14–13. A double or poly-coil. Coil edges 1 and 3 of Fig. 13b–13 lie in the top of one slot, while coil edges 14 and 16 lie in the bottom of another slot.

**9. Multiplex Lap Windings.** Many windings in practice consist of two, three, four or more simplex windings wound in parallel on the armature core. The windings are then called duplex, triplex and quadruplex, respectively. This is done to increase the current capacity of a machine without using larger copper conductors. This arrangement also helps to eliminate the slight variation in voltage, or "ripple," that occurs when a coil in one winding is short-circuited by a brush.

**Example 4.** Assume that we desire to wind a duplex 4-pole lap winding with 60 coil edges.

We determine the back pitch just as we did for a simplex winding. It would be $\frac{60}{4}$ or 15, an odd number. Now, if we were laying out a simplex winding, we would make the front pitch 13, or 2 less than the back pitch. But, in a duplex winding, we must leave additional winding spaces for another circuit through the winding; and therefore must leave an odd number for the other winding to go out on. So we make the front pitch 11 or **4 less** than the back pitch.

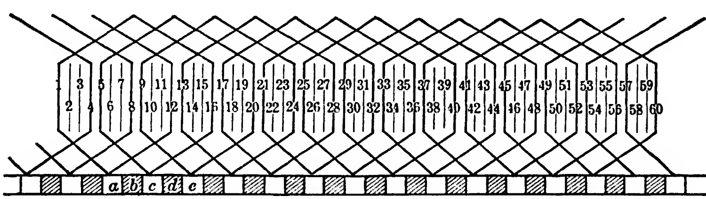


FIG. 15–13. The development of one winding of a duplex lap-wound armature.

Thus, in Fig. 15–13, we start from commutator segment *a*, go out on 1, span 15 coil edges and come back on 16 to commutator segment *c* (not *b*, for we must leave a segment for the other winding); go out on 5 (not 3), span 15 coil edges and come back on 20 to commutator segment *e*; then out again on coil edge 9 and so on.

Figure 16–13 is a two-layer circular diagram of the same winding. Only one-half of the complete winding is shown in each figure and there are enough winding spaces left in which to put another simplex winding having the same front and back pitches.

Note also that every other commutator segment is left vacant. The two windings are entirely separate and insulated from each other, and are tied together in parallel by the brushes.

The two circuits are clearly shown in the following winding tables.

| Winding Table for First Circuit as Drawn in Figs. 15–13 and 16–13 | | Winding Table for Second Circuit which can be Added to Figs. 15–13 and 16–13 | |
|---|---|---|---|
| 15 back, 11 front | | 15 back, 11 front | |
| 1 → | 16 | 3 → | 18 |
| ↙ | | ↙ | |
| 5 → | 20 | 7 → | 22 |
| 9 | 24 | 11 | 26 |
| 13 | 28 | 15 | 30 |
| 17 | 32 | 19 | 34 |
| 21 | 36 | 23 | 38 |
| 25 | 40 | 27 | 42 |
| 29 | 44 | 31 | 46 |
| 33 | 48 | 35 | 50 |
| 37 | 52 | 39 | 54 |
| 41 | 56 | 43 | 58 |
| 45 | 60 | 47 | 2 |
| 49 | 4 | 51 | 6 |
| 53 | 8 | 55 | 10 |
| 57 | 12 | 59 | 14 |
| 1 | 16 | 3 | 18 |
| 5 | 20 | 7 | 22 |
| | | 11 | 26 |
| etc. | | etc. | |
| repeating | | repeating | |

Note that no numbers in the second table appear in the first table, thus making it possible for two distinct windings in the same armature.

If we wish to use a triplex winding, we compute the back pitch just as in a simplex or a duplex.

$$\frac{60}{4} = 15.$$

A simplex and duplex would have a back pitch of 15.

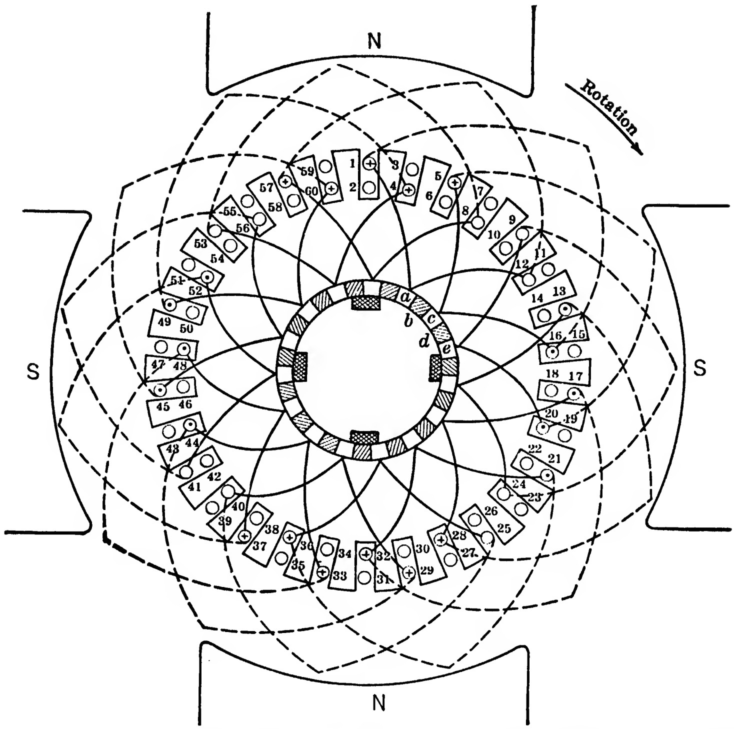



FIG. 16–13. Circular two-layer diagram of the four-pole duplex lap-winding of Fig. 15–13.

A simplex would have a front pitch of 2 fewer or 13. A duplex would have a front pitch of 4 fewer or 11.

A **triplex** would have a front pitch of 6 fewer or 9.

Rule for finding multiplex lap-winding pitches.

$$\text{Back pitch } y_b = \frac{C}{p} \text{ (approx.) and is } \textbf{odd.}$$

| | | |
|---|---|---|
| Front pitch for | simplex | = Back pitch − 2 |
| for | duplex | = Back pitch − 4 |
| for | triplex | = Back pitch − 6 |
| for | quadruplex | = Back pitch − 8 |
| | etc. | |

This may be stated as follows:

The **back pitch** is odd and approximately equal to the number of coil edges divided by the number of poles.

The **front pitch** is odd and is equal to the back pitch minus (or plus) twice the degree of multiplicity.

The brushes with a multiplex winding may cover as many segments as the degree of multiplicity. Thus with a duplex winding, the brushes are two segments wide; with a triplex, usually three segments, etc. They are equally spaced around the commutator and placed, as with a simplex winding, in such positions as to short-circuit coils which are cutting but few lines of force.

It is readily seen that in a duplex lap winding, there are two paths in parallel between each pair of brushes; and there would be 8 paths in parallel in a four-pole armature, or twice as many as in a four-pole simplex winding. In a six-pole duplex winding, there would be 12 paths in parallel, etc.

**Prob. 12–13.** Complete Fig. 15–13 or Fig. 16–13, putting in both circuits, poles and the brushes. Indicate polarity of brushes; that is, mark them plus or minus. How many paths are there through the armature from negative to positive brushes?

**Prob. 13–13.** Draw a diagram of a triplex lap winding for an armature, having 66 coil edges, to be used in a 4-pole frame. Put in poles and brushes. Indicate polarity of brushes. Tabulate the paths through the armature from negative to positive brushes.

**Prob. 14–13.** Draw a diagram of a duplex lap winding for the armature of Prob. 13–13, putting in poles and brushes and marking the polarity of the brushes.

**Prob. 15–13.** A generator is to induce 115 volts at 1800 rpm. There are 6 poles and $3.4 \times 10^6$ lines per pole. The armature is to have a duplex lap winding.

(*a*) How many conductors are needed?
(*b*) What back and front pitches would you use, with one turn per coil?
(*c*) What would be the back and front pitch if 2 turns per coil are used?
(*d*) How many commutator segments are needed in (*b*) and in (*c*)?
(*e*) How many paths through the armature in (*b*) and in (*c*)?

**Prob. 16–13.** (*a*) How many conductors would be needed for the generator of Prob. 15–13, if the armature is to be triplex lap wound?
(*b*) What is the back pitch?
(*c*) What is the front pitch?
(*d*) How many commutator segments?

**Prob. 17–13.** The resistance of the wire on a duplex lap-wound armature for a 6-pole machine is 4.18 ohms. What is the armature resistance exclusive of brush contact?

**10. Meaning of Reentrancy.** The winding shown in Figs. 15–13 and 16–13 is a **duplex-doubly-reentrant winding**; that is, it con-

sists of two entirely separate windings each of which closes on itself; or the winding as a whole closes **twice** upon itself. In case the number of coils is not a multiple of 2, the winding will close

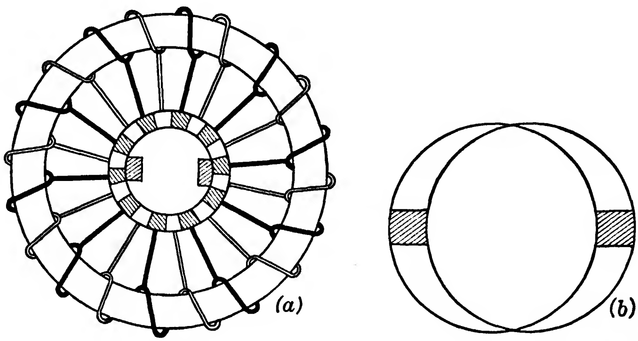



FIG. 17–13. (a) Duplex doubly reentrant ring winding. (b) Diagrammatic representation of a duplex doubly reentrant winding.

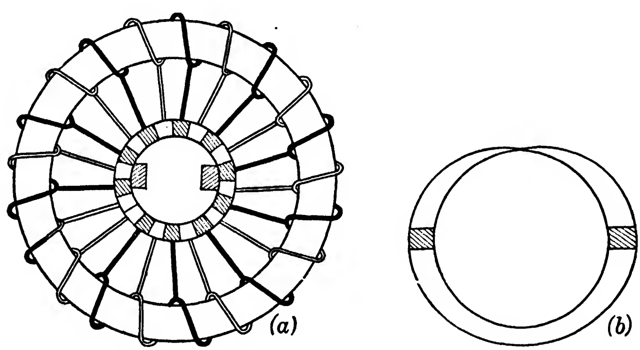



FIG. 18–13. (a) Duplex singly reentrant ring winding. (b) Diagrammatic representation of a duplex singly reentrant winding.

only **once** on itself and it is called a **singly-reentrant-duplex winding.** Similarly, in a triplex winding, if the number of coils is not a multiple of 3, the winding will close only **once** upon itself and is called a **singly-reentrant-triplex winding.** This can be more clearly shown in a ring winding. Figure 17*a*–13 consists of 20 coils and 20 commutator segments (a multiple of 2). There are two entirely separate windings, each of which closes on itself; and the winding is doubly-reentrant-duplex with four paths in parallel between the two brushes. This winding is also shown in diagrammatic form in Fig. 17*b*–13.

Figure 18*a*–13 has 21 coils and 21 commutator segments (not

a multiple of 2), and one winding merges into the other. Tracing through all the coils, it is apparent that the winding closes only once. There are, however, four paths in parallel between brushes; and it is a singly-reentrant-duplex winding. The diagrammatic representation of this winding is shown in Fig. 18*b*–13. All diagrams, Figs. 17–13 and 18–13, are duplex with four paths in parallel between the two brushes.

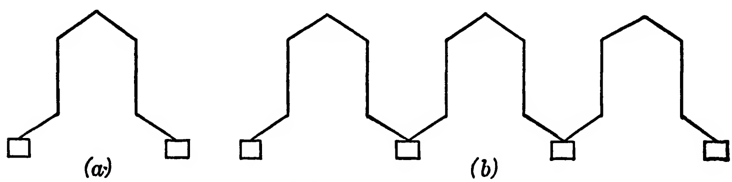



FIG. 19–13. (a) General shape of a wave-wound coil. (b) Wave-wound coils connected.

**11. Simplex Wave Winding.** Figure 19–13 shows the general shape of the coils in a wave-wound armature and the method of connecting these coils together. A wave-wound coil may also, as in the lap winding, have more than one turn per coil. It does not affect the form of the winding, as shown in Fig. 20–13.

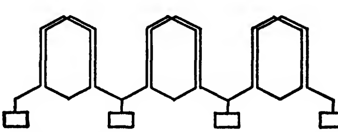


FIG. 20–13. Wave-wound coils of two turns.

Note the wave shape of the winding from which it gets its name.

It has been shown that in a simplex lap winding, a coil edge under one pole is directly connected to a second in a nearly similar position under the next pole. This second coil edge is then connected through the commutator segment **back** again to a **third** coil edge under the **original** pole, but removed two coil edges from the first.

In a **wave** winding, the second coil edge is connected **not back** to a third under the original pole, but **progresses forward** to a third coil edge under a **third pole** of the same polarity as the original pole. This makes no difference in the direction and value of the induced voltage. The arrangement is shown more in detail in Fig. 21–13. The winding is traced around the armature from commutator segment $x$, **out** through coil edge *ab* under a north pole, **in** through coil edge *cd* under a south pole, to commutator segment $v$; **out** on edge *ef* under the **next north** pole, **in** on edge *gh*

under a south pole to commutator segment $z$, next to the first segment $x$; and then to coil edge $a'b'$ under the original pole.

**Coil Pitch.** The distance $bc$ measured in coil edges is the **back pitch,** shown as $y_b$ in the figure. The **front pitch** is the distance $de$, shown as $y_f$. The **total pitch** is the distance $bf$, shown as $y$. The **commutator pitch** is shown as $y_c$ in the figure, and is measured in the number of commutator segments spanned.

For any wave winding, the choice of the pitches and number of coil edges necessary to make the winding close is more restricted than for a lap winding. For a simplex lap winding, both the

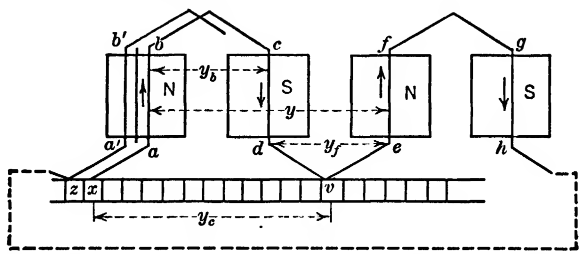



FIG. 21–13. Development of one "tour" around a wave-wound armature showing coil pitches and commutator pitch — a retrogressive winding.

front and back pitches must be odd and $y_f$ must differ from $y_b$ by 2. Similarly, in a wave winding **both pitches must be odd;** but unlike the lap winding, they may **differ from each other by two or more,** or **they may be the same;** that is, $y_f$ may be equal to $y_b$.

In laying out a diagram for a **lap winding,** we first determine the **back pitch** $y_b$, which is merely an approximation of the pole pitch. But in laying out a **wave winding,** it is necessary that we first determine the **average pitch.**

From Fig. 21–13, it is seen that the total pitch is equal to the sum of the front and back pitches, or

$$y = y_f + y_b \tag{4}$$

and the **average pitch** equals

$$y_{av} = \frac{y}{2} = \frac{y_f + y_b}{2}. \tag{5}$$

The **total pitch** is always **even.**

The **average pitch** may be **odd** or **even.**

A wave winding may have a back pitch of 21 and a front pitch of 19, and the average pitch would then be $\frac{21 + 19}{2}$ or 20.

Furthermore, in order that a wave winding may close, the **average pitch** must be a **whole number** and **agree exactly** with the formula

$$y_{av} = \frac{C \pm a}{p} \tag{6}$$

where $C$ = number of coil edges;
$p$ = the number of poles;
$a$ = the number of circuits in the armature or the multiplicity of the winding.

In a simplex wave winding, $a$ **must always be 2,** and the formula then is

$$y_{av} = \frac{C \pm 2}{2}. \tag{7}$$

It is **important to note that the value** $\frac{C \pm 2}{p}$ **must result in a whole number.**

If the average pitch in equation (7) above were $\frac{C}{p}$, after one "tour" around the armature, the winding would come back to coil side $ab$, Fig. 21–13, and close upon itself. But the winding must not close until all the coil edges have been used; therefore the constant 2 must be included so that the winding comes back to **another odd** or outgoing coil edge, two winding spaces removed from the first.

The (+) sign in the above equation indicates a **progressive** and the (−) sign a **retrogressive** winding. Note that this is just the reverse of the meaning of the signs in a lap winding. In Fig. 21–13, after one "tour" around the armature, the winding falls two coil edges short of reaching the starting point, and picks up the first commutator segment **next,** or to the left of the first; so that with successive tours, the winding gradually progresses to the left and is **retrogressive.** While in Fig. 22–13, a circular diagram of a six-pole machine, after one clockwise tour around the armature, the winding falls two coil edges past the starting point and picks up the next commutator segment **past** the first; and with succeeding tours, the winding proceeds to the right in a clockwise direction and is **progressive.**

It is noticed in the above figures that the ends of each coil span several commutator segments. In order that succeeding coil edges may each be connected in orderly manner to the commutator segments, the span of the segments between the ends of each coil must be the same. This is called the **commutator pitch.** Since there are half as many commutator segments as coil edges, the **commutator pitch** is **always equal** to the **average pitch,** or

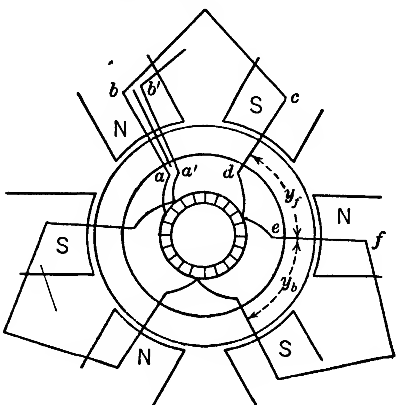



FIG. 22–13. Circular diagram showing one "tour" around the armature of a 6-pole progressive wave-winding.

$$y_{av} = y_c. \qquad (8)$$

**Example 5.** Assume that we desire to lay out a diagram of a simplex wave-wound armature for a 4-pole machine consisting of 36 coil edges.

The average pitch must equal

$$\frac{C \pm 2}{4} = \frac{36 \pm 2}{4} = 8\tfrac{1}{2} \text{ or } 9\tfrac{1}{2};$$

but since this does not result in a **whole number,** the winding will not close; and, therefore, a simplex wave winding for this number of poles cannot be wound with 36 coil edges. In a case like this, we will add two more coil edges and use a winding having 38 coil edges.

If this is done,

$$\frac{C \pm 2}{4} = \frac{38 \pm 2}{4} = 9 \text{ or } 10;$$

so the winding is possible, and will close on itself.

Note that the average pitch may be either **odd** or **even** and that the total pitch is 18 or 20.

From the values above, any of the following combinations would be satisfactory.

| Total Pitch | Average Pitch (= commutator pitch) | Back Pitch | Front Pitch | |
|---|---|---|---|---|
| 18 | 9 | 9 | 9 | Retrogressive |
| 18 | 9 | 7 | 11 | |
| 18 | 9 | 11 | 7 | |
| 20 | 10 | 11 | 9 | Progressive |
| 20 | 10 | 9 | 11 | |

Let is select an average pitch of 10, a progressive winding, and make the back pitch 11 and the front pitch 9. The commutator pitch must be equal to the average pitch, or 10.

Figure 23–13 is the development of the winding using these pitches. Figure 24–13 is a two-layer circular diagram of the same winding as that of Fig. 23–13. From commutator segment 19, we go **out** on coil edge 1, **back** on 12 (a back pitch of 11), to a commutator segment 10 (commutator pitch of 10); then forward and **out** on edge 21 (a front pitch of 9) and **back** on 32 to commutator segment 1 (1 (20) − 10 = 10) and **out** on coil edge 3 (two winding spaces removed from coil edge 1), and so on. We progress through the whole winding in this way and finally come back to coil edge 1.

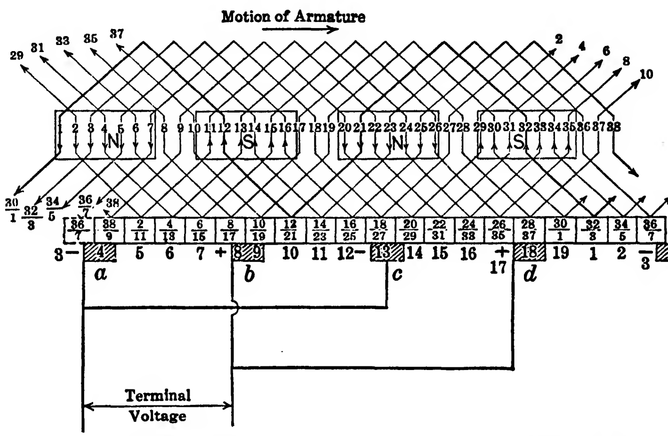



Fig. 23–13. The development of a wave-winding, showing the poles and the location of the brushes.

The table below shows that the winding closes on itself after going through all the coil edges.

1–12–21–32–3–14–23–34–5–16–25–36–7–18–27–38–9–20–29–2
–11–22–31–4–13–24–33–6–15–26–35–8–17–28–37–10–19–30–1

and repeat.*

* For a simple wave winding it can be shown: (1) That if the number of **pairs of poles** is **even,** the number of **coils** and **commutator segments** must be **odd.** The **average pitch** may be **odd or even.** That is, in machines having 4, 8, 12 or 16 poles, the number of coils and commutator segments must be odd.

(2) That when the number of **pairs of poles is odd** — that is, in machines with 6, 10 or 14 poles — the **number of coils and commutator segments may be odd or even.**

(*a*) If the number of coils is **even,** the average pitch must be **odd.**

(*b*) If the number of coils is **odd,** the average pitch must be **even.**

**Prob. 18–13.** Draw the development of a simplex wave-wound armature having 34 coil edges. Armature to be used in a 4-pole frame.

**Prob. 19–13.** A 6-pole generator is to induce 220 volts when running at 900 rpm. The flux per pole is $5.2 \times 10^6$ lines. If a simplex wave winding is to be used, specify:
(*a*) Average pitch.
(*b*) Back pitch.
(*c*) Front pitch.
(*d*) Number of commutator segments.

**Prob. 20–13.** A simplex wave-wound armature having 122 coil edges, 2 turns per coil, is used in a 4-pole generator having $4.3 \times 10^6$ lines per pole. If the speed of the armature is 1500 rpm what voltage is generated?

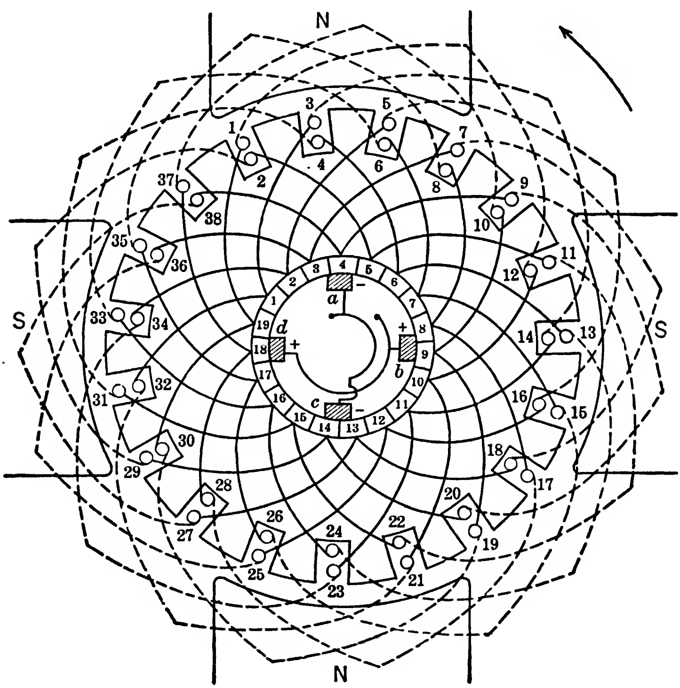



FIG. 24–13. A circular two-layer diagram of the wave-winding of Fig. 23–13.

## 12. Brushes and Armature Paths in Simplex Wave Windings.

As in the lap winding, the poles cover 60 to 80 per cent of the armature, and the brushes must be so placed that they short-circuit coils which are cutting as little flux as possible, as shown in Figs. 23–13 and 24–13.

If the armature were lap wound, four brushes would be needed in a four-pole machine. But in a wave-wound armature, **only two brushes are needed**; although any multiple of two may be used up to the number of poles.

**A simplex wave-wound armature always has only TWO paths. The number of paths remains the same, regardless of the number of brushes.**

Note the paths through the winding in Figs. 23–13 and 24–13. Starting with the negative brush *a*, we have one path as follows:

*a*–36–25–16–5–34–23–14–3–32–21–12–1–30–19–*b*

There are 14 coil edges in this path from negative brush *a* to positive brush *b*.

We have another path from the same brush,

*a*–9–20–29–2–11–22–31–4–13–24–33–6–15–26–35–8–*b*

There are 16 coil edges in this path from negative brush *a* to positive brush *b* but note that the first two edges, that is, edges 9 and 20, are short-circuited by brushes *a* and *c*. The short circuit has the path *a*–9–20–*c*–*a*. So that no emf is added to the terminal voltage by these two edges. Thus there are only 14 active edges in this circuit from negative *a* to positive *b*, which is the same number of edges that are in the other circuit.

There are two other edges coming to brush *a* at this instant; but starting with them and going through the winding, we shall find that they are in short circuits only, and do not form part of other armature paths. In fact, although there are four brushes, it will be found that the two paths traced above are the only ones through the armature. The rest are all short-circuit paths. Thus following out the other leads from the negative brush *a*, we have *a*–7–18–*c*–*a* which forms a short circuit, and *a*–38–27–*c*–*a* which forms another short circuit.

Also we may follow out from brush *d*,

*d*–28–17–*b*–*d*; a short circuit.
*d*–37–10–*b*–*d*; a short circuit.

Brushes *a* and *b* are all that are needed to collect the current from the two circuits through the armature. The other two brushes merely serve to break up the short circuit through the armature into a number of smaller short circuits and thus diminish the sparking.

This is better illustrated in Fig. 25–13, which shows that part of a six-pole wave winding short-circuited by the brush *a*. The three brushes, *a*, *a′*, *a″*, are all of the same polarity, and are connected. They are also directly **connected through one coil** of the winding. Therefore, brushes *a′* and *a″* do not increase the paths in the winding, but merely break up the short circuit from three coils in series to three short circuits of one coil each.

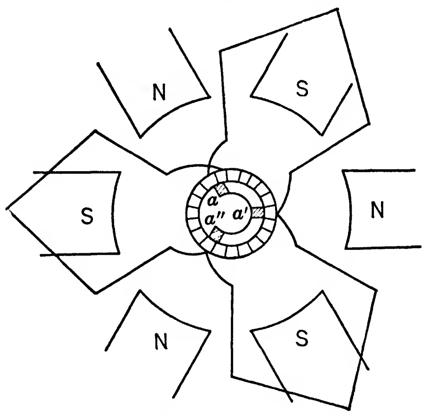


FIG. 25–13. One brush short-circuits three coils in series. The additional brushes merely break up the short-circuited path into one of three coils in parallel.

For this reason, except in the case of small machines of about 6 kw and in railway motors, it is customary to have the same number of brushes as poles. Since railway motors generally have four poles, are totally enclosed and the brushes can be reached only by a hand hole in the cover, only two sets of brushes set at 90 degrees are used.

**Prob. 21–13.** Put the poles and four brushes on the windings of Prob. 18–13 and trace

(*a*) All the paths through the winding from negative to positive brushes.

(*b*) All the short-circuit paths.

**Prob. 22–13.** (*a*) How many paths are there through the armature of Prob. 19–13, if 6 brushes are used?

(*b*) How many paths are there if only 2 brushes are used?

**13. Multiplex Wave Windings.** Wave windings, like lap windings, may be wound in duplex, triplex, quadruplex, etc., combinations. A duplex wave winding is merely a parallel combination of two simplex wave windings; and there would be four parallel paths in it. A triplex winding consists of three simplex windings with six paths, two in each winding. The brushes have to be wide enough to cover a sufficient number of commutator segments to collect the current from all the paths. Thus the brushes for a duplex wave would each cover two segments; for a triplex, three segments, etc.

In using the formula (Equation 6)

$$y_{av} = \frac{C \pm a}{p}$$

**to determine the average pitch,** be sure that $a$ is made

2 for a simplex,
4 for a duplex,
6 for a triplex, etc.

Moreover wave windings may also be wound with different degrees of reentrancy. The number of coils in a doubly reentrant winding must be a multiple of **2**; in a triply reentrant winding the number of coils must be a multiple of **3**. Furthermore, in machines of an **even** number of pairs of poles, the number of coils divided by the multiplicity of the winding must be an **odd** number; while in machines of an odd number of pairs of poles it may be an even number.

**Example 6.** What pitches may be used for a triplex wave winding consisting of 216 coil edges, if used with a 4-pole machine?
**Solution.**

$$\text{Average pitch} = \frac{216 \pm 6}{4} = 55\tfrac{1}{2} \text{ or } 52\tfrac{1}{2}.$$

Since this does not result in a whole number, we will use 218 coil edges.

$$\text{Average pitch} = \frac{218 \pm 6}{4} = 56 \text{ or } 53.$$

The number of coils with 218 coil edges is $\frac{218}{2}$ or 109. Since this is **not** a multiple of 3, this winding is singly reentrant.
We can use

| Back Pitch | Front Pitch | Average Pitch |
|---|---|---|
| 53 | 53 | 53 |
| 51 | 55 | 53 |
| 55 | 51 | 53 |
| 55 | 57 | 56 |
| 57 | 55 | 56 |
| 53 | 59 | 56 |

**Prob. 23–13.** Draw a diagram for a duplex wave winding for an armature to be used in a 4-pole machine. Use 36 coil edges. Place the poles and brushes in proper position. Specify the pitches you use.

**Prob. 24–13.** Trace all the paths in the windings of Prob. 23–13.

**Prob. 25–13.** Draw a duplex wave winding for a 4-pole machine having 64 coil edges. Specify what pitches you use and whether this is a progressive or retrogressive winding. Place the poles and brushes.

**Prob. 26–13.** Draw a diagram of a duplex wave winding for a 6-pole machine having 70 coil edges. Place the poles and brushes in proper position. Specify the pitches used. Can this be a progressive winding?

**14. Forced Windings. "Dummy Coils."** It is often desirable, for particular windings, to make use of armature cores which have already been designed for other windings, and which have more slots than can be filled by the desired coil edges. In this case, a "dummy coil" is used in order to fill all the slots, and make the armature mechanically secure and symmetrical. The ends of the dummy coil are cut off and taped and are not connected to the commutator.

Such a winding is called a **forced winding.**

**Example 7.** Assume it is desired to wind a two-layer, 4-pole simplex wave winding on an armature core having 20 slots.

**Solution.**

This calls for a winding with 40 coil edges, two per slot; or for 20 coils and 20 commutator segments.

From the formula, the average pitch would be $\frac{40 \pm 2}{4} = 9\frac{1}{2}$ or $10\frac{1}{2}$, but since these values are not whole numbers, this winding is impossible. (A 4-pole simplex wave winding cannot be wound with an even number of coils.)

Therefore, a winding of 38 coil edges or 19 coils, **with one dummy coil,** would be used.

The average pitch would now be $\frac{38 \pm 2}{4} = 9$ or $10$.

The number of segments in the commutator would now be 19 instead of 20.

Thus, in the above example, if 10 is taken as the average pitch, the back pitch may be 11, the front pitch 9 and the commutator pitch 10. A simplex wave winding now may be **forced** on the armature core with 20 slots.

A partial development of this winding is shown in Fig. 26–13.

**Prob. 27–13.** Specify the back and front pitches, and arrangement of a wave winding, to be used on an armature having 58 slots. Speed = 1200 rpm. Number of poles = 4. Flux per pole = $3.3 \times 10^6$ lines. Generated voltage = 600 volts. Two coil sides per slot.

**Prob. 28–13.** Specify all desirable pitches for lap and wave windings which would be used on a core having 75 slots, two coil sides per slot.

Number of conductors = 300.
Number of poles = 8.

**Prob. 29–13.** How many slots would be used for the core of the armature of Prob. 19–13, and how would the coil edges be arranged?

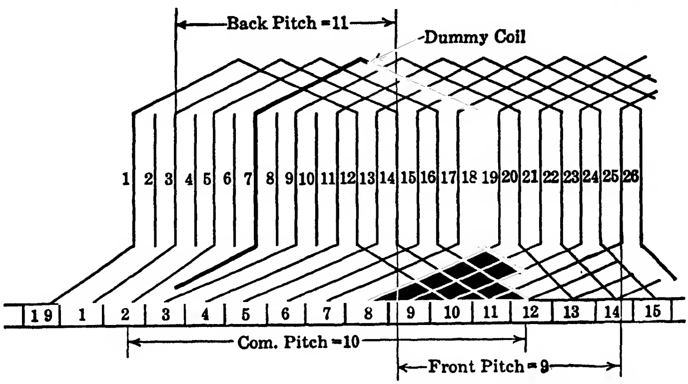



FIG. 26–13. Partial development of the "forced" winding of Example 7, showing one dummy coil. In laying out front pitch do not count either coil edge in any slot containing a dummy coil edge.

**15. Number of Commutating Poles.** It has been shown in Art. 39, Chapter X, that the function of the commutating poles is to set up a commutating flux, and therefore a voltage in the coils, at the instant they are short-circuited by the brushes. This voltage must be sufficient to reverse the current in these coils against the induced emf of self induction.

It was also stated that in large machines, the number of commutating poles is equal to the number of main poles; while in small machines, it might be only one-half the number of main poles.

In Fig. 27–13, the three coils of a six-pole wave winding are short-circuited by the brush *a*. The coil sides *ab*, *cd*, *a'b'*, *c'd'*, etc., are cutting the flux of each of the six commutating poles *N'S'*, *N''S''*, etc. Each of these poles may be of sufficient strength to set up in the two coil sides of each coil the required voltage to reverse the current in the coil. Now it is apparent that if the strength of the poles *S'S''S'''* is doubled, by doubling the turns on these poles, the voltage induced in coil sides *ab*, *a'b'*, *a''b''*, etc., will be doubled; and commutating poles *N'N''N'''*

can be dispensed with, and the same total voltage will be induced in each coil.

By this means, manufacturing costs on small machines may be reduced. It is to be noted that when there are only half as many commutating poles as main poles, the commutating poles are all of the same polarity.

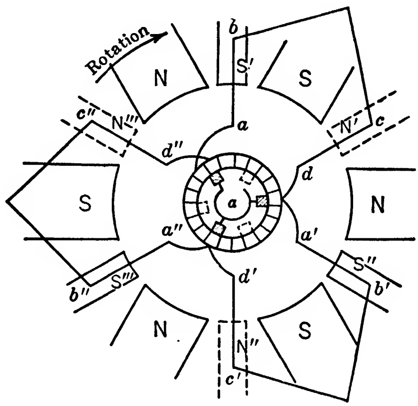



FIG. 27–13. The conductors of the coils short-circuited by the brushes, are cutting the flux set up by the commutating poles N′S′, etc.

## SUMMARY OF CHAPTER XII

**To produce SYMMETRICAL ARMATURE WINDINGS the following rules must be followed.**

**(1) GENERAL.**

**(a) The total number of conductors must fulfill the equation**

$$E = \frac{\Phi ZN}{60 \times 10^8} \times \frac{P}{a}.$$

**(b) There are two coil edges to a coil.**

**(c) The number of coil edges equals the number of conductors divided by the number of turns per coil.**

**(d) The number of commutator segments equals the number of coils.**

**(e) Both the front and back pitches of all kinds of windings must be odd.**

**(2) LAP WINDINGS.**

**(a) The back pitch equals APPROXIMATELY $\frac{C}{p}$.**

**(b) The front pitch is 2 more or fewer than the back pitch in simplex windings, 4 in duplex, 6 in triplex, etc.**

(c) **The number of paths through the winding equals the number of the brushes times the multiplicity of the winding.**

(d) **The number of the brushes equals the number of the poles.**

**(3) WAVE WINDINGS.**

(a) **The average of the front and back pitches must exactly equal $\frac{C \pm a}{p}$, which must be an integer.**

(b) **There may be two brushes or any multiple of two, up to the number of the poles.**

(c) **The number of paths through the armature equals TWO times the multiplicity of the winding, regardless of how many brushes are used.**

**THE NUMBER OF COMMUTATING POLES may be the same as or half the number of main poles.**

**When the number of commutating poles is the same as the main poles, the necessary voltage to reverse the current is induced in both edges of the short-circuited coils. With only half as many commutating poles as main poles, the necessary voltage must be induced in one coil edge only; and hence these poles must have twice the strength used in the former case, and must all be of the same polarity.**

## PROBLEMS ON CHAPTER XIII

**Prob. 30–13.** An 8-pole, 400-volt direct-current generator runs at a constant speed of 800 rpm. It has an effective air-gap flux of $3.38 \times 10^6$ lines per pole. The armature has 56 slots, each capable of holding a maximum of 8 conductors of the size needed to carry the current. There must not be more than 30 volts between adjacent commutator segments. Calculate the correct values for:

(*a*) The total number of conductors.
(*b*) The type of winding.
(*c*) The front and back pitches.
(*d*) A winding table.

**Prob. 31–13.** A 6-pole simplex armature winding has 38 conductors.

(*a*) Show by a winding diagram how these conductors may be connected to form a lap winding, indicating the location and external connections of the brushes.

(*b*) Show by a similar diagram how these conductors may be connected to form a wave winding indicating the location and external connections of the brushes.

(*c*) What will be the relative values of the armature resistances of the lap and wave windings in (*a*) and (*b*)?

(*d*) What will be the relative values of the electromotive forces generated in the two windings (*a*) and (*b*) for the same speed and flux per pole in each case?

(*e*) For the same back electromotive force and the same flux per pole in each case, what will be the relative speeds of the armatures in (*a*) and (*b*) when the machines are used as motors?

(*f*) With the same current per conductor in each case, what will be the relative current outputs of the armatures in (*a*) and (*b*)?

(*g*) What will be the relative $I^2R$ losses in the armatures of (*a*) and (*b*) under conditions of (*f*)?

**Prob. 32–13.** The following data are given for a direct-current generator:

| | |
|---|---|
| Generated voltage | 550 volts. |
| Number of poles | 8. |
| Flux per pole | $2.5 \times 10^6$ lines. |
| Speed | 300 rpm. |

What winding pitches would you recommend

(*a*) if the armature is to be lap wound?
(*b*) if the armature is to be wave wound?

State in each case

1. Front pitch.
2. Back pitch.
3. Number of turns per coil.
4. Number of commutator segments.
5. Number of slots in core punching.
6. Number of dummy coils (if any).

**Prob. 33–13.** The flux per pole of a certain 6-pole generator is $1.2 \times 10^6$ lines at full load. The speed is 900 rpm. The number of armature conductors is limited to 650. The wire with which the armature is to be wound has a maximum current-carrying capacity of 40 amperes. What maximum electric power can be generated, if the armature winding is

(*a*) simplex lap?
(*b*) duplex wave?

**Prob. 34–13.** An armature of a certain 6-pole generator is to be duplex wave wound. It must have approximately 660 conductors. Specify

(*a*) front and back winding pitches that you would recommend.
(*b*) turns per coil.
(*c*) number of commutator segments.

**Prob. 35–13.** An 8-pole, 7.5-kw, 230-volt simplex lap-wound generator has 8 conductors per slot. If this is changed to a wave-wound machine using the same total number of conductors, what will be its voltage, current and power rating?

**Prob. 36–13.** If the same machine of Prob. 35–13 were lap wound with the same weight of armature copper but with 2 conductors per slot, what would be the voltage, current and power rating?

**Prob. 37-13.** Design the simplest correct winding for a motor as follows:

115 volts
10 kw
1200 rpm no-load speed
Flux = $1.11 \times 10^6$ lines per pole
43 slots
4 poles

State in your results:

(*a*) The type of winding.
(*b*) Number of armature conductors.
(*c*) Number of turns per element.
(*d*) The winding pitches.
(*e*) The number of commutator segments.

**Prob. 38-13.** A 550-volt, direct-current motor of 175 horsepower at 1800 rpm is to have an armature core having 144 slots. Other data are as follows:

Number of poles, 4.
Flux per pole may be regulated between $1.5 \times 10^6$ and $1.7 \times 10^6$ lines.
Limiting current per path, 125 amperes.

Find:

(*a*) Type of winding.
(*b*) Number of coils.
(*c*) Front and back pitches.
(*d*) Number of commutator segments.
(*e*) Voltage between commutator segments.

**Prob. 39-13.** It is desired to design a 4-pole, 550-volt railway motor which will operate at 1200 rpm, when the effective flux per pole is $3 \times 10^6$ lines. There is available an armature core having 57 slots.

(*a*) How many conductors are necessary if a wave winding is used?
(*b*) How many coil edges per slot?
(*c*) How many turns per coil?
(*d*) How many commutator segments?
(*e*) State back and front pitches.

**Prob. 40-13.** A 4-pole, duplex lap-wound generator has the following rating: 125 volts, 60 amperes, 900 rpm. There are two turns per coil. This armature is rewound with twice the number of conductors, the same total amount of copper, but with a simplex wave winding with four turns per coil. The speed rating is changed to 1000 rpm. The flux per pole is the same as before.

What will be the current, voltage, and power rating of the machine as rewound?

# CHAPTER XIV

## BATTERIES AND ELECTROCHEMICAL ACTION

By far the greatest source of electrical power is the generator, in which a voltage is induced by the rotation of an armature in a magnetic field. In this machine, mechanical energy is converted into electrical energy.

Another source of electrical power is the electric battery in which a voltage is generated by chemical action. In the battery, chemical energy is converted into electrical energy.

In order to understand the action of a battery, it is necessary to consider the flow of electricity through fluids. In previous chapters, we have considered the flow of electricity in solids or metals. In any metallic conduction, such as in copper or aluminum, we know there are present in the atoms of the material, a great many "free" electrons, that is, electrons which can be easily detached from their particular atoms. A small applied voltage will detach these electrons and set them in motion to produce an electric current. In insulators, the electrons are more securely attached to the atoms of the material and it requires a high electric stress to tear even a few electrons loose and set up an electric current.

The flow of electricity in fluids differs from the flow in solids, in that parts of the fluid actually move and carry the electrons with them and thus produce a current.

**1. Electrolyte, Electrodes, Etc.** When two conductors are immersed in a fluid so that they do not touch each other, and a current is caused to flow through this circuit, as in Fig. 1–14, the two conductors are called the **electrodes**; and the fluid, which must also be a conductor of electricity, is called the **electrolyte.** The electrode, $B$, at which the **current enters the fluid** is called the **positive electrode** or the **anode.** The electrode, $A$, at which the **current leaves the fluid,** is called the **negative electrode** or the **cathode.**

**2. Flow of Electricity in Fluids; Electrons; Ions; Atoms.** Some fluids, like pure water, are such poor conductors that

they may be considered practically as insulators. However, if only a small amount of acid or salt or alkali be added to the liquid, the solution becomes a good conductor; and easily breaks up into its component parts. These component parts are not the atoms of the material, however, for any normal atom contains the same number of protons as electrons and neutrons, and has no electrical charge. Instead, the solution breaks up into what may be termed **deformed atoms, or groups of deformed atoms** of the material, called **ions.** Some of these **ions** have more electrons than the normal atom should have, and thus are negatively charged, and are called **negative ions;** and others have **fewer electrons** than the normal atom should have, and are therefore positively charged, and are called **positive ions.**

For instance, if hydrochloric acid, the molecule of which consists of one atom of hydrogen and one of chlorine (chemical symbol HCl), is diluted with water, the solution breaks up into hydrogen (H) ions, deficient in electrons, and therefore positively charged; and chlorine (Cl) ions having an excess of electrons, and therefore negatively charged. If, for example, two copper electrodes are immersed in this dilute hydrochloric acid, negative (Cl) ions will drift to both electrodes and give them a negative charge; after which the ions move aimlessly through the solution.

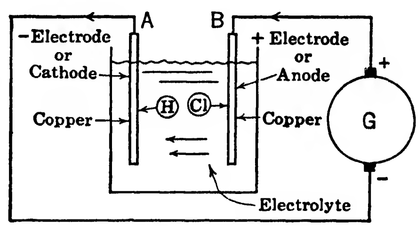



FIG. 1–14. The current enters the liquid, or electrolyte, at plate B, the positive electrode, or anode. It leaves the electrolyte at plate A, the negative electrode, or cathode.

If, now, a current is caused to flow from the copper electrode $B$ to electrode $A$, Fig. 1–14, under the action of an impressed voltage, the hydrogen or positively charged (H) ions drift as a positive current to the negative electrode or cathode $A$, and give up their charge, i.e., take electrons from the copper conductor; while the negatively charged (Cl) ions, with an excess of electrons, travel against the positive current to the anode, or positive electrode $B$, and give up their charge, i.e., give electrons to the anode. Thus, under the action of the impressed voltage, a stream of electrons is carried from the cathode $A$, through the solution to the anode $B$; the flow of the positive charges in one direction

plus the flow of the negative charges in the opposite direction constitutes an electric current.

Thus, the conduction or flow of current in a liquid (electrolytic conduction) is accompanied by an actual transfer, or movement of parts of the solution itself.

**3. Batteries. Primary and Secondary Cells.** When two electrical conductors, or electrodes, consisting of **different materials,** are placed in an electrolyte so that they do not touch each other, and the electrolyte or liquid acts chemically on one of them, then a battery cell is formed and an emf will be set up, or generated, between the electrodes. The electrode having the higher potential is called the **positive plate,** or **pole;** the other, the **negative plate** or **pole.**

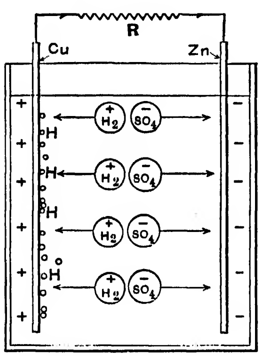



FIG. 2–14. The electric and chemical actions in a battery cell made of copper, zinc and sulphuric acid.

The amount of the generated voltage between the two plates depends entirely upon **what** the electrolyte is, and upon what the materials of the two plates are. The size, distance apart of the plates, or the amount of the electrolyte have nothing whatever to do with the emf of a battery, any more than the **size** of the conductors cutting lines of force affects the pressure developed by a generator. The number of **lines cut per second** determines the voltage of a generator; and the **material of the plates and electrolyte** determines the voltage of a battery cell.

When the liquid is dilute sulphuric acid, one plate is zinc, and the other copper, the electric pressure set up is **always about one volt.** The copper plate is positive, and if the cell is connected as in Fig. 2–14, current will flow out from the copper plate through the external circuit *R* and back to the zinc or negative plate as indicated. Inside the battery, the current enters the electrolyte at the negative plate and leaves it at the positive plate.*

* Since the current flows from the zinc plate to the copper plate **in the liquid,** the zinc is said to be electrochemically positive to the copper; while **outside** the cell, the copper is positive to the zinc. Note that the zinc or **negative plate, or pole,** is the **anode, or positive electrode;** while the **positive plate, or pole,** is the **cathode,** or **negative electrode**

The chemical action takes place between the zinc and the sulphur and oxygen, and forms zinc sulphate.

When one plate is zinc and the other carbon, and the electrolyte a solution of sal ammoniac, or ammonium chloride ($NH_4Cl$), the pressure is always nearly 1.5 volts, regardless of the size of the plates. The carbon is the positive plate; and the chemical action takes place between the zinc and the sal ammoniac and forms zinc chloride.

The important thing to remember is that in order to have any voltage set up between the plates of a cell, they must be of different materials; and at least one of them must be acted on chemically by the electrolyte, as current is taken from the battery.

In all types of battery cells, the chemical action either consumes one of the plates, or else changes the chemical composition of one or both plates. Either effect is generally accompanied by a loss in weight in the plate.

Battery cells are divided into two classes: **Primary cells** and **Secondary cells, or Storage cells.**

In the **primary cell,** the chemical action gradually consumes the negative plate; and it is necessary to renew both the plate and the electrolyte from time to time.

In the **secondary cell,** the plates and the electrolyte undergo a chemical change, as current is taken from the battery. This chemical action can be reversed; and the plates and electrolyte restored to their original chemical state, by sending a current in a reversed direction through the cell.

**4. Action of a Primary Cell.** Since the action of both primary and secondary cells is very much the same when delivering current, it is well to consider the action of a primary cell, it being the simpler.

Perhaps the very simplest cell is one composed of zinc and copper electrodes, with dilute sulphuric acid as the electrolyte. Figure 2–14 represents such a cell. **Cu** is the chemical symbol for copper and **Zn** for zinc.

The chemical symbol for the molecule of sulphuric acid is $H_2SO_4$, a combination of two atoms of hydrogen ($H_2$), one of sulphur (S) and four of oxygen ($O_4$). This breaks up into positively charged (H) ions, deficient in electrons, which drift to the copper plate giving it a positive charge; and negatively charged ($SO_4$) ions, having a surplus of electrons, which travel to the zinc plate and give it a negative charge. The fact that the charges

on the plates are of opposite kinds of electricity causes a difference of potential of approximately 1.1 volts between the plates. Therefore, if the copper plate is joined by an external circuit to the zinc plate, a current will flow through the external connection from the copper to the zinc; and inside the cell, a constant stream of positive ions will travel with the current to the copper plate; and another stream of negative ions will travel against the current to the zinc plate.

When current is taken from the battery, the following chemical action takes place. Bubbles of hydrogen gas are formed at the copper plate, or cathode, as the hydrogen ions give up their charge and take electrons from this plate. As the $(SO_4)$ ions give up their charge of excess electrons to the zinc plate, or anode, they unite with the zinc forming zinc sulphate $(ZnSO_4)$ which falls to the bottom of the tank. Thus the action of taking electrical energy from the cell gradually consumes the zinc electrode or anode.

If the plates are not joined externally, the chemical action goes on only until the plates are charged to such an extent that a voltage is set up between them, which is sufficient to prevent the flow of the ions.

The action of all primary cells is similar to this. The electrolyte is always broken up into oppositely charged parts or ions. One part gives up its charge to one plate; the other gives up its charge to the other plate. The difference of potential thus set up depends entirely upon the composition of the plates and the electrolyte.

In a well-designed cell, the chemical action takes place only when the cell is delivering energy. The rate at which electrical energy is delivered by the cell, then, determines the rate at which the zinc is being consumed; just as the rate at which steam energy is delivered by a boiler determines the rate of coal consumption. Zinc may, therefore, be said to be a fuel, the chemical energy of which is turned into electrical energy by means of a battery cell; just as coal is a fuel, the chemical energy of which is converted into heat by means of oxygen, a furnace, water and a boiler. Of course energy, especially in large amounts, cannot be obtained as cheaply from zinc as from coal.

**5. Internal Resistance.** All cells have an internal resistance which tends to reduce the terminal voltage when current is being taken from the cell. This resistance includes the resistance of the plates, the contact resistance of plates and electrolyte, and the resistance of the electrolyte. It depends upon the length and

area of the volume of electrolyte between the plates and upon its temperature and density. The cross section of the current path inside the cell should be made as large as possible; this includes the area of the electrodes in contact with the electrolyte. Increasing the size of the electrodes or plates reduces the resistance and increases the size of the cell, and hence its current capacity. **It should be noted, however, that increasing the size of the plates does not change the value of the emf.** This depends solely upon the materials of the two plates and the electrolyte, as has been said.

The internal resistance of the different primary cells may vary from about 0.1 ohm to 200 ohms or more.

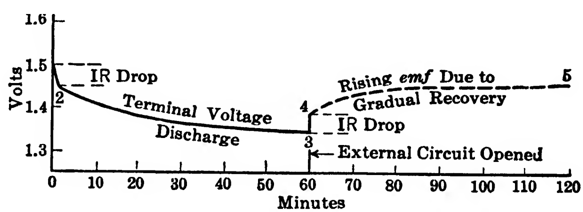



FIG. 3–14. Drop in terminal voltage and recovery of a primary cell due to the effect of polarization.

**6. Polarization.** The collection of hydrogen gas on the positive plate or cathode of a primary cell is called **polarization.** When the hydrogen (H) ion gives up its charge to the positive plate, it is likely to cling to the plate and form a layer of hydrogen gas around it, as indicated in Fig. 2–14. As hydrogen gas is a very poor electrical conductor, this hydrogen layer greatly increases the internal resistance, and decreases terminal voltage and the current output. Furthermore, there is an opposing or counter emf set up by the contact of the hydrogen with a copper or carbon electrode, which still further lowers the terminal voltage. The effect of polarization is approximately shown by the curve of Fig. 3–14. The emf of the cell is shown as 1.5 volts. When the battery is closed on an external circuit of constant resistance, the terminal voltage falls to point 2 on the curve, due to internal resistance drop. As the battery continues to supply current, hydrogen gas gradually accumulates and increases the internal resistance; and the terminal voltage (and current) gradually decrease, as shown by the curve from 2 to 3. If at 3, the external circuit is opened, the voltage almost immediately rises from

3 to 4, this rise in voltage representing the *IR* drop in the cell. With further lapse of time, the battery recovers practically to its original emf as shown by the curve from 4 to 5.

Any method of getting rid of these hydrogen bubbles is called "depolarizing" the cell. If the positive plate or cathode has a roughened surface, these hydrogen bubbles form at the projections and cannot cling as readily to the surface. They may be removed mechanically by brushing them off or by agitating the electrolyte, which is commercially impracticable. In practice, the usual method of depolarizing a cell is by chemical means. That is, some chemical, called a "depolarizer," which has a surplus of oxygen in it, is introduced into the electrolyte. The oxygen then very easily unites with the hydrogen and forms water ($H_2O$). Bichromate of potash, manganese dioxide, or carbon and manganese peroxide are common depolarizing agents.

When the depolarizer acts quickly, and does not allow the hydrogen bubbles to accumulate on the plate, the cell can be run continuously without polarizing, and without much decrease in terminal voltage. Such a cell is called a **closed-circuit cell.** When the depolarizer acts more slowly, and allows the bubbles to gradually gather on the plate in spite of the depolarizer, the cell can be used only intermittently, in order to allow the depolarizer time in which to act on the hydrogen and clear the plate. Such a cell is called an **open-circuit cell.** The curve of Fig. 3–14 is approximately that of an open-circuit cell.

It should be noted also that if a closed-circuit cell is allowed to stand too much on open circuit, the depolarizer may act chemically on the electrolyte and ruin it.

**7. Requirements of a Primary Cell.** In general, the requirements of a primary cell are: (1) the emf must be of such a value that the cell can deliver considerable energy with a moderate current; (2) cost of electrolyte and negative plate should not be expensive or need frequent replacement; (3) there should be no chemical action nor deterioration when the cell is not in use.

Because of the above requirements, zinc is almost universally used as the negative pole (the principal exception being the cadmium anode used in the Weston Standard Cell), and either carbon or copper for the positive pole.

**8. Types of Cells.** Many types of primary cells, usually called "wet" cells, have been invented, have served their purpose and have been superseded.

Of the better-known cells, some use only one fluid for the electrolyte, while others employ two different fluids. In the two-fluid cells, the electrodes are each immersed in a different fluid or electrolyte. These fluids are separated, either by some form of porous cup inside the battery jar or by gravity, due to the difference in density of the two fluids.

**The Gravity or Crow-Foot Cell** is a typical two-fluid cell, using copper and zinc as electrodes. The electrolytes are copper sulphate and zinc sulphate. The positive electrode, consisting of thin copper strips riveted together, is placed in the bottom of the jar, together with copper-sulphate crystals, and the jar partially filled with the copper-sulphate solution. The zinc sulphate is then carefully poured in. Since the copper sulphate is the heavier liquid, it remains in the bottom of the jar. The zinc electrode, cast in the form of a heavy crow-foot, is hung on the top edge of the jar in the zinc-sulphate solution. An insulated copper wire carried up through the solution connects the copper cathode to the outside circuit.

There is no polarization in this cell, but it deteriorates if left on open circuit. It is cheap and reliable, and gives an emf of about 1.1 volts. It is quite extensively used for closed-circuit telephone and telegraph work.

**The Le Clanché Cell** is another well-known primary cell. In this cell, the electrodes are carbon and amalgamated zinc with an electrolyte of sal ammoniac ($NH_4Cl$). The cathode consists of a porous carbon cup filled with manganese-dioxide crystals. The zinc anode is shaped as a cylinder around the carbon cup, and insulated from it. The manganese dioxide acts as a depolarizer, but on continued closed-circuit use, the cell polarizes, and it is suited for open-circuit work only. The open-circuit emf is about 1.5 volts, but the cell has comparatively high resistance and, with polarization, the terminal voltage is nearer 1 volt when the cell is in use.

(See any Electrical Engineering Handbook for further description of the various primary batteries.)

**9. Dry Cell.** The dry cell is merely a modification of the Le Clanché cell; but because it is portable and light, it has practically replaced all other types of primary cells for intermittent service.

The electrolyte is held by some absorbent non-conducting material, such as blotting paper or plaster of paris, and the entire

cell sealed to prevent evaporation. The cell, therefore, is not really dry and the name "dry cell" is really a misnomer, but is used in contradistinction to the ordinary "wet" battery. A cross section of the cell is shown in Fig. 4–14. The container is a cylindrical cup of sheet zinc, which is also the anode, and to which is soldered the negative terminal of the cell. This cup is lined with blotting paper or plaster of paris. The cathode is a carbon rod, set in the center of the zinc cup, which is packed with a mixture of powdered coke, carbon and manganese dioxide, the latter being the depolarizing agent. The electrolyte is sal ammoniac, with a little zinc sulphate or zinc chloride added, to reduce the deterioration of the zinc anode on open circuit. The top of the cell is sealed with waterproof cement, wax or some tar compound to prevent evaporation.

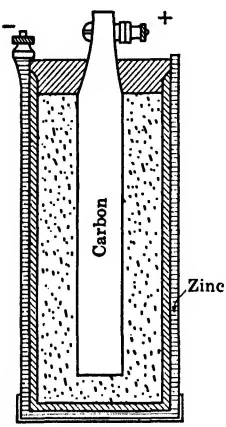



FIG. 4–14. Sectional view of a dry cell.

The dry cell is an open-circuit battery and cannot be used to supply a large or steady current because the manganese dioxide cannot clear up the hydrogen bubbles as fast as they are formed. The emf of a new dry cell on open circuit is from 1.5 to 1.6 volts, but decreases with age. If the emf is much less than 1.5 volts, it indicates the cell has begun to deteriorate. The internal resistance of a new cell is about 0.1 ohm, but this increases with age to several times this value. A standard size dry cell (about 6 inches high and $2\frac{1}{2}$ inches in diameter), when new, should give a current of about 15 amperes on short circuit $\left(\frac{1.5}{0.1} = 15 \text{ amperes}\right)$. This is a common test to check the condition of a cell.

**10. Electrochemical Equivalent. Electrolysis.** The rate at which the negative plate, generally of zinc, is consumed depends upon how much current is being supplied by the battery. When no current is being supplied, all chemical action ceases, if the metal is pure and there is no local action; and thus no material is consumed. The negative plate thus acts as fuel, just as coal acts as fuel in a fire box under a boiler. The rate at which coal is consumed depends upon how much heat energy must be converted into mechanical energy per hour. Similarly, the rate at which the negative plate is consumed depends upon how much

chemical energy must be converted into electrical energy per hour. When one ampere is drawn from the cell, it consumes a definite quantity of zinc per hour. If two amperes are drawn from the cell, twice as much is consumed per hour.

Now this process is reversible. If one ampere were forced through the cell in the opposite direction, theoretically this same amount of zinc per hour would be recovered from the electrolyte and deposited back on the negative plate again. If the current were doubled, twice as much zinc would be deposited per hour. This reversed process, in which the liquid is broken up and metal deposited, is called **electrolysis.**

The definite quantity of a substance, which is consumed per hour when one ampere is drawn from the cell, or which is deposited when the current is reversed, is called the **electrochemical equivalent** of the substance. The following table gives the electrochemical equivalents for the more common elements, in grams of the element deposited per ampere-hour.

ELECTROCHEMICAL EQUIVALENTS OF ELEMENTS

| Element | Symbol | Grams per ampere-hour | Ampere-hours per gram |
|---|---|---|---|
| Aluminum | Al | 0.3370 | 2.967 |
| Chromium | Cr | 0.6466 | 1.546 |
| Copper | Cu | 1.1858 | 0.843 |
| Gold | Au | 2.4523 | 0.408 |
| Hydrogen | H | 0.0378 | 26.5 |
| Iron | Fe | 0.6944<br>1.0415 | 1.440<br>0.960 |
| Lead | Pb | 3.8632 | 0.259 |
| Nickel | Ni | 1.0944 | 0.914 |
| Oxygen | O | 0.2984 | 3.351 |
| Silver | Ag | 4.0248 | 0.248 |
| Zinc | Zn | 1.2193 | 0.820 |

By means of this table, we can determine the amount of zinc, or any other metal, necessary to produce a given current for a given time.

**Example 1.** How many ounces of zinc would be consumed in 8 hours by a cell delivering 20 amperes? 1 ounce = 28.4 grams.

**Solution.** From the table, 1.219 grams of zinc are consumed in 1 hour by a current of 1 ampere.

$8 \times 20 \times 1.219$ grams = 195 grams or 6.87 ounces consumed in 8 hours by 20 amperes.

Also by means of this table, it is possible to determine the amount of copper or other metal that would be deposited in a given time by a given current from a salt solution of the metal.

**Example 2.** How much copper would be deposited from a solution of copper sulphate in 12 hours by a current of 2 amperes?

**Solution.** From table, 1.19 grams are deposited in 1 hour by 1 ampere.

$2 \times 12 \times 1.19 = 28.6$ grams deposited in 12 hours by 2 amperes.

**Prob. 1–14.** How many grams of silver are deposited per day of 24 hours from a solution of silver nitrate by a current of 10 amperes?

**Prob. 2–14.** The zinc plate of a cell weighs $\frac{1}{2}$ pound. How long will it last, if the cell is required to deliver 3.2 amperes?

**Prob. 3–14.** It is desired to deposit 100 pounds of copper in 24 hours. What current must be used?

**11. Electroplating.** In the process known as electroplating, advantage is taken of the fact that, if a current is passed through an electrolyte containing a salt of a metal, the metal of the positive electrode (the plate at which the current enters the electrolyte), goes into solution and is deposited on the negative electrode or cathode.

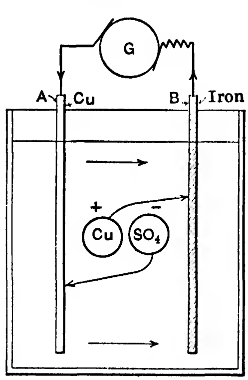



FIG. 5–14. Action in an electroplating vat.

The action is practically that discussed in Art. 2.

It is desired to copperplate a piece of iron as indicated in Fig. 5–14. The iron *B* is placed in a solution of copper sulphate ($CuSO_4$) together with a piece of copper, *A*. The electrolyte is broken up into ($SO_4$) ions negatively charged, and (Cu) ions positively charged. When current is forced through the vat from *A* to *B*, the positively charged (Cu) ions go **with the current** and are deposited on the iron cathode or negative plate, *B*; while the negatively charged ($SO_4$) ions go **against the current** and unite chemically with the copper plate or anode, *A*, forming more copper sulphate

($CuSO_4$). Thus the copper plate is gradually dissolved and conveyed to the iron. The electrolyte itself does not change chemically during the action; but the copper in solution is transferred from one plate to the other, under the action of the impressed voltage.

If it is desired to nickel plate the iron, a piece of nickel is used for the anode and nickel ammonium sulphate for the electrolyte.

In the electrodeposition of metals for plating articles, the cathode is always the article to be plated; the electrolyte, a dissolved salt of the metal to be plated on the cathode; and the anode, a piece of the same metal or an inert conductor. In the latter case, there is a chemical voltage and the electrolyte must be renewed from time to time. There are many " kinks " to good plating technique. Voltages from one to six are generally used.

**12. Electrotyping.** The object of electrotyping is to reproduce the printers' set-up type, engravings, etc. A wax impression is first taken of the set-up type. Since wax is a non-conductor of electricity, a thin coating of graphite is now given the wax mold. The whole mold with its coating of graphite is immersed in copper sulphate together with a copper bar. An electric current is now sent through from the copper to the graphite. This causes copper to be deposited on the graphite just as it was deposited on the iron in the cell of the previous paragraph. The current is allowed to run long enough to deposit a plate of sufficient thickness to be handled safely. Then the wax mold is removed and the remaining copper plate is an exact reproduction of the type.

**13. Electrolytic Refining of Metals.** The electrolytic refining of metals is merely an application of the principles of electrolysis. Copper, nickel and other metals are refined by electroplating them. Electrolytic copper thus refined is very pure, and is used to large extent in the electrical industry for copper conductors, bars, sheets, etc.

If both electrodes are of the same metal there is very little chemical voltage present. The only electrical energy used is that due to forcing current through the resistance of the electrolyte. However, there will always be a minute chemical voltage due to differences in purity of the metals.

In copper refining, the electrolyte is a solution of copper sulphate. Ingots of the impure or crude copper from the reducing

furnace are used as the anode; and thin strips of refined or pure copper, called "starting sheets," are used as the cathode.

Many copper ores contain small amounts of gold, silver and platinum; and by the addition of certain chemicals to the electrolyte, these are dissolved, and are collected in the sludge at the bottom of the tank. One reason for electrolytic refining of copper is to obtain these by-products which may pay the greater part of the cost of the process. The copper obtained is 99.95 per cent pure and many thousands of tons are refined in this manner annually.

The electrolyte is used hot in large tanks which are usually connected in series. An average current density of about 20 amperes per square foot of cathode is used, and the voltage per tank is about 0.3 volt.

From the electrochemical equivalent of copper, 1.185 grams of copper are deposited per ampere-hour. This is $\frac{1.185}{28.4 \times 16}$ or 0.0026 pound per ampere-hour.

Thus, to deposit a ton of copper requires $\frac{2000}{0.0026}$ or about 770,000 ampere-hours.

The current required to deposit, for example, $\frac{1}{10}$ of a ton of copper per tank per day of 24 hours, is $\frac{770{,}000}{10 \times 24}$ or 3200 amperes. If this were delivered by a 15-volt generator, the power required would be $15 \times 3200$ or 48 kilowatts. At 0.3 volt drop per tank, $\frac{15}{0.3}$ or 50 tanks can be connected in series. Fifty tanks at $\frac{1}{10}$ of a ton per tank per day give 5 tons of refined copper per day. Several series of tanks may be used on one large generator.

**Prob. 4–14.** How long will it take to refine 1 ton (2000 pounds) of copper, if 10 vats are connected in series and a current of 150 amperes can be used?

**Prob. 5–14.** An iron casting is to be copperplated and then nickelplated. The current in each case is to be 4 amperes. How long must it remain in each vat in order to have 3 pounds of copper and 6 ounces of nickel deposited on it?

**Prob. 6–14.** Two electroplating vats are arranged in series, one for gold plating and the other for silver plating. If a current flowing through the vats deposits 1.4 ounces of silver in a given time, how much gold is deposited at the same time?

**14. Definition of Ampere. Voltameter.** Since one ampere flowing for one hour through an electrolytic vat deposits a definite quantity of metal on the negative plate, we can determine very accurately the value of an unknown current by weighing the plate before and after an accurately timed run. On account of this precise method of determining the current strength, the following definition of an ampere has been established:

**An ampere is the rate of flow of a steady current which deposits 4.025 grams of silver per hour from a solution of silver nitrate diluted to a standard density with distilled water.**

This measurement is made with a device called a **voltameter,** which consists of a glass jar containing the silver plates and the silver-nitrate solution. Copper voltameters are also used, consisting of two plates of copper immersed in a copper-sulphate solution. By means of a voltameter, it is possible to calibrate accurately galvanometers and ammeters; although the method has the disadvantage of requiring too long a time, if many parts of the scale are to be calibrated.

**Prob. 7–14.** An ammeter, being calibrated by a silver voltameter, registered 4.38 amperes for a current which deposits 39.2 grams of silver in 2 hours, 15 minutes. What is percentage error of the ammeter at this reading?

**Prob. 8–14.** A copper voltameter, used to calibrate an ammeter, has 1.5 grams of copper deposited on the cathode in 1 hour and 15 minutes. What value should the ammeter indicate during the process?

**15. Electrolytic Destruction of Metal Water Mains, etc.** It is customary in electric railways to use the track as the return circuit. The rails, not being insulated from the ground, allow the current to leak into the ground and follow any low-resistance path it can find, such as a water or a gas main, back to the generator, which, of course, is also grounded.

This action is shown in Fig. 6–14. Where the current enters the pipe at *A* no harm is done. But at the point *B*, where the current leaves the pipe, generally near the generator station, there is all the action that takes place at the positive plate in an electroplating vat. The pipe, being in moist ground, is in contact with water containing more or less salt, which causes it to act as an electrolyte. Thus, as the electric current leaves the pipe, there is a chemical action set up between the pipe and salt water, by which

the iron of the pipe is eaten away in places, and carried by the electric current to some outside substance, which for the instant is acting as the negative plate.

In a short time, heavy iron pipes have been eaten through by this action. Among the various precautions taken to avoid this destruction by electrolysis are the following: (1) The heavy bonding or welding of the rails to procure a return of low resistance. (2) The use of a second trolley wire for the return. (3) Laying in the ground a heavy copper return conductor to offer a low-resistance path to the return current.

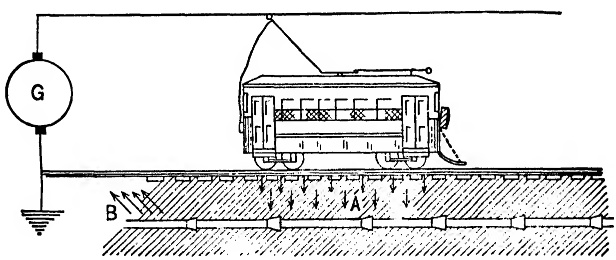



FIG. 6–14. The electric current damages the pipe by electrolysis at the point, B, where it leaves the pipe.

Method No. 2 is the most efficient, but also the most expensive, and is seldom if ever used. The introduction of the "trackless trolley," or trolley bus, equipped with rubber tires, however, necessitates the use of two trolley wires.

Stray currents must be kept out of reinforcing rods and steel supports in concrete structures; for the failure of these supports is likely to cause the collapse of the whole structure.

**16. Standard Cells.** The need of standards of electromotive force, current and resistance in practical work, such as the calibration of instruments, etc., is apparent. If two of these quantities are known, the other can be obtained by Ohm's law. Current standards are inconvenient to use and are difficult to maintain. Consequently, voltage and resistance standards are commonly used.

A resistance standard may be simply a strip or column of metal of definite length and uniform cross section at a known temperature. Secondary resistance standards are generally made of metals or alloys with negligible temperature coefficient.

A voltage standard is the emf of a **standard cell.** There are two types of standard cells, the Clark and the Weston cells. Since

the emf of a primary cell depends not only upon the material of the plates, or poles, and upon the electrolyte, but also on the impurities in these materials, great care must be taken in the construction of such cells, in order that their emf's will all be of specified value.

In the Clark cell, the poles are mercury and amalgamated zinc, and the electrolyte a saturated solution of zinc sulphate. The depolarizing agent is a paste of mercurous sulphate. The emf of this cell has been given as 1.434 international volts at 15° C, but this voltage changes considerably with temperature and the cell has been largely displaced by the Weston cell.

The Weston cell differs from the Clark cell, chiefly in the use of amalgamated cadmium as the negative pole in place of zinc, with cadmium sulphate as the electrolyte. The cell is placed in an H-shaped glass container. The positive pole is a pool of mercury, covered by a mercurous-sulphate paste as a depolarizer in one leg of the H, while the cadmium is placed in the other leg. There are two forms of these cells: the saturated, or normal cell, in which the electrolyte is a saturated solution and has crystals of cadmium sulphate in the electrolyte; the other form is the unsaturated cell. The unsaturated cell is the form supplied by the Weston Electrical Instrument Co. The emf of this cell is approximately 1.0186 international volts and the internal resistance is about 200 ohms. While the emf of all unsaturated cells is not exactly the same, so that each individual cell must be calibrated, its chief advantage is that practically there is no change in emf with change in temperature, as is the case with the saturated cell. Also, the unsaturated cell has longer life and does not polarize as rapidly as the Clark cell.

**Caution in use of Standard Cells.** No appreciable current (not more than 0.0001 ampere is recommended) can be taken from a standard cell without changing its emf, due to internal resistance and polarization. Thus the **voltage of such a cell should not be measured even with a voltmeter.** These cells should be used only where their emf is opposed or balanced by another emf as in the potentiometer. See Arts. 19 and 20, Chapter XVII.

**17. Storage Batteries.** In the ordinary primary cell, when the negative plate is almost consumed, it is customary to replace both it and the electrolyte to renew the cell. In the **secondary** or **storage cell,** chemical energy is restored by sending a current through it in the reverse direction from an outside source. In fact, some primary cells, such as the gravity cell, can also be renewed in

this way; but in this case only part of the negative plate can be restored, and the process is more expensive than replacing both plate and electrolyte.

When two or more secondary cells are connected in series, the combination is called a **storage battery or accumulator.** There are but two types of storage batteries in general use: the lead-acid battery and the iron-nickel-alkali, or Edison, battery.

When a battery furnishes current to an outside source, that is, when chemical energy is being converted into electrical energy, the battery is said to be **discharging**; and when current is sent through the battery in the reverse direction from an outside source, that is, when the chemical action is reversed and electrical energy is being converted into chemical energy, the battery is said to be **charging.**

Remember that a storage cell does **not** store electricity. It stores nothing but chemical energy. In **charging,** electrical energy is transformed into chemical energy. In **discharging,** chemical energy is changed back again to electrical energy.

The commercial storage battery can be repeatedly charged and discharged until the plates are worn out by the dislodging of the **active material.** The active material of a plate is that part which undergoes a chemical change when current flows in or out of a battery.

**18. The Lead Battery.** The lead-lead-acid storage battery is the type in most common use. In any cell, there must be two plates of **different materials** in an electrolyte.

When the lead cell is charged, the active material on the negative plate is pure spongy lead, chemical symbol Pb, and is gray in color. The active material on the positive plate is lead peroxide, chemical symbol $PbO_2$, which is dark brown in color.

The electrolyte is sulphuric acid, $H_2SO_4$, diluted with water, $H_2O$. The strength, or specific gravity, of the electrolyte varies with the use for which the battery is designed.

Since the amount of electrical energy which may be taken from a battery depends upon the amount of material in the battery which undergoes a chemical change, it is desirable to have as large a surface of active material in contact with the electrolyte as possible. For this reason, storage cells are built with a number of plates, all the positive plates being connected to one terminal, and all the negative to the other terminal. The greater the number of plates and their surface area, the greater the current capacity of the battery.

The positive and negative plates of a cell are interleaved and are kept from contact with one another by the use of separators in the form of sheets, which cover the entire surface of the plate. For many years, the standard separator has been thin sheets of specially treated wood with attached vertical wood or rubber channels. Perforated hard-rubber sheets are also used. There are also now on the market porous rubber sheets and porous glass sheets, which allow better circulation of the electrolyte and reduce the internal resistance of the cell.

To keep the positive plates from "buckling" or warping, due to the expansion of the active material, there must be the same

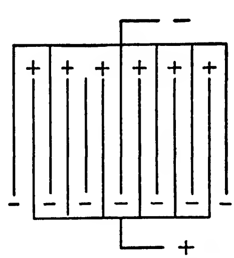



FIG. 7–14. The plates, or grids, in a lead storage cell are interleaved, there being one more negative than positive grid.

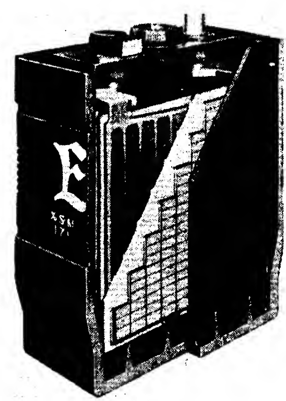


FIG. 8–14. Cutaway view of a lead storage cell. *The Electric Storage Battery Co.*

chemical action on both sides of these plates. Therefore, in all lead cells, there is always one more negative than positive grid. That is, the two outside grids, Fig. 7–14, are negative and each is worked chemically on one side only. Figure 8–14 shows a cutaway view of a lead battery.

The active materials, spongy lead and lead peroxide, are poor conductors of electricity and are not sufficiently hard to be made into plates; so it is necessary that they be supported by frames of some harder material which is a good conductor of electricity. The material of this framework must also have the property of **not acting** as a third plate, otherwise the acid will produce local action between the framework and the active material. The ma-

terial usually chosen for this framework is metallic lead, or an alloy of lead and antimony, which is mechanically strong and produces no local action.

The plates of the battery really consist of the active materials; the combination of frame and active material is called a **grid**, either positive or negative.

There are two types of grids in common use: one in which the active material is formed electrochemically from pure lead, and called the Planté type; the other is the Faure, or pasted grid, in which a paste of the active material is pressed into a supporting frame.

**19. The Planté Grid.** If two strips of ordinary sheet lead are immersed in dilute sulphuric acid in a glass jar and are connected through a suitable resistance to an emf, as in Fig. 9–14, a current will flow through the cell as indicated. After a short period of time, it will be noted that strip *A* in the figure will turn a dark brown color, due to the formation of lead peroxide; while strip *B* will not change in color, but the surface will begin to change from metallic lead to soft spongy lead. If the cell is disconnected from the line and a voltmeter is connected across the cell terminals, it will read approximately 2 volts, strip *A* being the positive pole and the other the negative; and a battery cell will have been formed electrochemically. The Planté grid is formed in this manner.

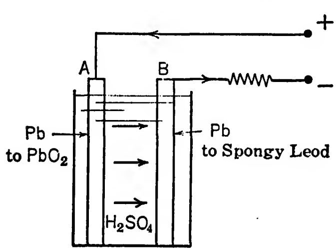



FIG. 9–14. Under the action of the current, strip A changes from lead to lead peroxide, and strip B changes from lead to spongy lead.

However, very little current can be taken from such a cell as described above, because very little active material has been formed. To obtain a greater surface exposed to the electrolyte, one form of Planté grid is made by a process called "spinning," which consists of passing it under revolving knives which work blocks or squares of the surface into grooves and ridges, leaving the edges and spaces between these squares to serve as the supporting frame. The Gould grid shown in Fig. 10*a*–14 is surfaced in this manner and is then formed electrically. Figure 10*b*–14 shows an enlarged cross-sectional view of the Gould plate.

Another form of Planté plate is the Manchester grid, Fig. 11–14,

in which corrugated ribbons of pure lead are coiled, and forced into holes in a lead-antimony frame. When the plate is formed,

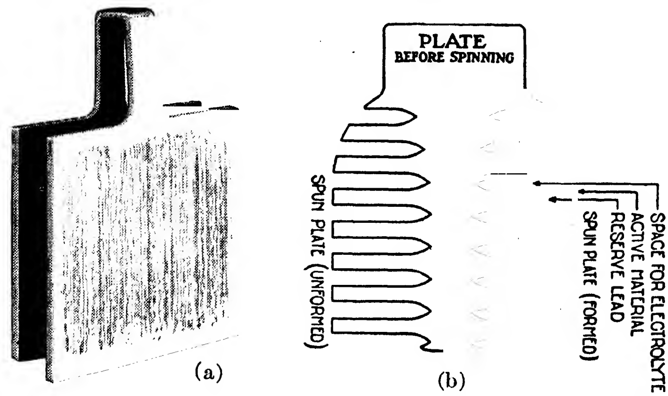



FIG. 10–14. (a) Planté formed plate. *Gould Storage Battery Co.* (b) Enlarged cross-sectional view of a part of a Gould plate.

the active material is more bulky than the original lead ribbons and is securely held in the frame. These plates are used as positives in connection with a pasted or Faure type negative.

For a given current output, Planté formed grids are heavier, more bulky and more expensive than pasted grids. However, they can stand greater above-normal rates of charge and discharge without injury. They are less likely to lose their active material, have longer life and are more dependable. They are, therefore, well suited for power-station service.

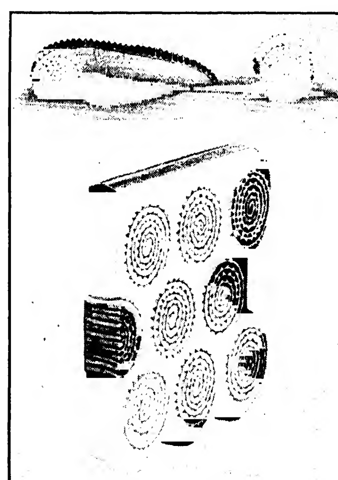
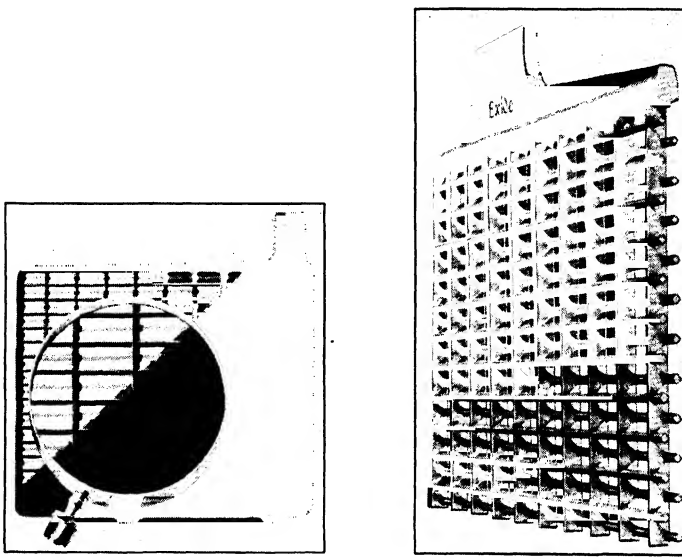


FIG. 11–14. A section of a Manchester grid. *Electric Storage Battery Co.*

**20. The Faure Grid.** In this type of grid, the frame consists of a lead-antimony lattice work, or frame, into which is pressed lead oxide in the form of a paste. When the cell is charged, the lead oxide in the positive grid is converted into lead peroxide; and in the negative grid into spongy lead. A Faure grid, partly filled

with active material, and an empty frame are shown in Fig. 12–14.

The pasted-type grid for a given output is cheaper, lighter and occupies less space than the Planté grid. The thickness of such a grid depends upon the service for which it is intended. When high discharge rates are desired, as for automobile starters, the cells of the battery consist of a greater number of thinner grids in order




(a) (b)

Fig. 12–14. (a) Partially filled Faure, or pasted, plate with lead antimony grid. (b) Empty grid for a Faure type pasted plate. *Electric Storage Battery Co.*

that the electrolyte may penetrate the active material at a higher rate. These thin grids are more liable to lose their active material and such construction shortens the life of the cell.

**21. Exide-Ironclad Battery.** Where a battery must be subjected to hard service, both mechanically and electrically, such as in trucks, mining locomotives, etc., the Exide-Ironclad battery is much used. See Fig. 13–14. The positive grid consists of a number of vertical hard-rubber tubes in which is packed the lead peroxide. Through each of these tubes runs a lead-antimony core which is part of the grid, and serves to conduct the current to the battery terminals. These rubber tubes have cut in them a series of very narrow slots, which permit free circulation of the electrolyte

and yet prevent the escape of the active material. This makes a very rugged type of grid. Rather thick heavy pasted plates are used as the negative grid.

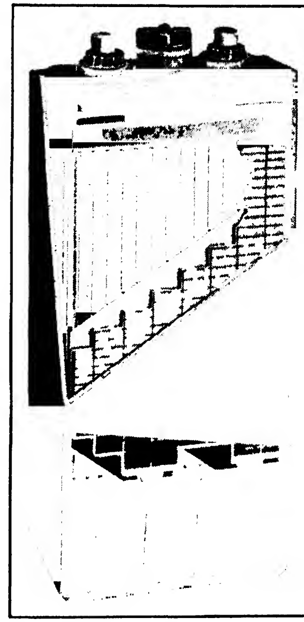


Fig. 13–14 Cutaway view of an Exide Ironclad Battery. *Electric Storage Battery Co.*

**22. Chemical Action on Discharge.** Consider Fig. 14–14. The sulphuric acid has broken up into positively charged ($H_2$) ions and negatively charged ($SO_4$) ions. The $SO_4$ ions unite with the lead plate forming lead sulphate ($PbSO_4$), giving up their negative charge to the lead plate. The hydrogen ions ($H_2$) carry their positive charges to the lead-peroxide plate, giving them up to the plate and uniting with the oxygen of the lead peroxide of the plate and forming water ($H_2O$). The hydrogen ions actually get electrons from the lead peroxide. This is equivalent to giving the lead peroxide a positive charge.

The sulphuric acid in contact with the peroxide plate is also broken up into ions of $H_2$ and $SO_4$. These $H_2$ ions unite with the oxygen of the lead peroxide and form more water. The $SO_4$ ions, instead of going over to the negative plate, unite with the lead, Pb, of the lead-peroxide plate and form lead sulphate, $PbSO_4$, on the positive plate. Thus both plates are being reduced to lead sulphate, $PbSO_4$. The cell continues to deliver current until the plates are entirely reduced to lead sulphate, when, of course, all action will cease, since there would be but one kind of material present, and a battery requires two kinds. The practical limit of discharge, however, is reached long before both plates are completely reduced to the same material.

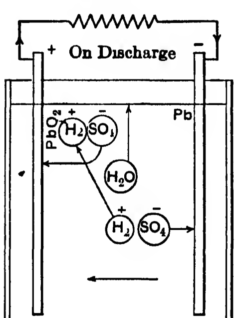



Fig. 14–14. The action in a lead storage cell on discharge.

Note **two things** which are taking place when a cell is discharging:

**First.** The acid is continually growing weaker. This results in a lower electromotive force.

**Second.** The active materials, lead and lead peroxide, are being replaced by lead sulphate, which has a much higher resistance.

**23. Chemical Action in Charging.** Refer to Fig. 15–14. Assume both plates to consist of lead sulphate, $PbSO_4$. When a current from an outside source *G* is sent through the cell, it breaks up the water which has been formed during **discharge** into positively charged hydrogen ions ($H_2$) and negatively charged oxygen ions (O). Part of the positively charged hydrogen ions are now attracted to the negative plate and unite with the $SO_4$ of the lead sulphate, forming sulphuric acid, $H_2SO_4$, and leaving pure spongy lead at the negative plate. The negatively charged oxygen ions (O) flowing against the current are attracted to the positive plate. Here they unite with the lead Pb of the lead-sulphate ($PbSO_4$) plate and form lead peroxide, $PbO_2$. The $SO_4$ part of the positive plate is finally united to the rest of the hydrogen ions liberated when the electric current broke up the water $H_2O$ into $H_2$ and O. This action forms still more sulphuric acid, and a positive plate of lead peroxide. When all the lead sulphate has been changed over to lead peroxide and pure lead, the battery is restored to the state it was in before it was discharged, and is now ready to furnish current again.

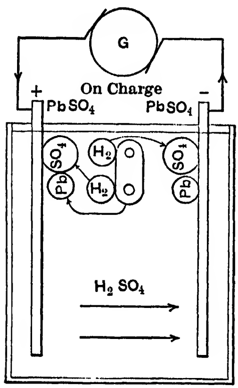



FIG. 15–14. The action in a lead storage cell on charge.

Note that the acid has been growing denser during charge; therefore, the electromotive force must have been increasing as the charging continued.

The chemical equations for the different actions on discharge and charge may be written as follows:

DISCHARGE

$$PbO_2 + Pb + 2\,H_2SO_4 \xrightarrow{\text{Discharge}} 2\,PbSO_4 + 2\,H_2O$$

$$PbO_2 + Pb + 2\,H_2SO_4 = 2\,PbSO_4 + 2\,H_2O$$

CHARGE

or

$$
\begin{array}{ccc}
\multicolumn{3}{c}{\text{DISCHARGING}} \\
\text{Positive Plate} & & \text{Negative Plate} \\
PbO_2 & \longleftarrow & Pb \\
+ & \longleftarrow & + \\
H_2 & \longleftarrow & SO_4 \\
+ & & \\
H_2SO_4 & & \\
\multicolumn{3}{c}{\text{Produces}} \\
PbSO_4 & & PbSO_4 \\
\multicolumn{3}{c}{+} \\
\multicolumn{3}{c}{2\,H_2O}
\end{array}
\qquad\qquad
\begin{array}{ccc}
\multicolumn{3}{c}{\text{CHARGING}} \\
\text{Positive Plate} & & \text{Negative Plate} \\
PbSO_4 & \longrightarrow & PbSO_4 \\
+ & \longrightarrow & + \\
O & \longrightarrow & H_2 \\
+ & & \\
H_2O & & \\
\multicolumn{3}{c}{\text{Produces}} \\
PbO_2 & & Pb \\
\multicolumn{3}{c}{+} \\
\multicolumn{3}{c}{2\,H_2SO_4}
\end{array}
$$

**24. Rating and Capacity of Batteries.** The size or capacity of storage batteries is practically always given in ampere-hours. The rating is based on the current which will discharge the battery to a safe or economical limit in 8 hours. This is known as the 8-hour rate. As an example, the normal discharge rate of a 160 ampere-hour battery is 20 amperes for 8 hours, or $20 \times 8 = 160$ ampere-hours. At higher rates of discharge, the ampere-hour capacity decreases. Thus the battery could not supply twice the current, or 40 amperes, for half the time, or 4 hours, due to the fact that at the higher discharge rates, the electrolyte cannot penetrate nor act chemically on all the active material, during this shorter period, before parts of the plates will have begun to sulphate. The above battery can supply only 34 amperes for 4 hours, and the ampere-hour capacity is reduced to $34 \times 4$, or 136 ampere-hours.

The following table gives the approximate ampere-hour capacity of a 160-ampere-hour battery with Planté grids at different rates of discharge.

| Rate | Amperes | Ampere-hour capacity | Per cent of capacity at 8-hr. rate |
|---|---|---|---|
| 8 hours | 20 | 160 | 100% |
| 6 hours | 25 | 150 | 94 |
| 4 hours | 34 | 136 | 85 |
| 2 hours | 55 | 110 | 69 |
| 1 hour | 90 | 90 | 56 |
| 30 min. | 136 | 68 | 43 |

**Ampere-hour Efficiency.** The ampere-hour charge required to restore a battery from a discharged condition is always greater than the ampere-hour discharge. This ratio of ampere-hour output to ampere-hour input is called the **ampere-hour efficiency.** However, **this must not be understood to mean** the efficiency of the battery, for ampere-hours do not represent energy.

**Example 3.** Assume the 160-ampere-hour battery considered above is discharged at normal rate of 20 amperes in 8 hours. To restore this battery to the fully charged condition at the 20-ampere rate would require more than 8 hours; or to charge the battery in 8 hours would require more than a 20-ampere charging rate. Assume that it requires 21.5 amperes to charge the battery in 8 hours.

The ampere-hour output $= 20 \times 8 = 160$
The ampere-hour input $= 21.5 \times 8 = 172$
Ampere-hour efficiency $= \frac{160}{172} \times 100 = 93$ per cent.

**25. Care of Lead Storage Cells.** From a study of the chemical action of a storage cell and the physical results of this action, we can determine what treatment such a cell should receive in order to give the most efficient service.

The main points to be considered are the following:

(1) Too rapid charging.
(2) Use of impure electrolyte.
(3) Use of too strong or too dilute electrolyte.
(4) Over-charging and over-discharging.
(5) Removing from service.

(1) **Too Rapid Charging.** If charge or discharge is carried on more rapidly than the 8-hour rate, the ampere-hour capacity of the cell is reduced somewhat as has been shown. It is to be understood that the 8-hour rate is merely that rate for the longest life and the greatest efficiency. No great harm is done, however, if higher charging rates are used, provided the current in amperes at no time is greater than the number of ampere-hours which can still be put into the battery. For instance, if a 100-ampere-hour battery is completely discharged, the charging rate might start at 100 amperes. But when 25 ampere-hours have been put into the cell, the charging current should not exceed 75 amperes, the current continually decreasing as the battery becomes charged. When higher rates than these are used, the chemical action is violent, excessive gassing occurs, the temperature rises rapidly and the active elements are not properly deposited on the plates; and are likely to form a deposit on the bottom of the jar. This deposit may

eventually rise high enough to reach the plates and short-circuit them.

(2) **Use of Impure Electrolyte.** This is the greatest source of trouble in a storage battery. Nothing but the purest sulphuric acid and distilled water should be used for the electrolyte.

Any metallic impurity in the acid or water is deposited on the plates as a pure metal and there causes local action, which consumes the material of the cell without producing any available energy.

Any acid impurity which will dissolve the lead of the framework is objectionable for very apparent reasons.

Too much care, therefore, cannot be exercised in selecting the materials which go to make up the electrolyte.

(3) **Too Light or Too Heavy Electrolyte.** The specific gravity at which a cell works best depends somewhat upon the use to which it is to be put. If the cell is to be used continuously, it can work at a higher density than if it is to stand unused for any length of time.

It is of advantage to have as high a density as is practicable, because of the **decreased internal resistance** and the **increased electromotive force** which the cell has when the acid is of high density, as previously explained. But if the cell is to stand inactive, the strong acid tends to change the active material into lead sulphate more rapidly than if weaker acid is used. The specific gravity of the electrolyte also depends upon the amount of space available in the cell for electrolyte. When there is no space limitation, and plenty of electrolyte may be used, it is best to keep the specific gravity down to about 1.200 when fully charged. When there is little space and much less acid can be used, the specific gravity may run as high as 1.300.

When studying the chemical action of the cell, we saw that the acid grew lighter as the cell was discharged, and heavier as the cell was recharged. One method, then, of telling what condition a cell is in, with regard to charge or discharge, is to test its specific gravity with a hydrometer. When fully charged, it should have a specific gravity of from 1.20 to 1.30, according to the conditions explained above. On discharge, the specific gravity should never fall below 1.17.

For example, suppose a cell had a density of 1.22 when fully charged. If at a certain time it showed a density of 1.17, it would not be wise to discharge it any further.

(4) **Over-charging and Over-discharging.** When all the active material has been changed into lead peroxide and spongy lead, there is nothing gained in continuing the charging, and a waste of electric power is going on.

There are three ways in which one may tell when a cell is fully charged:

(*a*) The plates when charging at normal rate begin to liberate gas in large quantities. The cell is said to "boil."

(*b*) The specific gravity rises to the value which it should have when fully charged and has remained constant for an hour or so during the charging.

(*c*) The impressed voltage has been constant for an hour during the charge.

**Note.** The voltage of a cell on open circuit shows **practically nothing** about its condition as to charge or discharge.

The emf (voltage on open circuit) between lead and lead peroxide in sulphuric acid of 1.2 density is 2.05 volts (approximately). Since the emf of a cell does not depend upon the area of the plates, as long as there is **any** lead left on one plate and lead peroxide on the other, the emf will be about 2.05 volts. Therefore always take the voltage of a cell when it is either charging or discharging at its **normal** rate.

**Over-discharge** is sometimes a source of trouble in a storage cell.

It weakens the acid and forms lead sulphate in excess, thus raising the internal resistance so that the terminal voltage becomes very low.

It weakens the acid so that the emf is lowered, under working conditions, to a marked extent.

There are two ways in which one may tell when a cell has been discharged as much as is practicable:

(*a*) The specific gravity falls to a value depending upon the value when the cell is fully charged. (See previous paragraph.)

(*b*) The voltage, **on discharge at normal rate,** falls to 1.75 volts.

**Caution. Never continue discharging a lead cell after the voltage falls to 1.75 volts at normal discharge current rate.**

(5) **Removing from Service.** No lead battery should be allowed to stand, even for short periods, in a discharged state; for sulphate formation continues, even when the cell is inactive, and the sulphate is difficult to remove electrically.

If a charged battery stands for a considerable period without use, the active material is acted on more or less by the acid, and

sulphate is formed. Therefore, an idle battery should be charged occasionally, say at least once a month.

When a battery is to stand idle for long periods without even the attention of occasional charges, it should be taken out of service and be put in " dry storage " as it is called. To do this, the battery should be fully charged, and then the electrolyte drained off (and saved) and the cells filled with distilled water and allowed to stand for a day. (One manufacturer has recommended short-circuiting the cells after the electrolyte has been replaced with water.) The idea here is to cleanse the plates of the acid. The water is now drained off and the plates may be stored indefinitely without injury to the grids.

To put the battery back in service, the broken or warped separators should be replaced and the cell refilled with the electrolyte (of proper density). It should now be charged. This should be done in accordance with recommendations of the manufacturer. Lacking these, it may be taken through one or more cycles of charge and discharge at normal rate.

**26. Curves of Charge and Discharge.** The changes in the physical properties of a lead storage cell are best brought out by

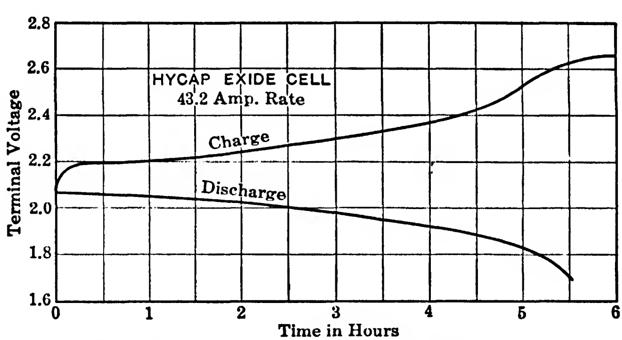



FIG. 16–14. Effect of time of charge and discharge upon the terminal voltage of a lead storage cell.

curves plotted from a series of readings. Figure 16–14 shows a set of curves plotted from data taken on an " Exide " Cell.

The charge and discharge took place at the $5\frac{1}{2}$-hour rate. Note the rapid rise in voltage during the first and the last hour of charge. During the intervening three or four hours, the voltage was fairly constant.

On the discharge, the voltage falls rapidly during the last hour,

but remains nearly constant during the remaining time. This makes the storage cell a first-rate source of constant potential.

Figure 17–14 shows the variation, on discharge, in specific gravity, emf and internal resistance.

NOTE:

(1) That the specific gravity of the cell increases on charge, and decreases on discharge.

(2) That the internal resistance increases on discharge slowly at first but rapidly as the cell is nearly discharged.

(3) That the emf of the cell rises slightly on charge and falls slightly on discharge.

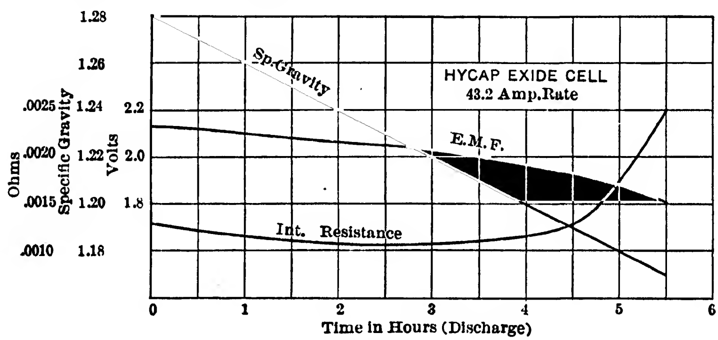



FIG. 17–14. Test of a lead battery on discharge.

A method by which the true ohmic resistance $R$ can be found is as follows: Take terminal voltage $V_1$ and current $I$ on discharge. Suddenly open the switch while the cell is discharging and record the value which the voltmeter registers on the swing of the needle. (The voltmeter will read instantly $E$, the emf of the cell, and then gradually rise as depolarization takes place.) Then,

$$R = \frac{E - V_1}{I};$$

where $R$ = true ohmic resistance of cell;
$E$ = emf of cell;
$V_1$ = terminal voltage on discharge;
$I$ = rate of discharge.

**Prob. 9–14.** What is the internal resistance of a cell which has a terminal voltage of 2.00 volts when discharging at a 12-ampere rate, and emf of 2.05 volts?

**Prob. 10–14.** What voltage would have to be impressed on the cell of Prob. 9 in order to charge it at a 12-ampere rate?

**Prob. 11–14.** A cell has an internal resistance of 0.003 ohm. What would be the difference in the terminal voltage on charge and discharge at 20 amperes?

**Prob. 12–14.** A 240-ampere-hour battery of three cells has an internal resistance of 0.0018 ohm per cell. What would be the difference in terminal voltage on charge and discharge at normal rate?

**27. Efficiency.** The efficiency of a storage battery is the ratio of the watthours output to the watthours input. This ratio will vary for different charge and discharge rates. The normal efficiency is the ratio at the normal 8-hour rate of charge and discharge.

The impressed voltage on charge is greater than the emf of the cell by the amount of the $IR$ drop; and the terminal voltage on discharge is less than the emf of the cell by the amount of this drop. This difference between the terminal voltages on charge and discharge is further increased by the fact that the emf itself is less on discharge than on charge, due to the electrolyte changing its density.

The average charging voltage =

$$\text{average emf of cell} + IR \text{ drop of cell.}$$

The average discharging voltage =

$$\text{average emf of cell} - IR \text{ drop of cell.}$$

**Example 4.** Assume the average charging voltage of the cell of Example 3 to be 2.32 volts and the average terminal voltage on discharge to be 1.95 volts. What is the efficiency of the cell?

**Solution.**

$$\begin{aligned} \text{Average watthours output} &= 1.95 \times 20 \times 8 = 312; \\ \text{Average watthours input} &= 2.32 \times 21.5 \times 8 = 398; \\ \text{Efficiency} &= \tfrac{312}{398} \times 100 = 78.3 \text{ per cent.} \end{aligned}$$

Good efficiency for a fair-size battery cell may be considered as about 75 per cent, while that of large batteries for central stations may be as high as 85 per cent.

**28. Advantages in Use of Storage Cells.** In general, the uses to which a storage battery may be put are three:

(1) To help the generators carry the "peak" of the load. That is, the battery is charged when the load is light. Then, when the load is too heavy for the generators to carry alone, the battery is placed in parallel with the generators to take part of the load. This use allows the installation of much smaller generators, engines and boilers.

(2) To take care of a long-continued light load and allow the engine and generator to be shut down.

The battery is here charged while the generator is taking care of its minimum load, and only thrown in when the generator is shut down. The engine is run steadily at its most efficient load, but no fuel is wasted in keeping it running to supply a light load. Often the battery takes care of its load with no attendant present.

This use saves in the cost of both fuel and labor.

(3) As an auxiliary source of supply in case of accident. Many companies now supply power under heavy forfeits to keep the line "alive" at all times. This necessitates some reserve source of supply which can be drawn upon at a moment's notice. Storage batteries meet this demand wherever a source of direct current is required.

**29. Use on Constant Potential Line. Rheostat Control and End-cell Control.** The voltage of a storage cell is seen to fall off as the cell discharges. If a battery of cells is to be used to supply power to lamps, some method must be devised to keep the voltage across the line constant.

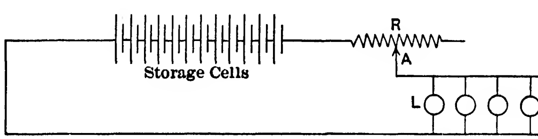



FIG. 18–14. The resistance R controls the voltage across the lamps.

One method shown in Fig. 18–14 is to use a resistance $R$ in a series with the cells. This resistance can be cut out as the voltage falls. This method is called **rheostat control.**

Suppose 115 volts were required across the lamps $L$. When the cells were nearly discharged, each would have a voltage of 1.75 volts on normal rate of discharge. There would be required then to furnish this pressure $\frac{115}{1.75}$ or 66 cells.

When the cells were fully charged, each would have a terminal voltage on discharge of about 2 volts. If 66 cells were used, the line voltage would then be $66 \times 2 = 132$ volts, which is $132 - 115 = 17$ volts in excess of pressure required. A resistance $R$ is, therefore, generally inserted between the line and the fully charged cells which will take up this 17 volts in the $IR$ drop across it and leave only 115 volts across the line.

As the voltage gradually falls on discharge, an operator from time to time cuts out some of this resistance $R$ and keeps the voltage across the line constant.

Another method is to cut in and out a certain number of the cells at the end of the series, as more or less voltage is required by the load. This is called **end-cell control.**

In Fig. 19–14 cells $A$, $B$, $C$, $D$, $E$, $F$, $G$ are called the **end cells.** As the cells become partly discharged and the voltage of each falls, the arm, $M$, of the end-cell switch is moved out toward $G$, throwing more cells across the line and thus raising the voltage. (See any Electrical Handbook for description of this switch.)

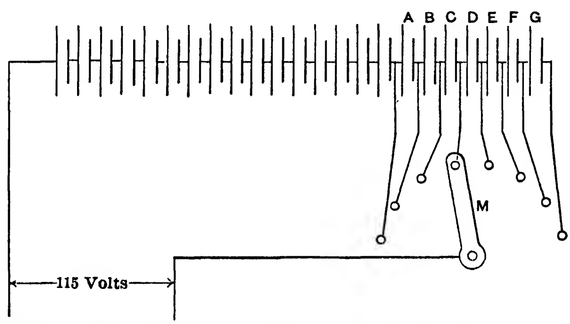



FIG. 19–14. The method of end-cell control of lead storage batteries.

**Prob. 13–14.** Assume each lamp in Fig. 18–14 takes 10 amperes at 112 volts. Assume terminal voltage of each of the 64 cells used in series to be 1.8 volts when low, and 2.0 volts when fully charged. What value would resistance $R$ have to be in order to keep the voltage across the lamps constant at 112 volts?

**Prob. 14–14.** How many end cells might be used in Prob. 13–14, instead of resistance $R$?

**Prob. 15–14.** Assume each cell to have an internal resistance of 0.002 ohm, and emf of each cell to range from 2.2 to 2.0 volts. How many cells would have to be used to maintain 110 volts across a set of lamps as in Fig. 18–14, if each lamp takes 4.5 amperes?

**Prob. 16–14.** If a resistance $R$ were used to control voltage in Prob. 15–14, what value would it have to be?

**Prob. 17–14.** How many end cells could be used in Prob. 15–14 to maintain constant voltage at lamps?

**30. Methods of Charging Storage Cells.** In charging a battery where the **end-cell** method of control is used, all the cells are thrown in at first. Since the end cells are used a very much shorter

time than the rest, they are not discharged to any great extent, and require very little charging. As soon as one of the end cells becomes fully charged (as determined by methods previously described), it is cut out by moving the switch arm back toward $A$, Fig. 19–14.

It has been shown that the voltage required to charge a cell is always greater than the terminal voltage of the cell on discharge, mainly on account of the internal resistance and polarization of the cell.

The equations previously given, showing the voltage which a cell will give on discharge and the voltage which is required to charge it, bring out this fact plainly.

$$V_c = E + IR_1; \quad (1)$$

$$V_d = E - IR_1; \quad (2)$$

where $V_c$ = terminal voltage required to charge;
$V_d$ = terminal voltage on discharge;
$E$ = emf of cell;
$I$ = current;
$R_1$ = internal resistance (virtual, including the effect of polarization).

If we subtract (2) from (1),

$$V_c - V_d = 2\,IR_1.$$

That is, the voltage required to charge is greater than the terminal voltage on discharge by twice the $IR_1$ drop of the cell.

Now since the charging voltage is always higher than the discharge voltage, it has been shown that some pressure higher than the line pressure must be used to charge. If the line can be disconnected from the generator during charge, then the voltage of the generator can be raised sufficiently to get the required charging pressure. But if the generator must supply the line and charge the battery at the same time, a small low-voltage generator, designed to carry whatever current goes through the battery, is often connected in series with the main generator and batteries. This small generator is called a **booster.** Its purpose is to raise the voltage across the cells. Figure 20–14 shows the method of using a booster.

Assume the generator $G$ must maintain 115 volts across the line and at the same time charge battery $S$, of 64 cells. The battery would require from 132 to 160 volts to charge (depending largely upon the degree of discharge). The small generator $B$ is put in series with the generator across the battery and raises the voltage enough to charge the cells.

There are many types of automatic and hand-controlled boosters, for which see any Handbook on Electrical Engineering.

For small installations, the battery is usually divided into two or three parallel parts during the charging. The excess voltage of the generator can then be taken up by a rheostat. On discharge, of course, the parts are again connected in series.

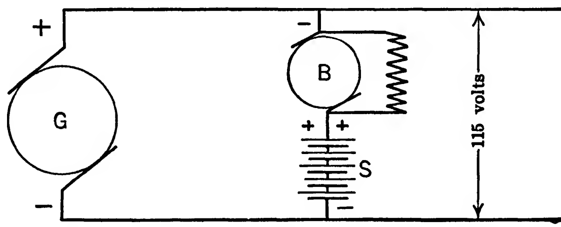



FIG. 20–14. The booster method of charging of lead cells.

**31. Floating Batteries.** There are certain conditions under which storage batteries may be used continuously on the line and be charged and discharged automatically without any special apparatus such as a booster, etc.

Suppose the batteries are placed directly across the line **at some distance** from the generator, or in such a way that there is considerable resistance in the line wires between the generator and batteries. This resistance would cause a large line drop when a heavy current was sent through the line. If the line drop were enough to cause the impressed voltage across the cells to fall below the emf of the cells, they would begin to discharge current into the

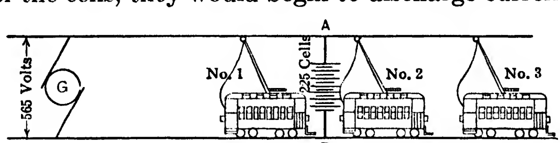



FIG. 21–14. The battery is "floating" on the line AB.

line and so aid the generator. If only a small current were sent through the line, the line drop would be small and the impressed voltage across the cells would be higher than their emf. The cells would then charge and take current from the line.

In Fig. 21–14, there is 0.80 ohm in the line (trolley and track) from generator to car No. 1. When the car is taking 100 amperes, there is a line drop of $0.80 \times 100 = 80$ volts. The voltage across the line at this point is then $550 - 80 = 470$ volts.

Assuming the emf of one cell to be about 2.0 volts, if we place a bat-

tery of 242 cells across the line at this point, we should have an emf of $242 \times 2.0 = 484$ volts; that is, 14 volts $(484 - 470)$ more than the impressed voltage. The battery would then discharge into the line and help the generator to furnish current to the three cars.

If, however, car No. 1 were taking only 10 amperes, and cars Nos. 2 and 3 were taking no current, there would be a line drop of only $10 \times 0.80 = 8$ volts. The voltage across the points $AB$ would then be $550 - 8 = 542$ volts, which is 58 volts $(542 - 484)$ higher than the emf of the storage battery across the line at that place. The cells would then charge and draw current from the line. (Of course in taking more current from the line, the cells would cause a still greater drop in the line voltage from generator to cells, so that there would exist considerably less than the 58 volts difference between the battery emf and the line voltage.)

Batteries can be "floated" only at a point where there is a large fluctuation in the voltage impressed across them by whatever method it is obtained.

**Prob. 18–14.** In a system arranged as in Fig. 21–14, the generator terminal voltage is 565 volts. Trolley is No. 00 copper wire. Track has resistance of 0.03 ohm per mile. How far from generator should a set of 225 lead storage cells be placed in order to "float" satisfactorily? Maximum current in section out to cells is 150 amperes. Average emf of each cell 1.98 volts.

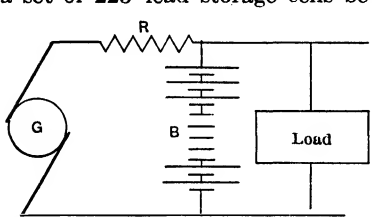



FIG. 22–14. The diagram of a floating battery B.

**Prob. 19–14.** In a system arranged as in Fig. 22–14, generator has a resistance of 0.2 ohm, emf of 120 volts. $R = 0.36$ ohm. $B$ consists of 56 cells of 2 volts emf. How many amperes must load use in order that the battery $B$ may deliver current to the line? Neglect line resistance.

**32. Edison Storage Battery.** A storage battery invented by Thomas A. Edison and manufactured by the Edison Storage Battery Company uses an alkaline, instead of an acid solution, as the electrolyte. See Fig. 23–14.

The positive plate consists at first of nickel hydrate, $Ni(OH)_2$, held in perforated steel tubes firmly attached to a steel grid.

The negative plate consists of iron oxide, $FeO$, held in pockets on a metal plate.

The electrolyte is a 21 per cent solution of caustic potash, $KOH$, in pure distilled water. Specific gravity, 1.20.

The retaining jar, instead of being either glass or vulcanite as in the case of other storage cells, is made of nickel-plated sheet steel. The chemical actions which take place within the cell are not yet clearly understood. The following are supposed to approximate the reactions:

When the cells are **first charged,** the nickel hydrate, $Ni(OH)_2$, is changed to nickel oxide, $NiO_2$; and the iron oxide, FeO, is reduced to metallic iron, Fe. The nickel oxide, $NiO_2$, soon decomposes to a lower oxide, $Ni_2O_3$. The cell may thus be considered to consist of a positive plate of nickel oxide, $Ni_2O_3$, and a negative plate of pure iron, Fe.

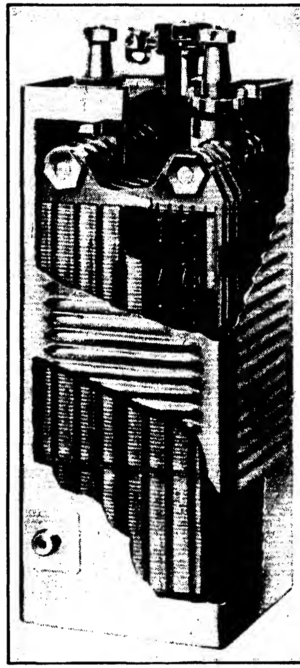


FIG. 23–14. A cutaway view of an Edison cell. *Edison Storage Battery Co.*

**33. Edison Storage Battery on Discharge.** While the nickel oxide, $NiO_2$, is going to a lower oxide of nickel, $Ni_2O_3$, oxygen is liberated which is negatively charged and goes over to the iron plate, gives up its negative charge, and unites with the plate to form iron hydroxide, $Fe(OH)_2$. The density of electrolyte does not change, which effect differs from that of the lead cell. The potassium hydroxide seems to act merely as a catalyzer, or carrier.

**34. Edison Cell on Charge.** The iron hydroxide, $Fe(OH)_2$, is broken up and the negatively charged oxygen leaves the iron plate, travels back against the current through the cell and unites with the positive nickel oxide, $Ni_2O_3$, and forms a higher oxide, $NiO_2$.

The following equation shows the chemical reaction:

$$Fe + Ni_2O_3 \cdot 1.2\,H_2O + 1.8\,H_2O \underset{\text{CHARGE}}{\overset{\text{DISCHARGE}}{\rightleftarrows}} Fe(OH)_2 + 2\,Ni(OH)_2$$

**35. Physical Changes in the Edison Cell.** As stated above, the specific gravity of the electrolyte lowers slightly on charge; but the change is not enough to be used as an indication of the condition of the cell. Some other method, then, must be used to determine the condition of this form of storage cell as to charge and discharge.

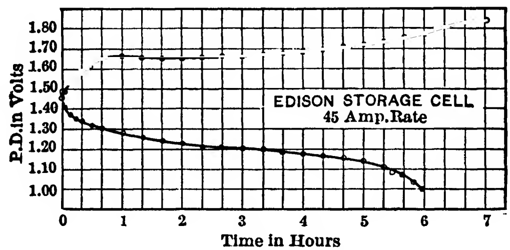



Fig. 24–14. The relation of terminal voltage to time of charge and discharge in an Edison cell.

Figure 24–14 shows some curves plotted from data taken on an Edison storage cell, type A6.

Note that on discharge, the voltage falls very rapidly at first as in a lead cell. Unlike a lead cell, however, there is no value of voltage at which it remains nearly constant, but the curve has a general downward trend.

Near the point of complete discharge, the voltage falls off rapidly again, as in a lead cell. The condition of the cell may thus be determined closely by the voltage on discharge.

The average terminal voltage on discharge at normal current is 1.2 volts. The lowest voltage is 1 volt.

**36. Comparison of Lead and Edison Cells.** Average voltage per cell on discharge: Lead cell, 1.95 volts; Edison cell, 1.2 volts. Watthours per pound of cell complete: Lead cell, 8.5 watthours; Edison, 16.8 watthours.

Internal resistance: The Edison cell has a slightly higher internal resistance; the efficiency is therefore lower.

It is claimed for the Edison cell that it can stand for an indefinitely long time, charged or discharged, and not be injured. A lead cell should not remain any length of time **uncharged,** and only a few months **charged.**

The temperature range at which an Edison cell will work best is limited to the region around 70° F. Any condition of use, which tends to lower greatly the temperature, decreases the efficiency

to a marked extent. The lead cells are not so limited in this respect.*

**37. Other Electrochemical Processes.** Electrochemistry is a profession by itself. It has rapidly become of fundamental importance in our complex life. A large number of chemical processes are now conducted by the aid of electricity and can only be given passing mention here.

Fused salts, in place of salts dissolved in water, may be decomposed by electrolytic action. This often requires the high temperature obtained from the electric furnace. In other cases, moderate temperatures may be used.

Aluminum is manufactured in this manner by electrically decomposing an ore of aluminum such as bauxite, $H_3AlO_3$. Aluminum requires the largest amount of electricity per pound of any metal obtained commercially by electrolysis. One company operates a battery of thirty-six 2500-kilowatt converters in a single plant to supply current for producing aluminum. The cost of aluminum is largely determined by the cost of electrical energy. Metallic sodium and potassium are similarly produced, but by the use of much higher temperature.

Gaseous hydrogen and oxygen are commercially produced in one process by the electrolytic decomposition of water.

The fixation of atmospheric nitrogen for the production of fertilizers and explosives is of vast importance to the agricultural and military interests.

Carborundum, calcium carbide, fused quartz, silicon, graphite, phosphorus and steel alloys of various sorts are typical important electric-furnace products.

Immense quantities of caustic soda and chlorine are now produced commercially from the electrolysis of sodium chloride solution in specially constructed cells. A chlorine plant at Edgewood, Md., has a capacity of $12\frac{1}{2}$ tons of gaseous chlorine per day.

The growth of the electrochemical industries has kept pace with that of electric power development. Because of the nature of the processes, power for this purpose can often be obtained very cheaply by using the power when other demands are low.

* For further information concerning storage batteries, the student is referred to Vinal, " Storage Batteries."

## SUMMARY OF CHAPTER XIV

When two electrical conductors are immersed in a conducting liquid, so that they do not touch each other, they are called ELECTRODES, and the liquid is called an ELECTROLYTE. When the current flows through this circuit, the conductor at which the current enters the liquid is called the ANODE; and that at which it leaves the liquid is called the CATHODE.

ELECTROLYTIC CONDUCTION is a name given to the carrying of electric charges through a fluid by the flow of ions.

Ions are molecules or atoms with more or fewer electrons attached than normal conditions demand. In a solid conductor, the atoms and molecules do not flow, the current consisting of a flow of electrons only.

ELECTRIC BATTERIES are devices for transforming chemical energy into electrical energy. They require two unlike conductors, called positive and negative plates, and an electrolyte which acts chemically upon one of the plates.

A PRIMARY CELL is an electric battery in which the worn-out element is replaced by another. The negative element is generally of zinc.

A STORAGE CELL is an electric battery in which the worn-out element is replaced by the electrolytic action of an electric current forced through cell in reverse direction. The elements are generally of lead and lead compounds.

ACTION IN CELL. The electrolyte is broken up into two oppositely charged parts called ions. The positively charged ions give up their charges to positive plate, and negatively charged ions give up their charges to negative plate, part of which unites with the ions to form a new compound. This constitutes the chemical action in the cell. A difference in potential between the plates is thus produced.

POLARIZATION. Hydrogen bubbles collect on the positive plate and increase the resistance of cell, and lower the emf by setting up a counter emf.

DEPOLARIZATION. Some oxidizing agent is used as a depolarizer to remove the hydrogen by uniting with it to form water. Cells which are depolarized rapidly are used for closed-circuit work, those which are depolarized more slowly, for open-circuit or intermittent work.

ELECTROLYSIS. When an electric current is sent through a solution containing a metal salt, it will deposit the metal on the negative plate. If chemical action takes place between the electrolyte and positive plate, the positive plate is consumed.

ELECTROCHEMICAL EQUIVALENT. The amount of metal deposited on the negative plate, and the amount taken from the positive plate, by an ampere-hour of electricity is a constant, depending upon metals and electrolyte.

ELECTROPLATING. Use is made of electrolysis to plate conducting materials; to refine metals; to make electrotypes, etc., and to restore the active elements in a storage battery.

DESTRUCTION OF WATER MAINS, ETC., BY ELECTROLYSIS. The leakage currents from the return circuits in electric railways and

other sources may travel along iron pipes and steel reinforcing of concrete, and are likely to eat away portions of the iron at the point where they leave the iron to return to the generator.

STORAGE CELLS. Do not store electricity, but chemical energy. Common types composed of lead peroxide, positive plate; and spongy lead, negative plate.

Action on charge and discharge:

$$PbO_2 + Pb + 2\,H_2SO_4 \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftarrows}} 2\,PbSO_4 + 2\,H_2O$$

RATING OF STORAGE CELLS is given in ampere-hours, based on the current which will discharge the battery to a safe or economical limit in 8 hours. This is known as the 8-hour rate. A battery which will furnish 40 amperes for 8 hours is called a 320 ($40 \times 8$) ampere-hour battery.

AMPERE-HOUR EFFICIENCY is the ratio,

$$\frac{\text{ampere-hours output}}{\text{ampere-hours input}}.$$

Ampere-hours input is the charge required to restore the battery from a discharged condition to that of full charge.

EFFICIENCY OF A BATTERY is the ratio,

$$\frac{\text{watthours output}}{\text{watthours input}}.$$

CARE OF STORAGE CELLS. Storage cells, to have longest life and maximum efficiency, should not be subjected to:

(1) Too rapid charging.
(2) Use of impure electrolyte.
(3) Use of too dense or too light electrolyte.
(4) Over-charging and over-discharging.

CONDITION OF CELLS. Can be ascertained:

(1) By color of plates.
(2) By specific gravity of electrolyte.
(3) By terminal voltage at normal charging or discharging rate (not by the emf).

USE OF STORAGE CELLS.

(1) To help carry " peak " of load.
(2) To carry all the " light load."
(3) As a reserve supply of electrical energy.

PRACTICE IN USE. Voltage across cells may be controlled by:

(1) Resistance in series with cells.
(2) End cells.
(3) Floating the cells on the line at a point of considerable voltage fluctuation.

**EDISON STORAGE CELLS. Positive element, nickel oxide, changes to lower oxide on discharge.**

**Negative element, spongy iron, changes to iron oxide on discharge.**

**The electrolyte, 21 per cent solution of potassium hydrate, remains at same specific gravity throughout charge and discharge.**

**Edison cells are lighter per watthour than most lead cells, but have lower emf per cell. They can remain charged or discharged indefinitely without deteriorating. The efficient temperature range is limited.**

**A LARGE NUMBER OF CHEMICAL PROCESSES are now conducted by the aid of electricity. Aluminum, oxygen, caustic soda and chlorine are a few of the more common substances commercially produced by electrolytic action.**

## PROBLEMS ON CHAPTER XIV

**Prob. 20–14.** The zinc plate of a certain battery cell weighs 8 ounces. How many hours will the battery cell last, if it is required to deliver an average current of 2.5 amperes?

**Prob. 21–14.** A cell is desired which will deliver an average of 0.05 ampere for 2 months of 30 days. What weight should the zinc plate have?

**Prob. 22–14.** A 280-ampere-hour lead storage-vehicle battery has an average voltage of 32 volts. How much energy can be drawn from it?

**Prob. 23–14.** If the average voltage required during charge of the battery of Prob. 22–14 is 40 volts and the time of charge 10 per cent longer than the time of discharge at normal rate, what is the efficiency of the battery? What is the ampere-hour efficiency?

**Prob. 24–14.** How long must a current of 500 amperes run in an electro-refining vat to deposit enough copper to make one mile of No. 0000 wire?

**Prob. 25–14.** If the electric energy of Prob. 24–14 costs 1.25¢ per kilowatthour and the refined copper is worth 20¢ per pound, at what voltage must the refining vat be run to make the cost of energy equal 5 per cent of the cost of refined copper?

**Prob. 26–14.** If the internal resistance of a nickel-plating bath is 0.15 ohm, and 1 volt is applied, what energy in kilowatthours is used in plating one pound of nickel?

**Prob. 27–14.** If the voltage in Prob. 26–14 is increased to 1.2 volts, what will be the new amount of energy per pound?

**Prob. 28–14.** Ten thousand articles, each with a surface area of 1 square foot, are to be plated with 0.003-inch-thick nickel coat. Take the weight of nickel as 0.311 pound per cubic inch. If 1 volt is used in the solution of Prob. 26–14 and the energy cost is 3¢ per kilowatthour, what is the energy cost per article?

**Prob. 29–14.** If the labor cost for plating bath attendance is 4¢ per article under the conditions of Prob. 28–16, and this can be reduced to 3¢ if the time of plating be halved, will it pay to use a 2-volt supply?

**Prob. 30–14.** Hydrogen under standard conditions of temperature and pressure weighs 0.0898 ounce per cubic foot. How long will it take 1000 amperes to liberate enough hydrogen by electrolytic decomposition to fill a balloon which is spherical and 40 feet in diameter?

**Prob. 31–14.** At a voltage of 1.5, and at 2¢ per kilowatt hour, what is the cost of energy for filling the balloon of Prob. 30–14?

**Prob. 32–14.** Compare the energy costs of plating nickel and silver (assuming the resistance of the plating bath to be the same in each case) for equal weights deposited, and for equal thickness on an article. (Density of silver is 0.380 pound per cubic inch; of nickel, is 0.315 pound per cubic inch.)

**Prob. 33–14.** A dry cell developing 1.5 volts will supply 1 ampere intermittently for a total of 50 hours. The cell costs 25¢, new. What is the cost per kilowatthour of power obtained in this way?

**Prob. 34–14.** An iron water pipe receives a stray current of 5 amperes from a trolley system. Where this current leaves the pipe, electrolysis occurs. How long will it take to remove 10 pounds of the iron of the pipe?

**Prob. 35–14.** A battery of 30 lead storage cells in series is to be charged at normal rate of 15 amperes from a 115-volt generator. How much resistance must be placed in series with the cells, assuming 2.1 volts emf and 0.005-ohm resistance for each cell.

**Prob. 36–14.** If cells in Prob. 35–14 are to be discharged in series at normal rate, what will be the terminal voltage of the battery?

**Prob. 37–14.** A set of 80 lead storage cells, each having 2.1 volts emf and 0.003-ohm internal resistance, is to be charged in two parallel sets of 40 cells in series. Each cell has a normal discharge rate of 40 amperes. What must be the terminal voltage of the generator?

**Prob. 38–14.** What must be the emf of the generator in Prob. 37–14, if its internal resistance is 0.025 ohm?

**Prob. 39–14.** (*a*) What is the ampere-hour capacity of the set of cells in Prob. 35–14? (*b*) What is the kilowatthour capacity of the set?

**Prob. 40–14.** (*a*) What is the ampere-hour capacity of each cell in Prob. 37–14? (*b*) What is the kilowatthour capacity of the set?

**Prob. 41–14.** A battery of 40 lead storage cells in series is to be charged at normal rate of 30 amperes from a 115-volt circuit by means of a series resistance. The emf per cell is 2.1 volts when charged and 1.8 volts when discharged. The resistance per cell is 0.0035 ohm when charged, and 0.0038 ohm when discharged. (*a*) What resistance must be connected in series with the battery, when discharged, to charge it

at a normal rate? (*b*) When the cell reaches full charge, how much resistance in (*a*) must be cut out to keep the charging rate to normal value?

**Prob. 42–14.** A set of 90 lead storage cells, each having 2.0 volts emf and 0.004-ohm internal resistance, is to be charged from a 115-volt line. If each cell is to take its normal current of 20 amperes, what will be the best arrangement of cells in order to have the least power lost in the series resistance in the line? Show calculations.

**Prob. 43–14.** How could the cells in Prob. 36–14 be arranged in order to deliver 160 amperes, and not exceed normal current of each cell? At what voltage would they deliver this current? What would be the current per cell?

**Prob. 44–14.** An electric circuit consists of the armature of a separately excited generator, two connecting wires and a storage battery, all in series. The armature has a resistance of 0.15 and generates an emf of 115 volts. Each of the connecting wires has a resistance of 0.1 ohm. The battery has a resistance of 0.12 ohm and generates an emf of 110 volts opposed in direction to the emf of the generator. (*a*) Find the terminal voltage of the generator. (*b*) Find the terminal voltage of the battery.

**Prob. 45–14.** A storage battery of 250 cells in series is "floated" at one end of a 4-mile trolley line, the resistance of which (line and return) is 0.08 ohm per mile. At the other end of the line, a generator maintains a voltage of 575 volts. Each cell of the battery has an emf of 2.1 volts and an internal resistance of 0.001 ohm. What current will the battery supply to the line when there are 6 cars at the battery end of the line, each taking 60 amperes?

**Prob. 46–14.** What will be the terminal voltage of the battery in Prob. 45–14?

**Prob. 47–14.** What current will the battery in Prob. 45–14 take when there is no load on the line?

**Prob. 48–14.** What current will the generator be delivering when the 6 cars of Prob. 45–14 are halfway between the generator and the battery?

**Prob. 49–14.** What current will the battery be delivering to, or receiving from, the line in Prob. 48–14?

**Prob. 50–14.** If there are but 2 cars on line in Prob. 48–14 each taking 75 amperes: (*a*) What power is generator delivering? (*b*) What power is battery receiving or delivering?

# CHAPTER XV

## KIRCHHOFF'S LAWS AND APPLICATIONS

Many circuits, sometimes called "networks," occur in direct-current practice which cannot be classified simply as series, parallel or series-parallel circuits. In order to readily solve such circuits, it is necessary to amplify Ohm's law as developed in preceding chapters.

**1. Kirchhoff's Laws. An Extension of Ohm's Law.** Two simple but fundamental laws were first deduced from Ohm's law by the German scientist Gustav Kirchhoff, and are called Kirchhoff's Laws. These laws are simply a restatement of the relations existing in any circuit in accordance with the provisions of Ohm's law.

Kirchhoff stated:

**First Law. The algebraic sum of the current at any junction point in a circuit is always equal to zero.** This is equivalent to saying that there is always as much current flowing **away** from a point in a circuit as there is flowing to the point.

**Second Law. Around any one path in a closed circuit, the algebraic sum of the electromotive forces equals the algebraic sum of all the resistance drops.** In other words, the sum of all electromotive forces and all the resistance drops, taken with their correct signs, is zero. It is also true that if there is no electromotive force in any given circuit or part of a circuit, the sum of the $IR$ drops through one path, between any two junction points, is always equal to the sum of the $IR$ drops through any other path, between these same two points. This is in accord with the laws for parallel circuits, as discussed in Chapter I.

The **first law** can best be explained by referring to the diagram of Fig. 1–15.

Assume each lamp to take 1 ampere and consider the point $B$. Since there are $5 + 3$ or 8 amperes flowing away from $B$, there must be 8 amperes flowing to $B$ along $AB$.

Consider the point $E$. Since there are $5 + 3$ or 8 amperes flowing to $E$, there must be 8 amperes flowing away from $E$ along $EF$.

It is apparent that if the above conditions did not hold, there would be a storage of electricity (an accumulation or a scarcity of electrons) at these junction points, which is impossible in this circuit.

We have unconsciously used this law, again and again, considering it too obvious to need demonstration.

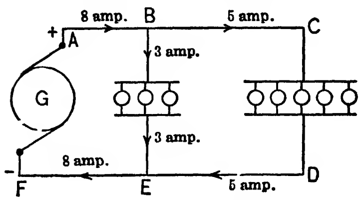



FIG. 1–15. The amount of current flowing to point B, equals the amount flowing away from it.

The **second law** is as obvious as the first, but more difficult to state clearly.

In the circuit $ABEFA$ of Fig. 1–15, the emf of the generator, $G$, must equal the $IR$ drop in $AB + BE + EF +$ the $IR$ drop in the generator itself.

Or, considering another complete circuit, $ABCDEFA$, on the diagram: the emf of the generator must equal the $IR$ drop in $AB + BC + CD + DE + EF +$ generator $IR$ drop.

Or, again, considering the circuit $BCDEB$, which contains no emf, the $IR$ drop in $BE$ must be equal to the $IR$ drop in $BC + CD + DE$, since the $IR$ drop in $BE$ is counter-clockwise in the circuit $BCDEB$, and the $IR$ drop in $BC + CD + DE$ is clockwise.

We have been accustomed to use this same principle under two separate rules: (first), that the $IR$ drop around any path in an electric circuit equals the sum of the impressed emf's; and (second), that the voltage drop along parallel parts between two points is the same. Thus we would have said that the voltage across $BE$ equals the voltage across $BC + CD + DE$; because both paths are in parallel between the points $B$ and $E$.

Therefore, Kirchhoff's laws state no new facts; but simply present new viewpoints from which to regard familiar facts.

However, in the application of these laws, certain rules and methods of procedure must be followed to avoid confusion; especially in complicated circuits, as will be explained in the next article.

**2. Application of Kirchhoff's Laws.** The **first law** states that the algebraic sum of the currents at any junction point must equal zero.

**Rule 1.** The currents flowing in either direction may be assumed positive and those flowing in the opposite direction as negative.

Thus, in Fig. 2–15, currents $I_1$ and $I_2$ are flowing to point $O$ and currents $I_3$, $I_4$ and $I_5$ are flowing away from it. If we assume $I_1$ and $I_2$ as positive, then $I_3$, $I_4$ and $I_5$ are negative; and

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0. \quad (1)$$

Substituting the values of the currents in the figure,

$$12 + 10 - 9 - 8 - 5 = 0;$$

or transposing equation (1),

$$I_1 + I_2 = I_3 + I_4 + I_5; \quad (2)$$

and substituting,

$$12 + 10 = 9 + 8 + 5.$$

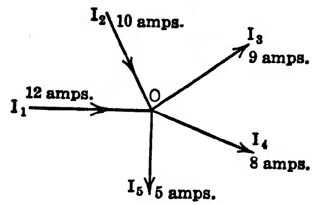



FIG. 2–15. The algebraic sum of the currents at point O equals zero.

**Rule 2.** In many networks, **the direction of the current is uncertain.** In such cases, it may be **assumed. If the solution results in a negative value, it simply means that the actual direction is opposite to that assumed. The numerical result, however, will be correct.**

The **second law** states that the sum of all the electromotive forces and all the $IR$ drops in a closed circuit, taken with their **proper sign,** is zero.

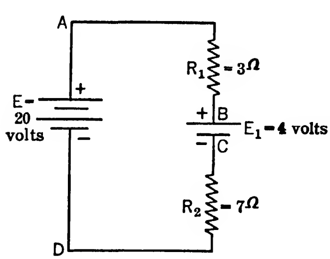



FIG. 3–15. The algebraic sum of the emfs and IR drops equals zero.

Thus, in the simple circuit of Fig. 3–15, the voltage of the battery, $E$, is impressed on a circuit, consisting of resistors $R_1$ and $R_2$ and another battery, so connected that its voltage $E_1$ is opposed to the impressed voltage. If the voltage $E$ is taken as positive, then $E_1$ is negative. The $IR$ drops in the resistors $R_1$ and $R_2$ are also each equivalent to a back voltage opposing $E$, and are, therefore, also negative; and we write

$$+E - IR_1 - E_1 - IR_2 = 0; \quad (3)$$

or

$$E = IR_1 + E_1 + IR_2; \quad (4)$$

or

$$E - E_1 = IR_1 + IR_2; \quad (5)$$

(the algebraic sum of all the emf's is equal to the algebraic sum of all the $IR$ drops).

Substituting the values of the emf's and resistances in the figure, in any one of the above equations, give the same result.

Using equation (5) and neglecting the internal resistance of the batteries,

$$20 - 4 = 3\,I + 7\,I;$$

or

$$16 = 10\,I \quad \text{and} \quad I = \frac{16}{10} \text{ or } 1.6 \text{ amperes.}$$

Substituting the value of $I$ above in equation (3),

$$20 - (1.6 \times 3) - 4 - (1.6 \times 7) = 0.$$

In the application of the second law to networks, the determination of the proper signs for the emf's and $IR$ drops sometimes is confusing, but is not troublesome, if the following simple rule is observed.

**Rule 3.** In tracing through a circuit,

a **rise** in voltage always indicates a (+) sign;
a **drop** in voltage always indicates a (−) sign.

Thus, starting from any point $D$, in Fig. 3–15, and tracing clockwise through the circuit: from the (−) terminal of battery $E$ to the (+) terminal, there is a **rise** in voltage and $E$ is **positive**; from $A$ to $B$, through resistance $R_1$, there is a **drop** in voltage and $IR_1$ is **negative**; from $B$ to $C$, the (+) terminal to the (−) terminal of battery $E_1$, there is a **drop** in voltage and $E_1$ is negative; and from $C$ to $D$ through resistor $R_2$, there is a **drop** in voltage and $IR_2$ is negative. These signs agree with those in equation (3) above.

We may trace through the circuit in the opposite direction, or against the direction of current flow. For, again starting at $D$, in Fig. 3–15: there is a **rise** in voltage from $D$ to $C$ and we call $IR_2$ **positive**; a **rise** in voltage from the (−) terminal to the (+) terminal of battery $E_1$, and $E_1$ is **positive**; similarly, a **rise** in voltage from $B$ to $A$, and $IR_1$ is **positive**; and finally, a **drop** in voltage from the (+) terminal to the (−) terminal of battery $E$, and $E$ is **negative**; and

$$+\,IR_2 + E_1 + IR_1 - E = 0;$$

or

$$IR_2 + E_1 + IR_1 = E;$$

the same as equation (4).

**Rule 4.** (*a*) Kirchhoff's first law must be applied to enough junction points in a network to include every unknown current at least once. (*b*) The second law must be applied to enough branches so that the voltage and *IR* drop relations in every branch are included at least once.

The following examples will illustrate the application of Kirchhoff's laws to the solution of networks as discussed above.

**Example 1.** It is desired to determine the current distribution in the circuit of Fig. 4–15. The value of the various resistances and emf's are marked on the diagram. (The internal resistance of the batteries will have little effect on the solution and will be neglected for the sake of simplicity.)

**Solution.** The direction of currents in the branches of this circuit is uncertain, but common sense leads us to assume that current in $AB$ flows to point $B$ as marked on the diagram. Let this current $= I_1$;

Let current in branch $BG = I_2$, and assume it to flow in the direction marked on the diagram;

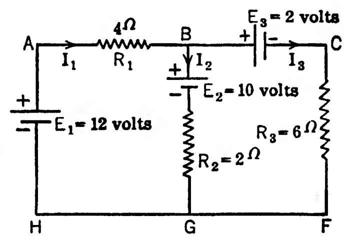


FIG. 4–15. A simple network in which the direction of current in at least one branch is uncertain.

Let current in branch $BCFG = I_3$, and assume it to flow in the direction marked.

Beginning at point $H$ and applying the **second law** to the circuit $HABGH$, according to the rule for signs, we write:

$$(a) \qquad +E_1 - I_1R_1 - E_2 - I_2R_2 = 0.$$

Substituting known values in the figure,

$$12 - 4\,I_1 - 10 - 2\,I_2 = 0.$$

Transposing,

$$12 - 10 = 4\,I_1 + 2\,I_2;$$

$$(b) \quad \text{or} \qquad I_2 = \frac{2 - 4\,I_1}{2} = 1 - 2\,I_1.$$

Also in circuit $HABCFGH$, starting at point $H$,

$$(c) \qquad +E_1 - I_1R_1 - E_3 - I_3R_3 = 0,$$

and substituting known values,

$$12 - 4\,I_1 - 2 - 6\,I_3 = 0;$$

transposing,

$$12 - 2 = 4\,I_1 + 6\,I_3,$$

$$(d) \quad \text{or} \qquad I_3 = \frac{10 - 4\,I_1}{6} = \tfrac{5}{3} - \tfrac{2}{3}\,I_1.$$

Applying the **first law** to point $B$ (also point $G$),

$$I_1 - I_2 - I_3 = 0;$$

or

$$I_1 = I_2 + I_3;$$

and substituting the values for ($b$) and ($d$),

$$I_1 = (1 - 2\,I_1) + (\tfrac{5}{3} - \tfrac{2}{3}\,I_1);$$

($e$) or $\frac{11}{3}\,I_1 = \frac{8}{3}$ or $I_1 = \frac{8}{11}$ ampere. *Ans.*

Substituting the value for $I_1$ in ($b$),

($f$) $I_2 = 1 - 2\,(\frac{8}{11}) = -\frac{5}{11}$ ampere. *Ans.*

**Note** that $I_2$ is **negative,** and, therefore, $\frac{5}{11}$ ampere flows in $BG$ in the **opposite** direction to that assumed.

Substituting the value of $I_1$ in ($d$),

$$I_3 = \tfrac{5}{3} - \tfrac{2}{3}\,(\tfrac{8}{11}) = \tfrac{13}{11} \text{ ampere.} \quad \textit{Ans.}$$

As a check on the solution, we may substitute the values of current, as computed above in ($a$) and

$$12 - 4\,(\tfrac{8}{11}) - 10 - 2\,(-\tfrac{5}{11}) = 0;$$

or substituting in ($c$),

$$12 - 4\,(\tfrac{8}{11}) - 2 - 6\,(\tfrac{13}{11}) = 0.$$

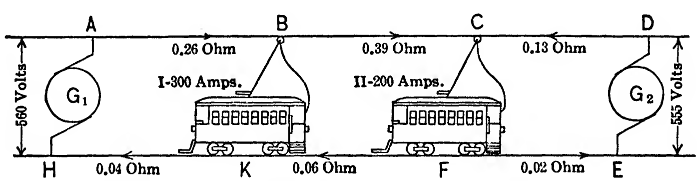



FIG. 5–15. A trolley line fed by two generators, one located at each end of the line.

**Example 2.** Assume trolley line, Fig. 5–15, to be fed by two generators, $G_1$ of 560 volts; and $G_2$ of 555 volts. Two cars are on the line; Car I taking 300 amperes, Car II, 200 ampere. The resistance of trolley and track as marked.

Find:

($a$) Voltage across each car.

($b$) Efficiency of transmission.

**Solution.** ($a$) Assume current to flow in direction indicated in different sections. The two cars take a total current of 500 amperes.

Let $x$ = current in section $AB$.

Then $x$ = current in section $HK$.

$x - 300$ = current in sections $BC$ and $KF$.

$500 - x$ = current in sections $CD$ and $EF$.

The above assumptions are made in accordance with the common-sense law that the amount of current which flows away from a point must be the same as that which flows to it. This, as we have seen, is called Kirchhoff's "first law."

Voltage across Car II equals

$$(555 - 0.15\,(500 - x). \tag{1}$$

Voltage across Car II also equals

$$560 - 0.30\,x - 0.45\,(x - 300). \tag{2}$$

Therefore

$$555 - 0.15\,(500 - x) = 560 - 0.30\,x - 0.45\,(x - 300). \tag{3}$$

These assumptions are made in accordance with the principle, stated in Chapter I, that the voltage across a parallel combination is the same as that across any and all of the parallel paths between the same two points. This, as we have seen, is part of Kirchhoff's "second law." Thus the voltage across Car II must be the same whether computed from the voltage of $G_2$ minus line drop, or from the voltage of $G_1$ minus the line drop from that generator.

Solving (3) for $x$,

$x$ = 239 amperes = current in $AB$ and $HK$.
Voltage across Car I = $560 - 239 \times 0.30$ = 488.3 volts.
Current in $CD$ = $500 - 239$ = 261 amperes.
Voltage across Car II = $555 - 261 \times 0.15$ = 515.8 volts.

Current in $BC = 239 - 300 = -61$ amperes (the minus sign means it must be flowing in the direction opposite to that marked).

If the solution is correct, substituting the value of $x$ in (1) and (2) should give the same result.

Thus, $555 - 0.15\,(500 - 239) = 515.8$ volts;
or $560 - 0.30\,(239) - 0.45\,(239 - 300) = 515.8$ volts. *Check.*

(*b*) Power delivered by $G_1 = 560 \times 239$ = 134,000 watts.
Power delivered by $G_2 = 555 \times 261$ = 145,000 watts.
Total power = 279,000 watts.

Power used by Car I = $488.3 \times 300$ = 146,500 watts.
Power used by Car II = $515.8 \times 200$ = 103,200 watts.
Total power used by cars = 249,700 watts.

$$\text{Efficiency of transmission} = \frac{249.7}{279} \times 100 = 89.7 \text{ per cent.} \quad \textit{Ans.}$$

**Example 3.** Determine the value of current in each branch and the impressed voltage on the circuit of Fig. 6–15, if the total current is 10 amperes. The value of the resistances is as marked on the diagram.

**Solution.**

**First:** Apply Kirchhoff's first law to the junction point $A$. Since there are 10 amperes flowing to $A$, there must be 10 amperes flowing away from it.

Let $\qquad x$ = current flowing from $A$ to $B$ as marked;
then $\quad 10 - x$ = current flowing from $A$ to $C$ as marked.

At point $B$, the direction of current in $BC$ is uncertain, but **assume** it to be flowing from $B$ to $C$ as marked.

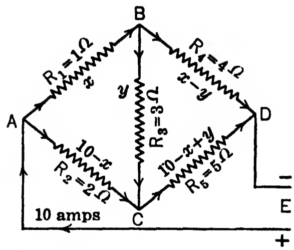



FIG. 6–15. A more complicated network.

Let $\qquad y$ = current flowing in $BC$;
then $\quad x - y$ = current flowing in $BD$.

At point $C$: $(10 - x)$ amperes and $y$ amperes are flowing to this point and current flowing from $C$ must be their sum.

Let $(10 - x) + y$ = current flowing in $CD$, as marked.

We have now considered enough junction points to include every unknown current at least once, according to Rule 4. If our assumptions are logical, the sum of the currents flowing to point $D$ must be equal to the current flowing away from it, or 10 amperes; and we see that

$$(x - y) + (10 - x + y) = 10.$$

**Second:** Apply the second law to the various branches in the circuit. In the circuit $ABD$:

$$E - xR_1 - (x - y)\,R_4 = 0;$$

or

$$E = xR_1 + (x - y)\,R_4.$$

Substituting known values,

(1) $$E = 1\,x + 4\,(x - y) = 5\,x - 4\,y.$$

In the circuit $ABCD$:

$$E - xR_1 - yR_3 - (10 - x + y)\,R_5 = 0;$$

or

$$E = xR_1 + yR_3 + (10 - x + y)\,R_5.$$

Substituting known values,

$$\begin{aligned} E &= 1\,x + 3\,y + 5\,(10 - x + y) \\ &= 1\,x + 3\,y + 50 - 5\,x + 5\,y \\ &= 8\,y - 4\,x + 50. \end{aligned} \tag{2}$$

In the circuit $ACD$:

$$E - (10 - x)\,R_2 - (10 - x + y)\,R_5 = 0;$$

or

$$E = (10 - x)\,R_2 + (10 - x + y)\,R_5.$$

Substituting values,

$$\begin{aligned} E &= (10 - x)\,2 + (10 - x + y)\,5 \\ &= 20 - 2\,x + 50 - 5\,x + 5\,y \\ &= 5\,y - 7\,x + 70. \end{aligned} \tag{3}$$

Now, since the sum of the drops through each path from $A$ to $D$ is the same, we have three expressions containing two unknowns, all equal to each other; and we proceed to solve for the currents.

Equating (1) and (2),

$$5x - 4y = 8y - 4x + 50,$$

(4) or $$9x - 12y = 50.$$

Equating (1) and (3),

$$5x - 4y = 5y - 7x + 70,$$

(5) or $$12x - 9y = 70.$$

$$\begin{array}{lrl} \text{Multiplying (5) by 12,} & 144x - 108y & = 840 \\ \text{Multiplying (4) by 9,} & 81x - 108y & = 450 \\ \hline \text{subtracting,} & 63x & = 390 \end{array}$$

and $$x = 6.19 \text{ amperes.} \quad \textit{Ans.}$$

Substituting in (5), $12\,(6.19) - 9y = 70.$

and $$y = 0.4755 \text{ ampere.} \quad \textit{Ans.}$$

$$x - y = 5.715 \text{ amperes.}$$
$$10 - x = 3.81 \text{ amperes.}$$
$$10 - x + y = 4.2855 \text{ amperes.}$$

To find the value of $E$ impressed:
Substituting the values of currents found above, in (1),

$$E = 5\,(6.19) - 4\,(0.4755) = 29.04 \text{ volts.} \quad \textit{Ans.}$$

As a check on results, substituting in equation (2),

$$E = 8\,(0.4755) - 4\,(6.19) + 50 = 29.04 \text{ volts.} \quad \textit{Check.}$$

Or in equation (3),

$$E = 5\,(0.4755) - 7\,(6.19) + 70 = 29.04 \text{ volts.} \quad \textit{Check.}$$

By means of the principles and procedure illustrated in the above examples, most problems involving current and voltage relations in d-c networks may be solved.

**Note.** As an aid in the solution of all problems involving Kirchhoff's laws, it is important to construct and letter a carefully drawn diagram. All known values should be placed on the diagram, together with an expression, or a symbol, for the current in each branch. The known, or assumed, direction of current should also be indicated.

**Prob. 1–15.** Solve the circuit of Example 1, if the terminals of battery $E_1$ in Fig. 4–15 are reversed.

**Prob. 2–15.** In Example 2, what current must be taken by Car II, in order that no current shall flow in $BC$ and $KF$? The current in Car I remains at 300 amperes.

**Prob. 3–15.** Under conditions of Prob. 2–15 find:
(*a*) Voltage across each car.
(*b*) Efficiency of transmission.

**Prob. 4–15.** In Fig. 7–15, each resistor takes 12 amperes. Resistance of $AB = BC = 0.04$ ohm. Resistance of $CD = MK = 0.05$ ohm. Resistance of $EF = KF = 0.03$ ohm. What is the voltage across Group I and Group II, if terminal voltage of $G_1 = 120$ volts and of $G_2 = 125$ volts?

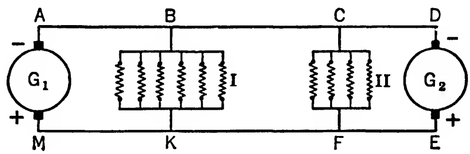



FIG. 7–15. Two generators supplying two sets of resistors.

**Prob. 5–15.** What is the efficiency of transmission of the system in Prob. 4–15?

**3. Parallel Combinations of Unlike Generators or Battery Cells.** When battery cells, of either **Primary** or **Storage** type, are used in series and parallel combinations, they are considered to have the same internal resistance and emf. This is approximately true, and the process of finding resulting voltage and current is a matter of simple addition. Even if the cells are unlike and are joined in series, the emf's and resistances merely add together. This case need not be considered.

But it is interesting to see what will happen if we have two or more cells of unlike emf and resistance, joined in parallel and feeding a line. Such combinations represent actual conditions in many telegraph and telephone circuits, and afford excellent practice in the application of Ohm's law as extended by Kirchhoff.

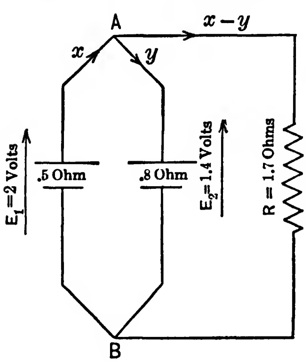



FIG. 8–15. Two parallel battery cells in series with the resistance R.

**Example 4.** Consider Fig. 8–15. Cell $E_1$ has 2 volts emf and 0.5 ohm internal resistance. Cell $E_2$ has 1.4 volts emf and 0.8 ohm internal resistance. When they are joined in parallel to feed a line of 1.7 ohms resistance

Find:

(*a*) Amount and direction of current through each cell and through the line.

(*b*) Terminal voltage (across $AB$).

**Solution.** Assume current flows as marked. There is probably a reversed current through $E_2$ on account of the higher emf of cell $E_1$.

We will assume this, and if the current value comes out a negative quantity, we have merely to reverse the arrowhead.

Let $\quad x =$ current through $E_1$.

Let $\quad y =$ current through $E_2$.

Then $x - y =$ current in line.

The voltage drop across the points $AB$ will be equal along the three paths.

$$\begin{aligned}\text{Thus voltage across } AB &= 2 - 0.5\,x\\ &= 1.4 + 0.8\,y\\ &= 1.7\,(x - y).\end{aligned}$$

Therefore

$$2 - 0.5\,x = 1.4 + 0.8\,y,$$

(1)
$$y = \frac{0.6 - 0.5\,x}{0.8},$$

and

$$2 - 0.5\,x = 1.7\,x - 1.7\,y,$$

(2)
$$y = \frac{2.2\,x - 2}{1.7}.$$

Since (1) = (2)

$$\begin{aligned}\frac{0.6 - 0.5\,x}{0.8} &= \frac{2.2\,x - 2}{1.7};\\ 1.02 - 0.85\,x &= 1.76\,x - 1.6;\\ x &= 1.00 \text{ ampere};\\ y &= \frac{0.6 - 0.5\,x}{0.8};\\ &= \frac{0.6 - 0.5}{0.8};\\ &= 0.125 \text{ ampere}.\\ x - y &= 1.00 - 0.125\\ &= 0.875 \text{ ampere}.\end{aligned}$$

$$\begin{aligned}\text{Thus current through } E_1 &= 1.00 \text{ ampere}.\\ \text{“} \qquad\qquad E_2 &= 0.125 \quad \text{“}\\ \text{“} \qquad\qquad R &= 0.875 \quad \text{“}\end{aligned}$$

$$\begin{aligned}\text{Voltage across } AB &= 2 - 0.5\,x;\\ &= 2 - 0.5;\\ &= 1.5 \text{ volts}.\end{aligned}$$

*Check*

$$\begin{aligned}\text{Voltage across } AB &= (\text{current through } R) \times (\text{resistance of } R)\\ &= 0.875 \times 1.7\\ &= 1.5 \text{ volts}.\end{aligned}$$

From the results of the computation above, we see that the current was backing up through cell $E_2$ on account of its low emf. This shows what is likely to happen in a battery of storage cells

when one cell or set of cells, joined in parallel with others, becomes worn out before the rest.

It is common practice, as we have seen, to operate shunt generators in parallel and compound generators in parallel. The necessity for using machines of approximately equal emf and resistance is apparent from an inspection of the above example. The distribution of current in line and generators, so used, is similar to that in the case of battery cells. Thus, if the expression "**Generators**" be substituted for "**Battery Cells,**" the method, computation and results would be the same.

**Prob. 6-15.** What resistance should be placed in series with $E_1$, Fig. 8-15, in order that the current in $E_2$ be zero?

**Prob. 7-15.** What resistance is required in series with $E_1$, in Fig. 8-15, to cause each cell to supply the same current? What current will each cell supply?

**Prob. 8-15.** Assume the following values for Fig. 8-15 and find current through cells and line.

$E_1$ has 2.2 volts emf and 0.40 ohm internal resistance.
$E_2$ has 1.8 volts emf and 0.70 ohm internal resistance.
$R$ has 0.12 ohm resistance.

**Prob. 9-15.** A series set of 5 cells each, having 1.40 volts emf and 0.25 ohm internal resistance, is joined in parallel to a series set of 6 cells of 1.3 volts emf and an internal resistance of 0.20 ohm each. The parallel combination feeds a line having a resistance of 0.45 ohm.

Find:
(*a*) Current through 0.45 ohm resistance.
(*b*) Voltage across 0.45 ohm resistance.

**4. Feeders.** In order to avoid a heavy line loss in power and a high voltage drop, it is customary in many low-voltage systems, i.e., 550 volts or under, to parallel the main conductor with a second conductor called a **feeder.**

Such a feeder is often run alongside of a trolley wire and joined to it at regular intervals.

This arrangement is merely a case of parallel conductors, but it presents some interesting problems in current, voltage, and power distribution.

**Example 5.** Trolley in Fig. 9-15 is No. 0, hard-drawn copper 0.530 ohm per mile. Feeder is No. 0000 annealed copper 0.258 ohm per mile. Track resistance is 0.04 ohm per mile.

Car No. I is 1 mile from generator station.
" II is 2 miles from Car I.
" III is 2 miles from Car II.

Feeder extends 4 miles from station and is tied to trolley every half mile.

Find:

(a) Volts lost between generator and I.
" " " I " II.
" " " II " III.

(b) Voltage across each car.

(c) Power lost in each section of line.

(d) Efficiency of transmission to three cars.

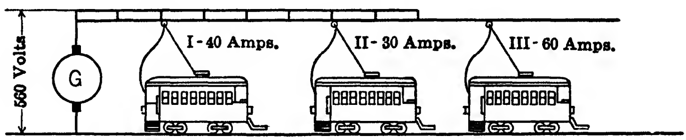



FIG. 9–15. A trolley line supplied by a feeder.

**Solution.** The feeder and trolley may be considered as two parallel circuits with a combined resistance per mile, found as follows:

| Resistance per mile | Conductance per mile |
|---|---|
| 0.530 ohm | 1.89 mhos |
| 0.258 ohm | 3.88 mhos |
| | 5.77 mhos $= \frac{1}{5.77}$, or 0.173 ohm. |

If the trolley and feeder had both been of the same kind of copper, it would have been possible to have considered them as one wire, the cross-section area of which was equal to the sum of the cross-section areas of each. Then the resistance per mile could have been found from the equation $R = \frac{Kl}{A}$, as explained in Chapter IV.

The trolley resistance between generator and Car I
$= 1 \times 0.173 = 0.173$ ohm.
" " " " Car I and Car II
$= 2 \times 0.173 = 0.346$ "
" " " " Car II and Car III
$= 0.173 + 0.530 = 0.703$ "
" total " " Gen. and Car I
$= 0.173 + 0.04 = 0.213$ "
" " " " Car I and Car II
$= 0.346 + 0.08 = 0.426$ "
" " " " Car II and Car III
$= 0.703 + 0.08 = 0.783$ "

Current between Car II and Car III $=$ 60 amp.
" " " I and Car II $= 60 + 30 =$ 90 amp.
" " " Gen. and Car I $= 90 + 40 =$ 130 amp.

Drop in line from Gen. to Car I
$= 130$ amp. $\times$ 0.213 ohm $=$ 27.7 volts.
" across Car I $= 560 - 28 = 532$ volts.
" in line from Car I to Car II
$= 90$ amp. $\times$ 0.426 ohm $=$ 38.3 volts.
" across Car II $= 532 - 38.3 = 494$ volts.
" in line from Car II to Car III
$= 60 \times 0.783 = 47$ volts.
" across Car III $= 494 - 47 = 447$ volts.

Power lost between Gen. and Car I $= 130^2 \times 0.213 = 3600$ watts.
" " " Car I and Car II $= 90^2 \times 0.426 = 3450$ watts.
" " " Car II and Car III $= 60^2 \times 0.783 = 2820$ watts.
Total power lost in line $= 9870$ watts.
Power delivered by Gen. $= 560 \times 130$ $= 72{,}800$ watts.
" " to Cars $= 72{,}800 - 9870$ $= 62{,}930$ watts.

or

Power delivered to Car I $= 532 \times 40 = 21{,}280$ watts.
" " " II $= 494 \times 30 = 14{,}820$ watts.
" " " III $= 447 \times 60 = 26{,}820$ watts.
" " to Cars $= 62{,}920$ watts.

$$\text{Efficiency of transmission} = \frac{62{,}930}{72{,}800} = 86.4 \text{ per cent.}$$

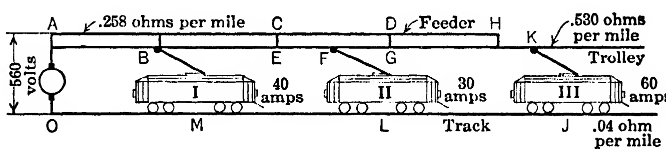



FIG. 10–15. Car II is half way between the tie-in points of the feeder and trolley line.

**Example 6.** When the cars do not happen to be exactly at a junction of the feeder and trolley, the problem becomes a little more involved.

Assume, in Fig. 10–15, the hard-drawn trolley wire 0.530 ohm per mile and feeder 0.258 ohm per mile, joined every mile to the trolley.

Distance between generator and Car I $= 1$ mile.
" " Car I and Car II $= 1\frac{1}{2}$ miles.
" " Car II and Car III $= 2$ miles.

Track resistance $= 0.04$ ohm per mile.

**Solution.**

As before, the combined resistance per mile of feeder and trolley $=$ 0.173 ohm.

As before, the current through section $AB = 130$ amperes.

Volts lost in line between generator and Car I $= 130 \times (0.173 + 0.04) = 27.8$.

Drop across Car I $= 560 - 27.8 = 532$ volts.

It is necessary now to investigate how the current divides among the sections $CD$, $EF$ and $FG$.

Apply the **first law** to the circuit $CDGFE$, and consider the point $CE$. 90 amperes is flowing to Cars II and III through $CE$.

Let current flowing away from $CE$ through $CD = x$.

Then current flowing away from $CE$ through $EF = 90 - x$.

Consider the point $F$, and assume current in $FG$ is in direction from $F$ to $G$. Since $90 - x$ amperes flows to this point and Car II takes 30 amperes, the current in $FG$ must be equal to the difference of the two currents, or

$$\text{current in } FG = 90 - x - 30 = 60 - x \text{ amperes.}$$

Applying the **second law**:

The drop from point $CE$ to point $DG$ = drop through $CD$ = drop through $EF$ + drop through $FG$.

Resistance of $CD$ $= 0.258$ ohm. Resistance of $EF = \dfrac{0.530}{2} =$ 0.265 ohm.

Drop along $CD$ $= 0.258\,x$ volts.

Drop along $EF$ $= 0.265\,(90 - x) = (23.85 - 0.265\,x)$ volts.

Drop along $FG$ $= 0.265\,(60 - x) = (15.9 - 0.265\,x)$ volts.

Hence, $0.258\,x = (23.85 - 0.265\,x) + (15.9 - 0.265\,x)$;

$0.788\,x = 39.75$;

$x = \dfrac{39.75}{0.788} = 50.4$ amperes = current through $CD$.

Current through $EF = 90 - 50.4 = 39.6$ amperes.

Current through $FG = 60 - 50.4 = 9.6$ amperes.

Trolley drop between Car I and Car II

= drop along $BC$ + drop along $EF$

$= (90 \times 0.173) + (39.6 \times 0.265) = 26$ volts.

Total drop between Car I and Car II

$= 26 + (90 \times 1.5 \times 0.04) = 31.4$ volts.

Drop across Car II $= 532 - 31.4 = 500.6$ volts.

Trolley drop between Car II and Car III

= drop along $FG + GH + HK$

$= (9.6 \times 0.265) + (60 \times 0.173) + (60 \times 0.265)$

$= 28.8$ volts.

Total drop between Car II and Car III

$= 28.8 + (60 \times 2 \times 0.04) = 33.6$ volts.

Voltage on Car III $= 500.6 - 33.6 = 467$ volts.

Power lost in line and efficiency of transmission may be found as in the previous example.

**Prob. 10-15.** An 8-mile electric railway has a No. 00 trolley wire of hard-drawn copper. A feeder, No. 0000 annealed copper wire, extends from the station 5 miles along the trolley wire and is tied to it every mile. At a certain instant, there are 4 cars on the line. Car I is 2 miles from station, taking 40 amperes. Car II is $3\frac{1}{2}$ miles from station, taking 60 amperes. Car III is 6 miles from station, taking 30 amperes. Car IV is $7\frac{1}{2}$ miles from station, taking 50 amperes. Track resistance is 0.05 ohm per mile. What must terminal voltage of generator be in order that Car IV may have a pressure of 500 volts?

**Prob. 11–15.** If the resistance of generator in Prob. 10–15 is 0.06 ohm, what emf must it generate? Shunt field = 150 ohms.

**5. The Three-Wire System.** The great saving of power or copper in the line by merely doubling the voltage was brought out in Chapter XII. This fact has led to wide establishment of 230-volt

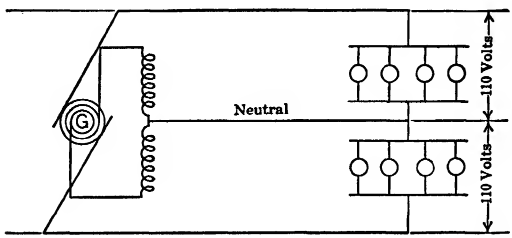



FIG. 11–15. A three-wire generator supplying a three-wire system.

to 240-volt circuits. But lamps of about 115-volt rating are the most common type of incandescent lamp, because they are cheapest and most durable. If they are to be used on a 230-volt circuit, they must be put two in series This would compel a customer always to burn at least two lamps at once. If he needed three lamps, he would have to use four, two parallel sets of two in series.

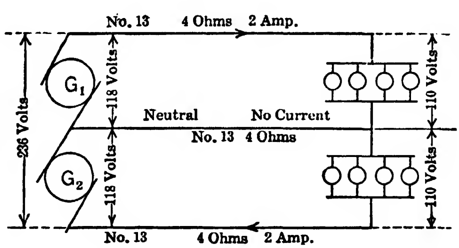



FIG. 12–15. A three-wire system supplied by two generators.

To avoid this feature, and still retain the advantage of transmitting at the higher voltage, a third wire, called a **neutral,** has been added to the usual arrangement, which produces the **three-wire system,** as shown in Fig. 11–15.

This neutral wire is usually the same size as each of the other two in order to be able to operate one side of the system at full load, even if the other side were out of commission. So, if we allow the same watts lost in the line, the total amount of copper is a little

greater than $\frac{1}{4}$ that of a two-wire system. In fact, there is just $\frac{3}{8}$ as much copper in the three-wire system.

A three-wire generator is used to supply such a system and has been described in Art. 41, Chapter X. However, in our study of this system, it is much simpler to consider the generator as separated into two 115-volt or 120-volt generators in series, as in Fig. 12–15, with the neutral connected to the junction of the two.

**6. Balanced and Unbalanced Three-Wire System.** When there are the same number of lamps burning on each side of the neutral, as in Fig. 12–15, the neutral carries no current, and the system is said to be **balanced.** It is only when the system is **unbalanced** that the neutral is of use and carries current, as in Fig. 13–15. The system may be said to be unbalanced, then, when the appliances on one side of the neutral are carrying more current than those on the other side; thus compelling the neutral to carry the

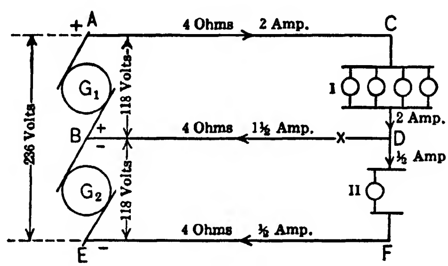



FIG. 13–15. An unbalanced three-wire system.

surplus. Thus, if four $\frac{1}{2}$-ampere lamps were turned on, as in Fig. 13–15, on the (+) side of the neutral, and only one on the other side, the neutral would have to carry $1\frac{1}{2}$ amperes back to the generator $G_1$, which is supplying the greater part of the power. If, however, the loads in Fig. 13–15 had been reversed, the neutral would carry $1\frac{1}{2}$ amperes in the **opposite** direction, and generator $G_2$ would supply most of the power.

**7. Voltage Distribution in a Three-Wire System.** In using the three-wire system, every effort is made to keep it balanced. In this case, the voltage, current and power distribution differ in no respect from that of a two-wire system, with lamps, or other electrical appliances, paired off, two in series.

Of course, on a large system, any slight deviation from a balance makes no noticeable difference; but it is instructive to see what

happens to the voltage distribution in a system unbalanced as much as that of Fig. 13–15.

This is best shown by drawing a "voltage diagram," as in Fig. 14–15. This represents the conditions in the circuit of Fig. 13–15.

The vertical lines *AB* and *BE* (equal to *A'B'* and *B'E'*) represent the 118 volts of the generator. The line *AA'* represents the top wire; *BB'*, the neutral wire; and *EE'*, the other outside wire.

Consider the voltage across Group I.

Draw the line *AC* sloping in the direction in which the current

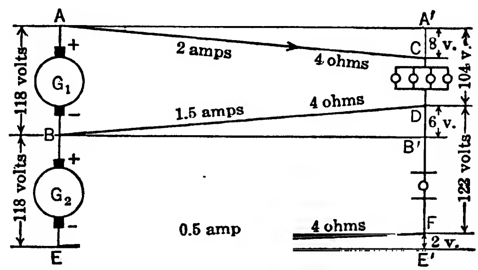



FIG. 14–15. "Voltage diagram" of the unbalanced three-wire system of Fig. 13–15.

flows. Since the voltage drops 2 × 4 or 8 volts in *AC*, the point *C* drops 8 volts below the level of *A*, represented by the vertical line *A'C*.

In the same way, the line *DB* is drawn sloping in the direction in which the current flows. The voltage drops 1.5 × 4 or 6 volts from *D* to *B* in the neutral wire; and point *D* is 6 volts **above** the level of point *B*, represented by the vertical distance *DB'*. The remainder of the vertical distance *AB* (or *A'B'*) is *CD*. This is 8 volts less than *A'B'* at one end, and 6 volts less at the other.

Total line drop from generator to Group I = 6 + 8 or 14 volts.

$$\text{Voltage across Group I} = 118 - 14 = 104 \text{ volts;}$$

or

$$= 118 - 6 - 8 = 104 \text{ volts.}$$

Consider the voltage across Group II.

The current in the neutral flows in a direction to the generator from *D* to *B*; and *D* is 6 volts **above** the level of *B* (or *B'*), as just stated. A current of 0.5 ampere flows from Group II to the generator in the bottom wire, and the line *FE* is drawn sloping

in the direction in which the current flows. The drop in this wire, from $F$ to $E$, is $0.5 \times 4$, or 2 volts. Therefore, point $E$ is 2 volts **below** the level of $F$; or $F$ is 2 volts above the level of $E$ (or $E'$).

The vertical distance $B'E'$ represents the voltage of the generator, $G_2$, or 118 volts. The lamps are connected between two points at the levels of $D$ and $F$; so the vertical distance $DF$ must be equal to the voltage on Group II. But $DF$ is 6 volts **longer** than $B'E$ at one end, and 2 volts **shorter** at the other; or a total of 4 volts longer than $B'E'$. The voltage at Group II is thus $118 + 4$ or 122 volts.

The voltage across Group II =

$$\text{voltage of } G_2 + \text{line drop in } BD - \text{line drop in } FE;$$

or

$$118 + 6 - 2 = 122 \text{ volts.}$$

Thus it is seen that in an extremely unbalanced system, as that in Fig. 14–15, the voltage distribution is disturbed; there being an excessive voltage drop on the heavily loaded side, while there may be an actual **rise** in voltage on the lightly loaded side. The lights on the heavily loaded side would give poor illumination; while the life of the lamps on the other side would be shortened, if they did not actually burn out.

In the solution of three-wire systems involving more complicated diagrams, the direction of the current and $IR$ drop in the different sections of the neutral is often confusing. These can readily be solved by the application of Kirchhoff's laws, as in the example below.

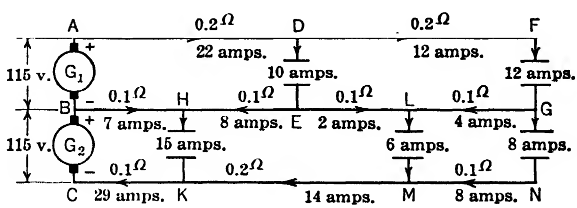



FIG. 15–15. A diagram of a more complex three-wire system.

**Example 7.** In the three-wire system of Fig. 15–15, the resistances of each section of the line and the currents taken by the several loads are as marked. Determine the voltage across each load.

**Solution.** The value and direction of the current in each section of the outside wires and in each load is apparent, and is marked on the diagram.

Applying Kirchhoff's first law to the junction points in the neutral wire:

At $G$, 12 amperes − 8 amperes = 4 amperes flowing in $LG$, from $G$ to $L$, and marked on the diagram.

At $L$, 6 − 4 = 2 amperes flowing in $EL$, from $E$ to $L$, as marked.

At $E$, 10 − 2 = 8 amperes flowing in $HE$, from $E$ to $H$, as marked.

At $H$, 15 − 8 = 7 amperes flowing in $BH$, from $B$ to $H$, as marked.

Also note that there is as much current flowing away from point $B$ as to it; or 22 + 7 = 29 amperes.

Voltage drop in section

$AD = 0.2 \times 22 = 4.4$ volts.
$DF = 0.2 \times 12 = 2.4$ volts.
$GL = 0.1 \times 4 = 0.4$ volt.
$LE = 0.1 \times 2 = 0.2$ volt.
$EH = 0.1 \times 8 = 0.8$ volt.
$HB = 0.1 \times 7 = 0.7$ volt.
$NM = 0.1 \times 8 = 0.8$ volt.
$MK = 0.2 \times 14 = 2.8$ volts.
$KC = 0.1 \times 29 = 2.9$ volts.

Applying the second law (and remembering Rule 3):

To circuit $BADEHB$, starting at point $B$,

$$+BA - AD - DE - EH + HB = 0.$$

Voltage across $BA - AD - EH + HB$ = voltage across $DE$.

Substituting values, 115 − 4.4 − 0.8 + 0.7 = 110.5 volts across $DE$. *Ans.*

To circuit $EDFGLE$, starting at point $E$,

$$+ED - DF - FG - GL + LE = 0.$$

Voltage across $ED - DF - GL + LE$ = voltage across $FG$.

Substituting values, 110.5 − 2.4 − 0.4 + 0.2 = 107.9 volts across $FG$. *Ans.*

To circuit $CBHKC$, starting at $C$,

$$+CB - HB - HK - KC = 0.$$

Voltage across $CB - HB - KC$ = voltage across $HK$.

Substituting values, 115 − 0.7 − 2.9 = 111.4 volts across $HK$. *Ans.*

To circuit $KHELMK$, starting at $K$,

$$+KH + EH - LE - LM - MK = 0.$$

Voltage across $KH + EH - LE - MK$ = voltage across $LM$.

Substituting values, 111.4 + 0.8 − 0.2 − 2.8 = 109.2 volts across $LM$. *Ans.*

To circuit $MLGNM$, starting at $M$,

$$+LM + LG - GN - MN = 0.$$

Voltage across $LM + LG - MN$ = voltage across $GN$.

Substituting values, 109.2 + 0.4 − 0.8 = 108.8 volts across $GN$. *Ans.*

**Proof.** Drop through any complete circuit, such as $ADEHKC$, from (+) terminal of $G_1$ to (−) terminal of $G_2$ must equal 230 volts.

$$4.4 + 110.5 + 0.8 + 111.4 + 2.9 = 230 \text{ volts.} \quad \textit{Check.}$$

**Prob. 12–15.** If there were 2 lamps in Group I and 8 lamps in Group II, Fig. 13–15, each taking 0.5 ampere, other data remaining unchanged, what would the voltage across each group become?

**Prob. 13–15.** What will be the voltage distribution in Fig. 13–15, if the wires all have a resistance of 1.5 ohms? The generators maintain a pressure of 115 volts each, and each lamp takes 0.4 ampere.

**Prob. 14–15.** Assume each lamp in Fig. 16–15 takes 3.2 amperes. Find:

(*a*) Current in *AB*, *CD*, and *EF*;
(*b*) Line drops in *AB*, *CD*, and *EF*;
(*c*) Line loss;
(*d*) Voltage across *BD* and *DE*;
(*e*) Efficiency of transmission;
(*f*) Resistance of each set of lamps.

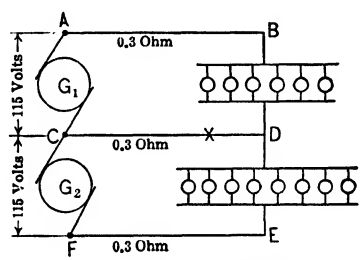



FIG. 16–15. A three-wire system.

**Prob. 15–15.** Suppose the neutral in Prob. 14–15 broke at *x*, what would be the voltage across *BD* and *DE*? (Assume the resistance of the lamps to be constant.)

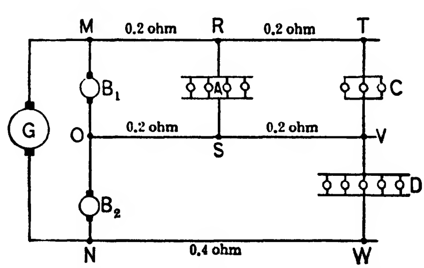



FIG. 17–15. An unbalanced three-wire system with a balancer set.

**Prob. 16–15.** Assume that each lamp in Fig. 17–15 takes 2 amperes and that the resistance of the lamps remains constant. The terminal voltage of the generator is 230 volts. Each small machine of the balancer set is so regulated as to maintain 115 volts across it.

Find:

(*a*) Line drop in each section;
(*b*) Voltage across each set of lamps;
(*c*) Efficiency of transmission.

**Prob. 17–15.** If a break occurred in the neutral between *O* and *S*, Fig. 17–15, what would be the values of (*a*), (*b*) and (*c*) of Prob. 16–15? The lamp resistances are the same as in Prob. 16–15.

**Prob. 18–15.** If a break occurs between *S* and *V*, Fig. 17–15, what will be the values of (*a*), (*b*) and (*c*) of Prob. 16–15? Lamp resistances remain as in Prob. 16–15.

**Prob. 19–15.** The load in *LM*, in Example 7, Fig. 15–15, is reduced to 4 amperes, and that in *HK* is reduced to 10 amperes.

Find:

(*a*) Line drop in each section;
(*b*) Voltage across each load;
(*c*) Efficiency of transmission.

**Prob. 20–15.** Prove the statement that a three-wire system requires $\frac{3}{8}$ as much copper as a two-wire system to transmit a given amount of power a given distance with the same line loss. The voltage between one side and neutral is to be the same as the voltage of the two-wire system; and the neutral to be the same size as either outside wire.

## SUMMARY OF CHAPTER XV

**KIRCHHOFF'S LAWS. (An Extension of Ohm's law.)**

**FIRST LAW: The algebraic sum of the currents at any point in a circuit is equal to zero. That is, the sum of all the currents flowing away from any point in a circuit is always equal to the sum of all the currents flowing to that point.**

**SECOND LAW: In any circuit, the algebraic sum of all the emf's equals the algebraic sum of all the IR drops. In any part of a circuit containing no source of emf, the sum of the IR drops through one path, between any two junction points, is always equal to the sum of the IR drops through any other path, between these same two points.**

**In the APPLICATION OF KIRCHHOFF'S LAWS, there are four general rules.**

**RULE 1: At any junction point, currents flowing in either direction may be assumed positive and those flowing in the opposite direction as negative.**

**RULE 2: When direction of current is uncertain, it may be assumed. If solution results in a negative value, it means that the actual direction is opposite to that assumed. The numerical value will be correct.**

**RULE 3: In tracing through a circuit, the SIGNS of the em'fs and IR drops are determined as follows:**

**A RISE in voltage always indicates a (+) sign.**

**A DROP in voltage always indicates a (−) sign.**

**RULE 4: (a) KIRCHHOFF'S FIRST LAW must be applied to enough junction points in a network to include every unknown current**

**at least once. (b) THE SECOND LAW must be applied to enough branches so that the emf and IR drop relations in every branch are included at least once.**

**NETWORKS are supplied with power at different points and from different sources. Power, voltage and current distribution can be solved by APPLYING KIRCHHOFF'S LAWS.**

**FEEDERS are used in low-voltage systems to reduce voltage drop in the mains. Power, voltage and current distribution can be solved by APPLYING KIRCHHOFF'S LAWS.**

**THREE-WIRE SYSTEM. By use of a neutral wire, the proper voltage for the standard 115-volt lamps can be obtained from a 230-volt three-wire generator, and makes available two standard voltages. Neutral wire is intended to carry very little current. This system requires three-eighths as much copper as a two-wire system of same capacity for the same lamp voltage. Power, voltage and current distribution can be solved by the APPLICATION OF KIRCHHOFF'S LAWS.**

## PROBLEMS ON CHAPTER XV

**Prob. 21–15.** An electric railway is in the form of a rectangle 7 miles by 3 miles. The generator station is in the middle of one of the long sides. The trolley wire is No. 0 hard-drawn copper. The track resistance is 0.03 ohm per mile. There is one No. 0000 annealed-copper feeder running from the generator station directly across the rectangle to the other side, where it is joined to the trolley wire. There are 2 cars on the line, one at the middle of each short side of rectangle: one is taking 110 amperes and the other 70 amperes. What is the voltage across each car, if the terminal voltage of the generator is 650 volts?

**Prob. 22–15.** What would be the voltage across each car in Prob. 21–15, if there were no feeder?

**Prob. 23–15.** There are 5 cars on the line in Prob. 21–15, one at each corner of the rectangle, and one at the junction of the farther end of the feeder and trolley, each car taking 60 amperes. What voltage will there be across each car?

**Prob. 24–15.** A 10-mile trolley line is supplied by a 600-volt generator at one end and has two feeders. One of No. 0000 annealed-copper wire extends from the generator 6 miles along the line, and is tied to the trolley every 2 miles. The other of hard-drawn copper, 600,000 circular mils in cross section, extends from generator along the trolley for 3 miles and is tied to trolley at the ends only. There are 3 cars on the line, distributed as follows: Car I, 3 miles from generator takes 100 amperes. Car II, 5 miles from generator, takes 60 amperes. Car III, 7 miles from generator, takes 80 amperes. Trolley wire is No. 0 hard-drawn copper. Track resistance is 0.03 ohm per mile. Generator voltage is 600. (*a*) What is the voltage across each car? (*b*) What is the efficiency of transmission?

**Prob. 25–15.** If the trolley wire in Prob. 24–15 breaks one mile from the generator, what will the voltage across each car become?

**Prob. 26–15.** Each of two storage batteries has an emf of 12 volts and an internal resistance of 0.008 ohm. If the two batteries are joined in series across a line of 1.2 ohms resistance, what current will flow?

**Prob. 27–15.** If the batteries of Prob. 26–15 are joined in parallel across the same line, what current will flow and how much will each battery deliver?

**Prob. 28–15.** Assume one of the batteries in Prob. 26–15 had an emf of 11.5 volts and an internal resistance of 0.006 ohm, and the other remained as in Prob. 26–15. What current would flow through the line, if connected as in Prob. 26–15?

**Prob. 29–15.** If batteries of Prob. 28–15 were connected as in Prob. 27–15, what current would each supply?

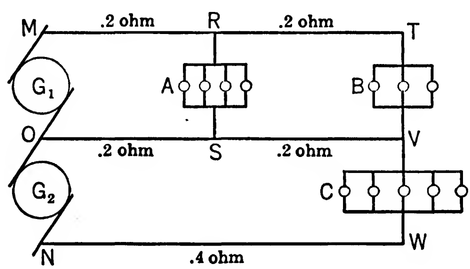



FIG. 18–15. An unbalanced three-wire system.

**Prob. 30–15.** Assume that each lamp in Fig. 18–15 takes 2.5 amperes and that resistance of lamps remains constant. Terminal voltage of each generator is 120 volts. Find:

(*a*) Line drop in each section;
(*b*) Voltage across each set of lamps;
(*c*) Power delivered by $G_1$ and $G_2$;
(*d*) Efficiency of transmission.

**Prob. 31–15.** If a break occurs in neutral between $O$ and $S$ of Prob. 30–15, what will be the values of (*a*), (*b*), (*c*) and (*d*)?

**Prob. 32–15.** If a break occurs in the neutral between $S$ and $V$ of Prob. 30–15, what will be the values of (*a*), (*b*), (*c*) and (*d*)?

**Prob. 33–15.** Figure 19–15 shows the wiring diagram of a building supplied by a two-wire system. Resistance of each section of the line wires is the same as marked on the diagram. Motor $M$ takes 60 amperes. Each lamp takes 3.5 amperes, and its resistance is constant. (*a*) Find voltage across each set of lamps. (*b*) Draw diagram indicating amount and direction of current in each section of line.

**Note.** The $IR$ drop across $NMR$ must equal $IR$ drop across $NSR$, since these are parallel circuits between the same two points $R$ and $N$.

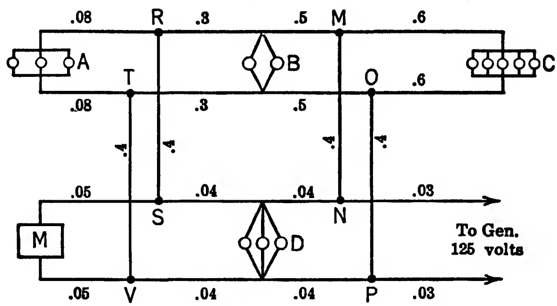



FIG. 19–15. A system of lamps and motors connected to a two-wire line.

**Prob. 34–15.** Find the voltage across the various sets of lamps of Prob. 33–15, when the motor is not running.

**Prob. 35–15.** Repeat Prob. 33–15 leaving out jumpers *MN* and *OP*.

**Prob. 36–15.** For operating a coil of 2 ohms resistance, four storage cells in parallel are used. Each had, when new, an emf of 2.2 volts and an internal resistance of 0.04 ohm. What current flows through the line, and what is terminal voltage of the battery?

**Prob. 37–15.** One of the cells of Prob. 36–15 was damaged. Its emf fell to 1.9 volts and its internal resistance rose to 0.085 ohm. The four cells are still used, as in Prob. 36–15. (*a*) What is current in line? (*b*) Terminal voltage of the battery?

**Prob. 38–15.** In a system arranged as in Fig. 7–15, each resistor takes 10 amperes. The resistance of the different sections of the line and of each generator armature is as follows:

| | |
|---|---|
| $AB$ = 0.05 ohm. | $CD$ = 0.06 ohm. |
| $BC$ = 0.04 ohm. | $EF$ = 0.04 ohm. |
| $KF$ = 0.05 ohm. | $G_1$ = 0.02 ohm. |
| $MK$ = 0.03 ohm. | $G_2$ = 0.025 ohm. |
| Emf of $G_1$ = 130 volts. | Emf of $G_2$ = 135 volts. |

Find:

(*a*) Voltage across Group I and across Group II.
(*b*) Terminal voltage of $G_1$ and $G_2$.
(*c*) Efficiency of transmission.

**Prob. 39–15.** The resistance per mile (trolley and track) of a 5-mile railway line is 0.15 ohm per mile. The generator at one end of the line develops 550 volts; while at the other end, a storage battery consisting of 240 cells in series is "floated." Each cell has an emf of 2.1 volts and an internal resistance of 0.001 ohm. (*a*) What current will the cells supply to the line, when there are 5 cars at the battery end of the line, each taking 65 amperes? (*b*) What will be the terminal voltage of the set of battery cells?

**Prob. 40–15.** What current will the cells in Prob. 39–15 take when there is no load on the line?

**Prob. 41–15.** When the 5 cars in Prob. 39–15 are halfway between the generator and the battery, and are each taking 65 amperes,

(*a*) What current will the generator deliver?
(*b*) What current will the battery deliver?
(*c*) What will be the voltage on the cars?

**Prob. 42–15.** If there are only two cars on the line in Prob. 41–15, each taking 50 amperes: (*a*) What current does the generator deliver? (*b*) What current does the battery receive or deliver?

**Prob. 43–15.** Six storage cells, arranged as in Fig. 20–15 in two parallel sets of three cells in series, are discharging through a resistor of 1.2 ohms. Each cell normally has an emf of 2.1 volts, and an internal resistance of 0.02 ohm; but one cell in set *A* has "gone bad" and has an emf of 1.8 volts and an internal resistance of 0.14 ohm. What current is supplied to *R* by each set of cells?

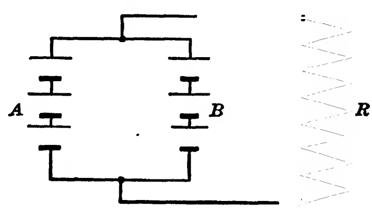



FIG. 20–15. Batteries A and B are arranged in parallel to feed the resistor R.

**Prob. 44–15.** If the resistance of *R*, Fig. 20–15, is reduced to 0.025 ohm, and other data remain as in Prob. 43–15, what current will flow in each set of cells and in what direction?

**Prob. 45–15.** If the resistance of *R*, Fig. 20–15, is increased to 2.6 ohms, and other data remain as in Prob. 43–15, what current will flow through each set of cells and in what direction?

**Prob. 46–15.** An electric railroad is supplied with power by sub-stations, 12 miles apart. The trolley wire is connected at frequent intervals with a heavy "feeder." The combined resistance of feeder and trolley may be considered as equivalent to a single conductor whose resistance is 0.17 ohm per mile. The return path through the track has a resistance of 0.025 ohm per mile. At a certain time, three cars are running in the section between these two sub-stations: Car I, taking 150 amperes, at a distance of 2 miles from one end; Car II, taking 200 amperes, at a distance of 4 miles from the other end; and Car III, taking 100 amperes, halfway between the other two. The voltage at each sub-station is 750. What is the voltage across the motors of each car?

**Prob. 47–15.** Figure 21–15 represents a method of raising line voltage. *T* is the trolley wire and *R*, the rails. *F* is a feeder which is connected to the trolley wire at a distance of 5 miles from the power station. *B* is a "booster"; i.e., a generator used to increase the voltage applied to the feeder. The terminal voltage of the generator is 600. The

trolley wire has a resistance of 0.064 ohm per 1000 feet. The rails form a return conductor whose resistance is 0.0061 ohm per 1000 feet. The booster armature has a resistance of 0.023 ohm and generates an emf of 100 volts. The total resistance of the feeder is 0.82 ohm. *C* is a train requiring a current of 500 amperes. What is the lowest voltage which the train can have in the region between the power station and the point of connection of the feeder and trolley?

**Suggestion:** Plot a curve for points at various distances from the power station and determine lowest voltage from the curve.

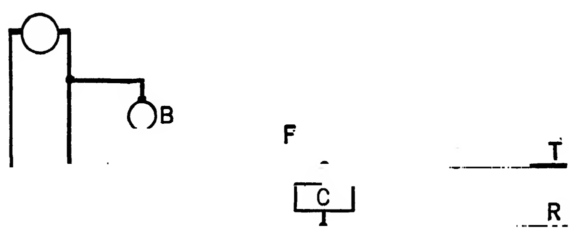



Fig. 21–15. The voltage across the train, C, is kept at a high value by the "booster" B, in the feeder line F.

**Prob. 48–15.** A single-track railroad line is 8 miles long and is supplied with power from generators at its two ends. The positive conductor consists of a third rail, reinforced by a copper cable in parallel, the combination having a resistance of 0.028 ohm per mile. The negative conductor is a pair of bonded track rails in parallel, with a combined resistance of 0.035 ohm per mile. The station at one end maintains 600 volts, and at the other a voltage of 575 volts, between positive and negative conductors.

At what point along the line would an electric locomotive, requiring 1200 amperes, receive a minimum voltage between third rail and track? What would the voltage at this point be, and what current would each station supply?

**Suggestion:** Make determination from plot of voltage and distance from one station.

# CHAPTER XVI

## INSULATORS, DIELECTRICS AND CONDENSERS

In the preceding chapters, only the flow of an electric current, or electricity in motion, has been considered. Some of the laws pertaining to electric charges, or electricity at rest, will be studied in this chapter. These laws become of primary importance in the high-voltage circuits in use today.

It was stated in Chapter I that a flow of current consists of the movement, or drift, of electrons through the molecules of the conductor under the action of an applied voltage, as indicated in Fig. 1–16. As these electrons leave one atom, or group of atoms, and pass on to the next group, their places are immediately taken by electrons passed on from a preceding group of atoms. Thus, as this procession of electrons moves on in the circuit, all the

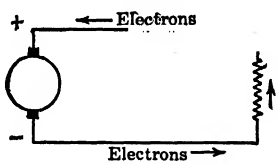



FIG. 1–16. Direction of flow of electrons in a closed circuit.

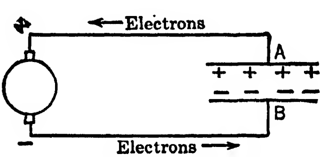



FIG. 2–16. The pressure on the circuit takes electrons from plate A, leaving it positively charged; and places electrons on plate B, making it negatively charged.

atoms of the conductor have associated with them their normal number of electrons. In other words, the free electrons are continually replacing each other.

Now, if a conducting electric circuit is open, or is connected to two insulated metallic plates, as in Fig. 2–16, under the action of the generator emf, electrons are taken from plate $A$ and electrons are forced out to plate $B$. Since no electrons can pass from plate $B$ to plate $A$ to take the place of those which have already been taken from this plate, plate $A$ is deficient in electrons and is positively charged; and plate $B$, with a surplus of electrons, is negatively charged. If the emf of the generator is constant,

there will be no further movement of electrons; and two charges of electricity at rest will exist on the two plates.

**1. Difference between Insulators and Dielectrics.** In Chapter I, it was stated that in conductors, there are many free electrons present in the atoms of the material. A very moderate voltage will set these electrons in motion. In certain substances, known as insulators, there are few, if any, free electrons present in the atoms of the material; and a high electric stress or difference of potential is necessary to tear even a very small number of electrons from the atoms and set up a current. Such materials as glass, porcelain, rubber, the varnishes, mica, dry paper, oiled linen and air are insulators.

The function of insulation, or an insulator, is to prevent a leakage or loss of current. However, there is no perfect insulating material known; for if the voltage across any insulator be made high enough, there will be a leakage of current. It is, therefore, permissible and convenient to consider an insulator to be a conductor of exceedingly high resistance, which prevents the leakage of an appreciable amount of electricity.

When a voltage is impressed upon an electric circuit, a difference of potential exists between the two insulated conductors, and an electric field of force is set up around them in somewhat the same manner that a magnetic field is set up around two magnetic poles. The insulator separating the two conductors is in this field and is acted upon by it. Furthermore, electrical energy is stored in this electrical field in a manner comparable to the storage of magnetic energy in a magnetic field. The amount of energy stored in an electric field depends, among other things, upon the magnitude of the voltage between the two conductors and upon the material used for insulation.

For instance, under the action of a given pressure between two insulated conductors, more energy will be stored in the electric field when the insulation is glass, porcelain or mica, than if the insulation is air. Glass, porcelain and mica are therefore said to be better **dielectrics** than air.

Thus a dielectric must be an insulator and prevent the leakage of an appreciable amount of electricity; must have the property of withstanding an electric stress, or voltage; and in addition must be a medium which lends itself to the storage of electrical energy.

The distinction between an insulator and a dielectric may be

explained as follows. Since an insulator may be considered as a conductor of extremely high resistance, the heat evolved, due to the passage of even a very small leakage current through the insulator, is dissipated energy which cannot be recovered. In a dielectric, the resistance or opposition to the flow of leakage current is elastic in nature, and electricity, or electrical energy, is stored in the field; and at least a part of this energy may be recovered.

**2. The Dielectric Field.** The field established in the space surrounding two insulated conductors at different potentials is called the " Dielectric," or " Electrostatic " Field.

Consider the two insulated conductors in Fig. 3–16, shown in sectional diagram, and assume there is no leakage current. Any

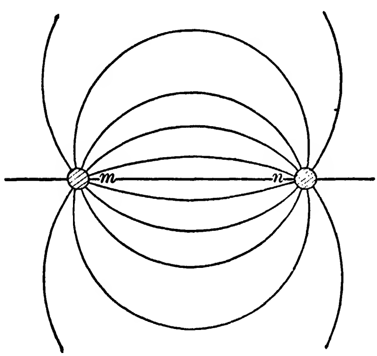



FIG. 3–16. The electric field between two conductors.

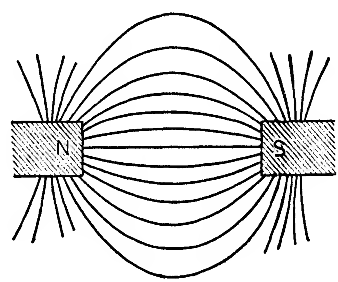



FIG. 4–16. The magnetic field between two magnet poles.

points in this field at different distances from the conductors will have different potentials with respect to those of the conductors themselves. This difference of potential results in a stress in the field and a storage of electricity. These stresses in the electric field exist in definite lines or streams, as do the stresses in the magnetic field, Fig. 4–16. These electrical lines of force, called dielectric-flux lines, act, as do magnetic lines, like stretched rubber bands and seem to repel one another. Also, as in the case of two magnetic poles, there is a force of attraction between the two conductors.

Unlike magnetic lines, however, these dielectric lines originate at the surface of the positively charged conductor, and end at the surface of the negatively charged conductor. They do not exist

inside the conductors, as magnetic lines exist inside a magnet. They are, therefore, not continuous and do not close on themselves. Furthermore, these lines of electric stress at the surface of the charged conductors are always at right angles to that surface, and follow the path a free positive electric charge between the two conductors would take. On surfaces of irregular shape, the dielectric-flux lines concentrate in regions where the radius of curvature of the surface is the least. The electric stress is the greatest where these lines are most dense; so that small radii of curvature or sharp points in any insulation or insulator are to be avoided.

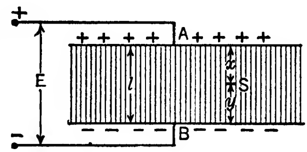


FIG. 5–16. The dielectric field between two plates. The potential gradient in this uniform field is E/l where l = distance between plates and E = voltage between plates.

Note that the electric field around two insulated conductors in Fig. 3–16 is not to be confused with the magnetic field due to the flow of current in these conductors. The magnetic field produced by an electric current is always at right angles to the electric field.

**3. Potential Gradient.** Consider the two parallel flat surfaces $A$ and $B$, Fig. 5–16, having a potential difference between them and separated by a dielectric. The dielectric field between them is uniform except at the edges of the plates. If $E$ is the voltage between the plates and $l$ the thickness of the dielectric in centimeters, then the potential at any point $S$, at a distance $x$ from plate $A$, or a distance $y$ from plate $B$, is $\frac{E \times x}{l}$ volts below that of $A$; or it is $\frac{E \times y}{l}$ volts above that at $B$.

The potential of point $S$, as it is moved from $A$ to $B$, decreases uniformly from the potential at $A$ to that at $B$. This fall in potential in a dielectric field, per unit length, is called the **potential gradient.** This is generally expressed in "**volts per centimeter,**" or it may be expressed in "**volts per inch.**" In the uniform field of Fig. 5–16 the potential gradient is $\frac{E}{l}$ volts per centimeter.

**Example 1.** What is the potential gradient of the dielectric field of Fig. 5–16, if there are 10,000 volts between the plates, and the thickness of the dielectric is 2.5 centimeters?

**Solution.**

$$\text{Potential gradient} = \frac{10{,}000}{2.5} = 4000 \text{ volts per centimeter.}$$

In Fig. 3–16, the dielectric field is not uniform and the potential gradient along the different dielectric-flux lines is not the same, but will be greatest along the line *mn*.

**4. Dielectric Strength.** In a dielectric, we have said that the electrons are securely attached to the atoms and hence cannot flow along the material when a moderate electromotive force is applied. However, if the electromotive force is raised to a sufficient value, the electrons can possibly be torn loose and the material thus broken down, or disrupted, in exactly the same way that air is broken down. When insulation is thus punctured, the electrons are torn loose from the atoms. The electric stress necessary to do this is usually very high and depends upon the material. The stress necessary to disrupt or break down an insulator or dielectric is called its **dielectric strength**; and is measured in "volts per centimeter." Thus a dielectric is said to break down at a potential gradient of so many volts per centimeter or per inch. See Table VI.

Glass is one of the best dielectrics from an electrical standpoint. It may have a dielectric strength of about 100,000 volts per centimeter. Mechanically, of course, it is open to considerable objection. Porcelain has nearly the same dielectric strength and is much stronger mechanically. For places where an insulator of small thickness is desired, mica is used. Its dielectric strength is about one-half that of glass.

The dielectric strength of a material varies with a great many factors. For instance, any such material as oil tends to absorb moisture; and the percentage of moisture content greatly affects the dielectric strength. This is particularly true of transformer oil. Even one-tenth of one per cent of moisture, diffused through the oil, will reduce the dielectric strength to less than ten per cent of full value. The temperature also has a large effect. In general, the higher the temperature of the material, the less will be its dielectric strength. It should also be mentioned that the length of time during which the voltage is applied to the specimen is a great factor; certain substances that will stand up one minute under a voltage of 120,000 volts per centimeter will break down under 80,000 volts per centimeter applied for half an hour.

**Prob. 1–16.** For rubber insulation, the relative dielectric strengths for various times of electrification are given in the following table.

| Time of electrification in minutes | Relative dielectric strength |
|---|---|
| 1 | 180 |
| 3 | 110 |
| 5 | 100 |
| 10 | 90 |
| 15 | 85 |
| 30 | 80 |

The 5-minute factory test of No. 00 rubber insulated cables, having $\frac{7}{64}$-inch insulation, is 11.5 kilovolts. What voltage will these cables safely stand for 1 minute? (1 kilovolt = 1000 volts.)

**Prob. 2–16.** Bakelite, prepared for the electrical trade, may have a dielectric strength of 380,000 volts per inch. In a certain machine, it was desired to replace some mica insulating sheets $\frac{1}{300}$-inch thick with bakelite sheets. How thick should the bakelite sheets be? Assume the same voltage will be applied in each case between the surfaces of the sheets.

**5. Condenser Action. Capacitance. The Farad.** When an insulator or dielectric is placed between two conductors, a **condenser** is formed. When a voltage is applied to the plates, it cannot cause the electrons to flow through the insulator, as described at the beginning of this chapter. It may, however, be thought of as **straining the bonds** by which the electrons are attached to the atoms of the insulator.

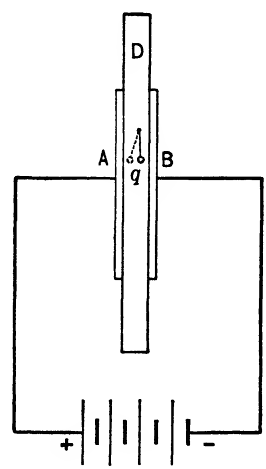


Fig. 6–16. The electron $q$ is forced away from B and toward A by the voltage between A and B.

The bonds, then, in a dielectric are more or less flexible; and while a moderate pressure cannot break them, it will flex or stretch them and cause the electrons to move a little in position. In Fig. 6–16, $D$ represents a plate of dielectric material between two metal plates (or electrodes) $A$ and $B$. When the positive terminal of a battery is connected to metal plate $A$ and the negative terminal to metal plate $B$, the electrons (one of which is represented by $q$), being negative, are repelled from the negative plate $B$, and are attracted to the posi-

tive plate $A$. The pressure is not strong enough to break the bond, which is represented by the short straight line, but it flexes it and the electron moves a little toward plate $A$. The insulating plate $D$ is made up of an enormous number of electrons, all of which are moved slightly, from the negative toward the positive plate; they stay in this new position as long as the pressure is maintained between the plates. The flexing of these bonds causes a momentary current, which stops as soon as the electrons have reached their new positions.

Thus, when a voltage is applied across $A$ and $B$, there is a momentary current, as the electrons move slightly, which will soon die out and remain zero as long as the pressure remains constant. When the pressure is removed by short-circuiting the battery, the electrons, due to the elasticity of the bonds, will resume their original positions. Thus, there will be a momentary current in the reverse direction. In general, whenever the electric stress across the dielectric of a condenser is changed, there will be a current in the condenser (called the displacement current), which is due to the slight motion of the electrons. When the voltage is steady, there is no current.

This property, which produces a current in a condenser when the voltage across it changes, is called **capacitance,** and it is measured in **farads. When a change of one volt per second across it produces a current of one ampere, the condenser is said to have a capacitance of one farad.**

The farad is a unit much too large for practical purposes; so the **microfarad** (one-millionth of a farad) is generally used. The capacitance of very small condensers is often given in micromicrofarads, $\mu\mu$f's (or one-millionth of a millionth of a farad).

This relation between change of voltage and current can be expressed approximately by the equation

$$i_{av} = C\frac{e_1 - e_2}{t_1 - t_2}: \tag{1}$$

in which $i_{av}$ = the average current in amperes;
$C$ = capacitance in farads;
$e_1$ = voltage in volts at time $t_1$;
$e_2$ = voltage in volts at time $t_2$;
$t_1 - t_2$ = time of change from $e_1$ to $e_2$ in seconds;
$\dfrac{e_1 - e_2}{t_1 - t_2}$ = average rate of change of voltage.

**Example 2.** A condenser has a capacitance of 10 microfarads. What average current will flow, when the voltage is raised from 5 to 65 volts in 2 seconds?

**Solution.**

$$i_{av} = C\frac{e_1 - e_2}{t_1 - t_2}$$

$$= \frac{10}{10^6} \times \frac{65 - 2}{2} = 0.0003 \text{ ampere.}$$

**Prob. 3-16.** A condenser $C$ with a capacitance of 20 microfarads is connected as in Fig. 7-16. The slide-wire is 120 inches long and of a total resistance of 24 ohms. If the slide $S$ is being moved to the right at the rate of 10 feet per second, what current will flow in condenser $C$? Assume that the current taken by the condenser does not change the voltage distribution along the slide-wire.

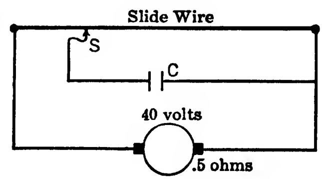



FIG. 7-16. A method for varying the voltage across a condenser.

**Prob. 4-16.** How much would the result in Prob. 3-16 be modified, if the slide-wire were replaced by another of the same length, but with a resistance of 120 ohms?

**Prob. 5-16.** What is the capacitance of a condenser, in which a current of 0.015 ampere flows, when the voltage across its terminals rises at a uniform rate from 100 volts to 375 volts in 0.14 second?

**6. Dielectric Constant.** The capacitance of such an arrangement, as is shown in Fig. 6-16, has been found to depend upon the material of the dielectric used, as stated in Art. 1. If $D$ is a sheet of glass, the capacitance will be found to be about twice what it is when $D$ is composed of mica. We explain this by saying that the **dielectric constants** of the two materials are different.

The **dielectric constant** of a material is a measure of its effectiveness when used in a condenser; just as the permeability of a material is a measure of the effectiveness of the material for use in a magnetic circuit. It will be remembered from the study of magnetic circuits that the permeability of iron is a number of thousands, and that of other materials much lower; but that the permeability of air is unity. In the same way, when we study condensers, we find that while the dielectric constant of most insulating materials is high compared with that of air, yet air or even a vacuum has a dielectric constant; that is, both will give

condenser effects when between parallel plates. When the dielectric of a condenser is stressed, we can visualize the movement of electrons which constitutes the displacement current. It has not yet been satisfactorily explained, however, why there is still a current when the dielectric is removed and replaced by a vacuum, as experiment shows is probably true. There is by no means as much range in the dielectric constant of various materials, however, as there is in the permeability.

The dielectric constant of air has been chosen as unity, and all other materials have been referred to air as a standard. The dielectric constant of a material on this basis is called its **relative dielectric constant.** Note, in Table V, that glass has a relative dielectric constant, ranging between 6 and 10, depending upon the grade of the glass. Mica has about **two-thirds** of this. Some liquids have high dielectric constants, that of glycerine being about fifty-six.

The dielectric constant of a material must not be confused with its **dielectric strength,** which is the voltage per centimeter that must be applied to rupture or break down the material. The **dielectric strength** is a measure of the insulating property of the material. The **dielectric constant** is a measure of its ability to store energy, or its **condensive** effect.

**Prob. 6–16.** A certain condenser using acetone as a dielectric has a capacitance of 1.84 microfarads. The pressure across it varies from 8 volts to 34 volts in 0.003 second. What average current will flow in this condenser under this condition, if the acetone is replaced by glycerine?

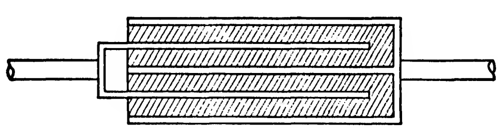


FIG. 8–16. A condenser consisting of several plates.

**7. Parallel-Plate Condensers.** A parallel-plate condenser may be constructed as shown in Fig. 6–16 or, in order to increase the amount of capacitance, as shown in Fig. 8–16. Such condensers may be built up of sheets of metal and plates of glass. They may be constructed of paraffined paper and tinfoil. The construction depends, of course, upon the voltage to which they are to be subjected.

The capacitance of such a unit is its current-per-volt change per second. Since this current depends upon the number of electrons in a cross-section which can move in position, it is to be expected that the capacitance will be proportional to the cross-sectional area of the sheet of dielectric between the plates. Also, since the extent to which the electrons will move depends upon the force, or volts per centimeter, to which they are subjected, it is to be expected that the capacitance of such a condenser will be inversely proportional to the thickness of the dielectric. Such is found experimentally to be the case. The capacitance is also proportional to the dielectric constant, $K$, of the material used. We may thus write for the capacitance

$$C = \frac{8.84\,KA}{10^8 \times l} \tag{2}$$

in which $C$ = capacitance in microfarads,
$K$ = relative dielectric constant (air = unity),
$A$ = area (one side) in square centimeters of all the dielectric actually between the plates,
$l$ = thickness of dielectric in centimeters.

Inexpensive condensers up to 8 microfarads are commonly made of alternate sheets of aluminum foil and waxed paper. These are wound into rolls and made very compact. Condensers of large capacitance (10 to 50 microfarads) are made by using an insulating layer, formed directly on the metal plates of the condenser, the space between being filled with an electrolyte in liquid or paste form. The thickness of the layer thus formed is very small and a high capacitance is thus obtained.

Figure 9–16 shows a variable condenser of the type commonly used in radio work, and for measurements with high-frequency alternating currents. It consists of a number of movable plates arranged on a shaft so that they can be moved between a set of fixed plates. Air is usually the dielectric, although occasionally a condenser of this type is immersed in a liquid dielectric to increase the capacitance obtainable. The condenser, shown in Fig. 9–16, is a precision type built especially for measurements requiring variable condensers of known capacitances.

**Example 3.** What is the capacitance of a condenser built with 2000 plates, in which the dielectric consists of sheets of paraffined paper,

0.005 cm thick? The part of each sheet actually between plates has an area of 16 × 20 sq cm.

**Solution.**

$$C = \frac{8.84\,KA}{10^8 l} = \frac{8.84 \times 2.1 \times 16 \times 20 \times 2000}{10^8 \times 0.005}$$

$$= 23.8 \text{ microfarads } (\mu f).$$

**Prob. 7–16.** A rolled aluminum foil and paper condenser consists of 20 strips of foil, 2 inches wide and 40 inches long, separated by paraffined paper 0.002-inch thick. What is the capacitance after rolling and connecting alternate sheets of foil together?

**Prob. 8–16.** If mica sheets 0.0015-inch thick and half the cross-section are substituted for the paraffined paper in Prob. 7–16, what is the capacitance of the condenser?

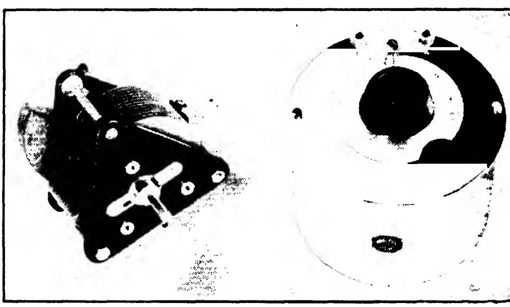


FIG. 9–16. A variable condenser. *General Radio Co.*

**8. Charge on a Condenser.** Electrons repel one another so powerfully that some are crowded out to the surface, but cannot leave it on account of the attraction of the metal for them, except when the temperature is high. These electrons act like an incompressible fluid in the body of the metal.

But if a sufficiently high electric pressure is exerted on the electrons, a few more can be crowded on a surface. This is especially true, if an oppositely charged plate is very near, for the surface electrons of the first plate are attached to it.*

When a dielectric is placed between the plates, some of the electrons belonging to the atoms in it are forced over toward one plate, and these atoms thus attract other electrons of the metal and crowd

* This partially explains why a vacuum can act as a dielectric of a condenser; although it does not explain the fact that a current passes in the vacant space between the plates when the voltage across them changes.

more out of the metal to the surface of the plate. The plate thus has a greater charge than it would have, if the dielectric were not present. For it must be remembered that a body is charged when it has a greater or smaller number of electrons on it than it normally has. If it has a greater number, it is negatively charged; if it has fewer, it is positively charged. When just sufficient electrons are present to satisfy or neutralize the positive nuclei, then we say that the body is not charged.

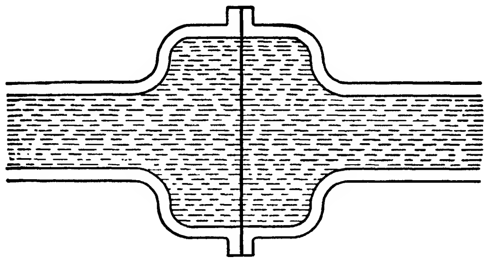


FIG. 10–16. A condenser is like a pipe with an elastic diaphragm.

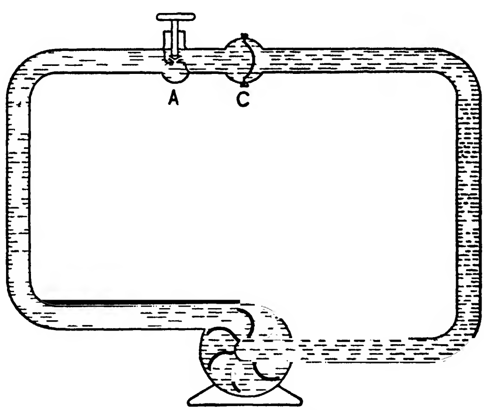



FIG. 11–16. A water analogy to the electric circuit of Fig. 12–16.

An electric condenser has an excellent hydraulic analogue in a flexible diaphragm stretched across a pipe, as shown in Fig. 10–16. If this pipe is connected to a centrifugal pump, as in Fig. 11–16, and the pump is started running, the diaphragm will be stretched as shown, and will stay stretched as long as the pump operates. During the period when the pump is accelerating, and hence increasing the pressure, there will be a flow of water in the pipe which

will cease soon after the pressure becomes constant. If the pump is stopped, there will be a flow of water back, and the diaphragm will spring back to its mid-position.

Similarly, if an electric condenser is connected in series with a generator, as shown in Fig. 12–16, and the generator started, there will be a flow of current around the circuit after the generator is started, if the switch $S$ is closed; but the flow will soon stop, if the generator speed, and hence the voltage, become constant. If the generator is stopped, and the voltage thus removed, there will be a flow back through the circuit for a short period of time.

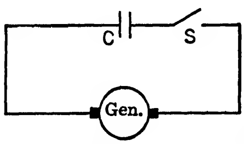


FIG. 12–16. A circuit containing a condenser.

Suppose that instead of stopping the generator, we first open the switch at $S$. This is equivalent to closing a valve at $A$, in Fig. 11–16. In such a case, the current cannot flow back even though the generator is stopped. We now say the condenser is charged. The electrons in the dielectric tend to spring back to their positions before displacement; but they cannot do so because the electrons trapped on the plates exert forces which keep the electrons in the dielectric displaced. The electrons on the plate thus act like the column of water trapped between the valve $A$ and the displaced diaphragm $C$, in Fig. 11–16. The water cannot escape; and since it cannot be compressed, the diaphragm cannot return to its mid-position. The difference in pressure of the water across the valve is that set up by the pump at the time the valve was closed. Likewise, when the switch $S$ in Fig. 12–16 is opened, the voltage across the condenser is equal to the generator voltage at the instant the switch was opened. We say the condenser is charged to this voltage.

The amount of the charge can be found as follows:

If the voltage rises at a uniform rate from $O$ to $E$ volts in $t$ seconds, the average rate at which the voltage changes will equal $\frac{E}{t}$ and the average current will equal

$$I = C \times \frac{E}{t}. \qquad (3)$$

The quantity of electricity put into the condenser will then be $I \times t$.

$$It = C\frac{E}{t}t,$$

$$= CE. \tag{4}$$

But $$It = Q,$$

therefore $$Q = CE, \tag{5}$$

in which

$Q$ = quantity of electricity in coulombs or ampere-seconds,
$C$ = capacitance in farads,
$E$ = emf in volts.

This may also be written as

$$C = \frac{Q}{E}. \tag{6}$$

Thus the capacitance may be written as the **charge per volt** across the terminals. We, therefore, have two equivalent definitions of capacitance:

(1) The capacitance is that factor by which the rate of voltage change is multiplied to determine the current. A condenser has one farad capacitance when a change of one volt per second will cause one ampere to flow in the condenser.

(2) The capacitance is the charge per volt. A condenser has one farad capacitance when one volt will put one coulomb charge upon it.

**Example 4.** How many coulombs of electricity will a condenser of 15 microfarads capacitance hold, when the pressure between its terminals is 200 volts?

**Solution.**

$$Q = CE = 0.000{,}015 \times 200 = 0.003 \text{ coulomb.}$$

**Example 5.** If the 0.003 coulomb, in Example 4, takes $\frac{1}{500}$ of a second to flow into the condenser, what is the average current?

**Solution.**

$$I = \frac{Q}{t} = \frac{0.003}{0.002} = 1.5 \text{ amperes.}$$

**Prob. 9-16.** What is the capacitance of a condenser that holds 0.0009 coulomb under a pressure of 81 volts?

**Prob. 10-16.** How many volts would be required to put 0.005 coulomb into the condenser of Prob. 9-16?

**Prob. 11-16.** What average current flows, if it takes 0.005 second to charge the condenser in Prob. 9-16?

**Prob. 12–16.** The condenser in Prob. 9–16 takes 0.005 second to charge completely on a 250-volt circuit. What average charging current flows?

**Prob. 13–16.** How many coulombs will a 20-microfarad condenser hold when charged to 1000 volts?

**Prob. 14–16.** If the condenser in Prob. 13–16 is charged in 0.005 second, what will be the average charging current?

**Prob. 15–16.** If the condenser in Prob. 14–16 is charged on a 1000-volt circuit, and then disconnected and a 20-ohm wire is placed across its terminals, what average power will be used in heating the wire? (Average voltage of discharge $= \frac{1000}{2}$.)

**Prob. 16–16.** How long will the discharge last in Prob. 15–16? (Use average current.)

**Prob. 17–16.** How much work (in watt-seconds) will be done on the wire in Prob. 15–16?

**Prob. 18–16.** If a 40-ohm wire were used in Prob. 15–16, what would be the average current, average power and total work done on the wire?

**9. A Comparison of Inductance and Capacitance.** The inductance of a circuit has been likened to the inertia of a moving train or of a flywheel on the shaft of an engine. The capacitance is like the elasticity of the shaft or connections. The inductance makes its presence known when we change the current in the circuit; the capacitance, when we change the voltage. The most useful equation (Equation 5–9, page 238) dealing with inductance is

$$e_{av} = L\frac{i_1 - i_2}{t_1 - t_2}.$$

The similar equation dealing with capacitance (Equation 1) is

$$i_{av} = C\frac{e_1 - e_2}{t_1 - t_2}.$$

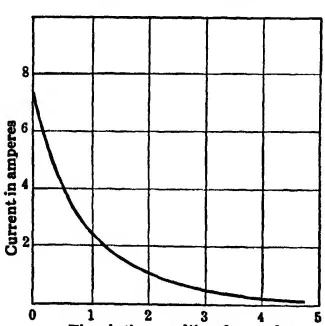


FIG. 13–16. The current curve when a condenser is charged by a constant voltage.

When a steady voltage is suddenly applied to an **inductance** with resistance in series, the current gradually **rises** to its normal value, as in Fig. 7–9, page 232.

When a steady voltage is suddenly applied to a **capacitance** with

a resistance in series, the current starts in at its maximum value and gradually falls to zero, as in Fig. 13–16.

Inductance and capacity produce opposite effects in a circuit, just as a flywheel and a spring on a line of shafting would produce opposite effects. This is well illustrated by the effect of the flywheel of an automobile engine, which hinders a quick starting by the starting motor; while the Bendix spring in the starter allows a certain play in the connection between the starting motor and the engine.

**10. Energy Stored in a Dielectric Field.** The energy stored in a magnetic field was given in Chapter IX, Art. 9, as

$$W = \frac{LI^2}{2}.$$

This is called magnetic energy.

When a dielectric is subjected to an electric stress, we have seen that electricity, and therefore energy, is stored in the dielectric or in the dielectric field, by a slight movement of the electrons. This, due to the elastic nature of the bonds holding these electrons to the atoms, is analogous to the stretching of a spring. When a spring is stretched, or put in tension, work is done upon it and energy is stored in it. And if the spring has no weight and develops no friction, all this energy can be recovered when the spring is released. Similarly, all the electricity, or energy, stored in a condenser may be recovered (neglecting $I^2R$ losses).

The quantity of electricity which flows into a condenser of $C$ farads capacitance, when it is charged to a potential of $E$ volts in time $t$, is

$$Q = CE \text{ coulombs.}$$

The potential on the condenser, which is the voltage which must be overcome, varies from 0 to $E$ volts; thus the average voltage on the condenser during the charging time is

$$E_{av} = \frac{E}{2} \text{ volts.}$$

The average charging current is

$$I_{av} = \frac{Q}{t} \text{ amperes.} \tag{7}$$

The work done, then, in charging the condenser equals the product

of average voltage, average current and time $t$, or

$$\begin{aligned}\text{Work} &= E_{av} \times I_{av} \times t \\ &= \frac{E}{2} \times \frac{Q}{t} \times t \\ &= \frac{E}{2} \times \frac{CE}{t} \times t \\ &= \frac{CE^2}{2} \text{ watt-seconds.} \qquad (8)\end{aligned}$$

The energy stored in the dielectric field of a condenser is that due to electricity at rest, and is therefore called **electrostatic energy.**

**Example 6.** How much energy is stored in the dielectric field of the 15-microfarad condenser of Examples 4 and 5, when charged to the same potential as in these examples?

**Solution.**

$$W = \frac{CE^2}{2} = \frac{0.000015 \times 200^2}{2} = 0.3 \text{ watt-seconds.}$$

**Prob. 19–16.** Compare the value of the energy stored in the dielectric field of the condenser, in Probs. 13–16 and 14–16, with the work done, in Probs. 17–16 and 18–16, upon the 20-ohm and the 40-ohm wires.

**11. Measurement of Capacitance. Direct Deflection of Ballistic Galvanometer.** In measuring capacitance, we make use of the fact that, in order to charge a circuit with electricity, electrons must flow along a conductor. If a ballistic galvanometer is inserted in series in the line, this flow of electrons can be made to give a momentary deflection, or **throw,** proportional to the number of electrons or the amount of the **charge.**

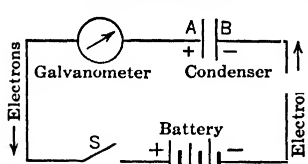


FIG. 14–16. The galvanometer measures the momentary current flowing in the condenser circuit.

Thus in Fig. 14–16, when the switch $S$ is closed, the electrons flowing along the upper conductor to charge plate $A$, must pass through galvanometer $G$. This will cause the galvanometer to "throw." Since the galvanometer is constructed so that the needle swings, or "throws," to a reading, proportional to the product of the moving force and time of application, and immediately returns to zero, it is said to be a **Ballistic** galvanometer. This term

distinguishes it from one which requires a steady current to cause its deflections to be proportional to the moving force.

A more common way of connecting the galvanometer is shown in Fig. 15–16. The switch $S$ is thrown down to charge the condenser plates $A$ and $B$, and then quickly thrown up and allowed to discharge through the galvanometer. A special charge and discharge switch is required for accurate results.

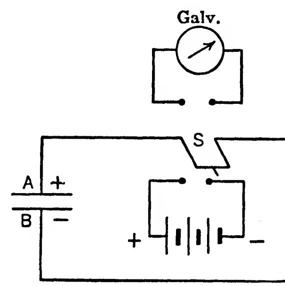



FIG. 15–16. The condenser AB is discharged through the ballistic galvanometer to measure the charge.

To measure the capacitance of any device, such as a given length of telephone cable, it is first charged for one minute, then suddenly discharged through the galvanometer and the throw noted. A condenser of known capacitance is then inserted in the place of the unknown, and charged for a minute, and the throw of the galvanometer noted as it is discharged. The capacitances of the two pieces are then in the same ratio as the galvanometer throws, **provided the throws have not too great a difference.** Since the known capacitance is usually a variable standard condenser, its capacitance can be adjusted until the galvanometer throw on discharge is very nearly equal to the throw caused by the unknown capacitance. In this way, a considerable degree of precision can be obtained.

**Prob. 20–16.** In a circuit arranged as in Fig. 15–16, a cable is charged to a given potential. On being discharged through a ballistic galvanometer, it causes the instrument to deflect 13.6 scale divisions. A standard condenser of 1.5 microfarads capacitance after being charged to the same potential, on discharge, causes the same galvanometer to deflect 11.8 scale divisions. What is capacitance of cable as tested?

**12. Capacitance Measurement. Bridge Method.** Another method of measuring capacitance is shown in Fig. 16–16. It resembles a Wheatstone bridge, with the exception that condensers $C_1$ and $C_2$, one known and the other unknown, replace two of the resistances. A source of alternating-current supply takes the place of the battery, and a pair of telephone receivers, $T$, replaces the galvanometer. The alternating supply is usually of a relatively high frequency (1000 cycles per second is commonly used) so that an easily audible tone can be heard in the receivers. This

current usually is supplied from either a magnetically driven tuning-fork oscillator or from a vacuum-tube oscillator. A balance is obtained, as in a Wheatstone bridge, by adjusting $R_1$ and $R_2$ until no sound is heard in the receiver $T$.

The following equation is then used to compute the capacity of the unknown, for instance, $C_2$:

$$\frac{R_1}{R_2} = \frac{C_2}{C_1}. \tag{9}$$

Note that the ratio of capacitances is the **inverted** ratio of the

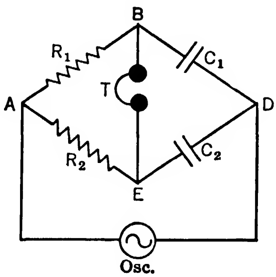



FIG. 16–16. A Wheatstone bridge arrangement to measure capacitance.

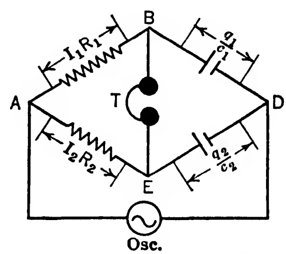



FIG. 17–16. Voltage drops in a capacitance bridge.

resistances. In this respect, it differs radically from the Wheatstone bridge equation for resistance measurements.

The proof of the above equation is as follows:

When there is no sound in receiver $T$ (Fig. 17–16), $B$ and $E$ must be at the same potential. The voltage from $A$ to $B$ ($I_1R_1$) must equal the voltage from $A$ to $E$ ($I_2R_2$), or

(1) $$I_1R_1 = I_2R_2.$$

Also the voltage from $B$ to $D$ must equal the voltage from $E$ to $D$. The voltage across a condenser equals the charge in it divided by its capacitance. (See Article 8.)

If $q_1$ be the charge on $C_1$, and $q_2$ the charge on $C_2$, at any instant, then

$$\text{Voltage } (B \text{ to } D) = \frac{q_1}{C_1};$$

$$\text{Voltage } (E \text{ to } D) = \frac{q_2}{C_2}.$$

Therefore,

$$\frac{q_1}{C_1} = \frac{q_2}{C_2},$$

or,

(2) $$C_2q_1 = C_1q_2.$$

But in order that the quantity ($q_1$) get to the condenser ($C_1$), it must flow through ($R_1$) as there is no flow along $EB$.

In like manner, the quantity $q_2$ must flow through $R_2$.

Thus we have the equation,

$$(3a) \qquad I_1 \text{ (current in } R_1) = \frac{q_1}{t};$$

and

$$(3b) \qquad I_2 \text{ (current in } R_2) = \frac{q_2}{t};$$

where $t$ = the time in seconds in which the charges flow through the resistances. This time must be the same in each resistance since the two resistances are in parallel across the same alternator.

Then, multiplying (3*a*) by $R_1$ and (3*b*) by $R_2$, we have,

$$I_1R_1 = \frac{q_1}{t}R_1;$$

and

$$I_2R_2 = \frac{q_2}{t}R_2;$$

substituting these values in (1),

$$\frac{q_1R_1}{t} = \frac{q_2R_2}{t};$$

or

$$(4) \qquad q_1R_1 = q_2R_2.$$

Dividing equation (4) by (2) we have,

$$\frac{q_1R_1}{q_1C_2} = \frac{q_2R_2}{q_2C_1};$$

or

$$\frac{R_1}{C_2} = \frac{R_2}{C_1};$$

This may be transposed to read

$$\frac{R_1}{R_2} = \frac{C_2}{C_1}.$$

**Prob. 21–16.** In a bridge arranged as in Fig. 16–16, no sound is heard in the telephone receiver when $R_1 = 1.297$ ohms and $R_2 = 3.000$ ohms. What is the capacitance of $C_1$ if the capacitance of $C_2$ is 2.461 microfarads?

**Prob. 22–16.** In Fig. 16–16, suppose that the oscillator and $T$ are interchanged in position. What will be the equation of the bridge for this condition?

**13. Locating a Break in a Telephone Cable.** An arrangement similar to that of Fig. 16–16 is often used in locating a break in one of a pair of telephone cables. In Fig. 18–16, $C_1$ represents a pair of good wires, $C_2$ represents a pair in which one of the wires is broken at $F$.

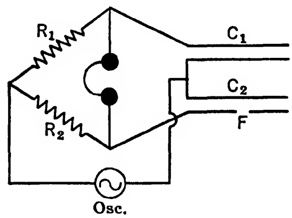



FIG. 18–16. A capacitance method for finding a break in a cable.

When bridge is balanced,

$$\frac{R_1}{R_2} = \frac{C_2}{C_1},$$

where $C_2$ represents the capacity of the faulty pair.

Now, if the same conditions exist with regard to the wires (except that one wire of one pair is broken), the capacitance of the faulty pair as far as the break will have the same relation to the capacitance of the good pair, that the distance out to the break has to the length of the good pair.

Thus:

$$\frac{l_2}{l_1} = \frac{C_2}{C_1}; \tag{10}$$

where
$l_2$ = distance to break;
$l_1$ = length of good pair;
$C_2$ = capacitance to break;
$C_1$ = capacitance of good pair.

Therefore,

$$\frac{R_1}{R_2} = \frac{l_2}{l_1}. \tag{11}$$

If the length of the good pair is known, the distance $l_2$ out to the break can easily be computed.

**Prob. 23-16.** In a cable test for a break, arranged as in Fig. 18–16, the length of the pair of good wires ($C_1$) is 2500 feet. A balance is obtained when $R_1$ = 14 ohms and $R_2$ = 30 ohms. How far out is the break in the faulty cable ($C_2$)?

**Prob. 24-16.** In a testing outfit, arranged as in Fig. 18–16, the telephone receiver gives no sound, when $R_1$ = 40 ohms and $R_2$ = 76 ohms. If the length of the good pair of cables, $C_1$, is 6000 feet, how far out is the break in the faulted pair, $C_2$?

**Prob. 25-16.** If the capacitance of the good pair in Prob. 24-16 is 2.83 microfarads, what is the capacitance of the faulty pair as far out as the break?

**14. Condensers in Parallel.** Consider two condensers, I and II, Fig. 19–16, joined in parallel across the mains, the voltage of which is $E$. The capacitance of condenser I is $C_1$; of condenser II, $C_2$. Find the combined capacitance of the two when so joined.

Let $C$ be the combined capacitance;

Let $Q_1$ = quantity of charge in condenser I;

Let $Q_2$ = quantity of charge in condenser II.

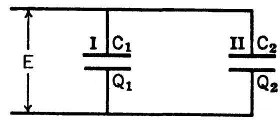


FIG. 19–16. Two condensers connected in parallel.

Then total quantity in both condensers $= Q_1 + Q_2$.

(a) $$Q_1 + Q_2 = CE.$$

(Quantity of total charge equals total voltage times total capacitance.)

But $$Q_1 = C_1E$$

and $$Q_2 = C_2E.$$

Therefore (b) $$Q_1 + Q_2 = (C_1 + C_2)\,E.$$

From (a) and (b) we have

$$CE = (C_1 + C_2)\,E,$$

or $$C = C_1 + C_2. \quad (12)$$

Thus the capacitance of condensers joined in parallel equals the sum of the capacitances of the separate condensers. Joining condensers in parallel is merely adding the plate area of one to that of the other.

**Example 7.** What is the capacitance of 4 condensers of 3, 0.2, 7 and 2.5 microfarads, respectively, when joined in parallel?

**Solution.**

$$C = C_1 + C_2 + \ldots.$$
$$C = 3 + 0.2 + 7 + 2.5 = 12.7 \text{ microfarads.}$$

**Prob. 26–16.** Two condensers of 3 and 5 $\mu$f capacitance, respectively, are connected in parallel. What voltage is required to force a charge of 0.0035 coulomb into the parallel combination?

**Prob. 27–16.** What is the charge on each condenser in Prob. 26–16?

**15. Condensers in Series.** Condensers in series present a peculiar phenomenon. Let condenser I of capacitance $C_1$, Fig. 20–16, be joined in series with condenser II of capacitance $C_2$. Let $E$ be voltage across combination, $E_1$ across condenser I, and $E_2$ across condenser II.

The charge $Q$ is sent into the condensers under the action of the voltage $E$. Since the two condensers are in series, the same charge must be sent into each, just as the same current is sent through resistances in series. If electricity acts as an incompressible fluid, then there must be as much displacement of electrons in condenser II as in condenser I when they are in series. Thus the charge sent into **each** is $Q$, and the charge sent into **the combination** is also the same, $Q$.

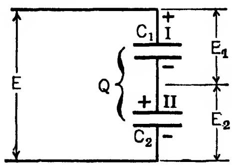


FIG. 20-16. Two condensers connected in series.

Let $C$ = combined capacitance of $C_1$ and $C_2$.

(a)
$$E = \frac{Q}{C};$$
$$E_1 = \frac{Q}{C_1};$$
$$E_2 = \frac{Q}{C_2}.$$

But

(b)
$$E = E_1 + E_2.$$

Therefore, from (a) and (b),

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2};$$

and

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}. \qquad (13)$$

Thus the reciprocal of the combined capacitance of condensers in series equals the sum of the reciprocals of the capacitances of the separate condensers.

**Example 8.** If the capacitance of condenser I in Fig. 20-16 is 2 microfarads and that of condenser II, 5 microfarads, what is the capacitance of the two condensers joined in series?

**Solution.**

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2};$$
$$\frac{1}{C} = \frac{1}{2} + \frac{1}{5} = \frac{7}{10};$$
$$C = \frac{10}{7} = 1.43 \text{ microfarads.}$$

**Prob. 28–16.** What charge is sent into condensers, of above example, if the voltage across the combination is 115 volts?

**Prob. 29–16.** What is the voltage across condenser I and across condenser II, Fig. 20–16, if voltage across the two in series is 110 volts? Capacitance of I = 6 microfarads, of II = 5 microfarads.

**16. Capacitance of Telephone and Telegraph Cables, etc.** One of the obstacles to sending telegraph messages across the ocean is the large capacitance of the submarine cable. The insulating material around the wire acts as a dielectric between two conductors, the wire and the water. The great length of the cable gives a sufficiently large area to the dielectric to produce a very high capacitance. In transmitting a message, then, it is necessary alternately to fill up and empty a piece of apparatus of great capacitance. The time necessary for these operations puts many difficulties in the way of efficient and rapid transmission.

Telephone cables being laid in pairs, and often in conduits, also possess considerable capacitance. This would make it almost impossible to operate long lines, were it not for the possibility of "loading" a line with inductance coils. As has been explained, **capacitance** and **inductance** have practically opposite effects on a circuit, inductance tending to choke back the flow of electricity, capacitance tending to increase the flow. Under the proper conditions, one may be made to neutralize the other. Thus the inductance of the "loading" coils on a telephone line practically neutralizes the capacitance effect of the long insulated cables.

Another striking example is a new submarine cable, which has a sheath of the metal Permalloy described in Art. 23, Chapter VIII. Because of the presence of the sheath, the self inductance of the circuit is increased to the extent of neutralizing most of the undesirable capacitance.

Sometimes condensers are used, in a similar way, to counteract the effect of inductance. Examples of this are frequently found in alternating-current practice.

**17. Capacitance of Cables.** The capacitance of submarine cables and of cables laid in metal sheaths is an important factor in telegraphy and telephony, as discussed above.

The capacitance of such lines may be computed by means of the same equation as for a plate condenser. A more precise equation,

however, for the capacitance of a cable is the following:

$$C = \frac{0.0388\,Kl}{\log_{10}\dfrac{D}{d}}; \tag{14}$$

where $C$ = capacitance of cable in **microfarads;**
$l$ = length of cable in miles;
$D$ = outside diameter of dielectric (insulation);
$d$ = inside diameter of dielectric in same units as $D$;
$K$ = relative dielectric constant of insulation.

**Prob. 30–16.** A No. 6 B & S copper wire, covered with 0.30 cm of gutta percha insulation, is held in a lead sheath. Find capacitance per mile of this cable and grounded sheath.

**Prob. 31–16.** A submarine telegraph cable is of copper 0.42 inch in diameter, and covered with 0.6 inch of gutta percha insulation. What is capacitance between cable and water of 100 miles of such cable?

**18. Capacitance of an Aerial Line.** The capacitance of twin wires may be found by means of the following equation:

$$C = \frac{0.0194\,l}{\log_{10}\dfrac{S}{r}}; \tag{15}$$

where $C$ = capacitance in **microfarads;**
$l$ = length of **one wire** in **miles;**
$S$ = space between wires;
$r$ = radius of each wire in **same units as** $S$.

**Prob. 32–16.** What is the capacitance per mile of a line consisting of two No. 2 B & S wires, separated by a space of 30 inches?

**Prob. 33–16.** What is the capacitance of a 60-mile line consisting of two No. 4 B & S copper wires hung 40 inches apart?

**19. Mechanical Force on a Condenser. Electrostatic Voltmeters.** When two bodies are charged electrically, there is always a small mechanical force acting between them, as already mentioned in Art. 2, called an electrostatic force. This force tends to pull the bodies together if the charges are opposite in sign, and to push them apart if the charges are alike in sign. Electrostatic attraction and repulsion are similar to magnetic attraction and repulsion, although the electrostatic forces are very small compared with magnetic forces. The force between two charged plates of a

condenser is given by the equation:

$$f = \frac{1.59\, KAE^2}{l^2 \times 10^{11}} \text{ ounces}; \tag{16}$$

in which $f$ = force between two similar plates in ounces;
$K$ = relative dielectric constant;
$A$ = area of one side of one plate in square inches;
$l$ = distance between plates in inches;
$E$ = electromotive force between plates in volts.

Note, by Example 9, how exceedingly small this force ordinarily is. Yet when the voltage, $E$, between the plates is very high, the force becomes large enough to be used to measure voltage. Thus, in Fig. 21–16, the electrostatic force is great enough on high voltages to actuate the delicately suspended vanes, and cause them to indicate the pressure between the movable and stationary vanes.

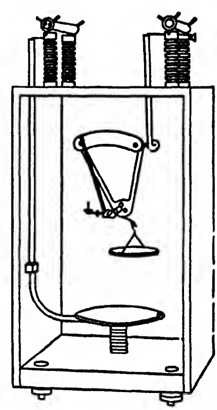


Fig. 21–16. An electrostatic voltmeter. *General Electric Co.*

**Example 9.** What is the force between two plates 64 centimeters square, separated 0.64 centimeter in air, and subjected to an emf of 100 volts?

**Solution.**

$$f = \frac{1.59\, KAE^2}{l^2 \times 10^{11}}$$

$$= \frac{1.59 \times 1 \times 64^2 \times 100^2}{0.64^2 \times 10^{11}} = 0.00159 \text{ ounce.}$$

**Prob. 34–16.** In an electrostatic voltmeter, similar to that shown in Fig. 21–16, the effective area of each plate is 18 square inches. The distance between the plates is 3 inches. What force acts on the plates when a potential difference of 40,000 volts is impressed between them?

**Prob. 35–16.** In designing an electrostatic voltmeter of the type shown in Fig. 21–16 for 75,000 volts, it is desired to produce a force of attraction on the plates of 0.003 ounce. Determine the other dimensions of the voltmeter, using two plates of the same size and having a factor of safety of 4 against flash-over between plates.

**20. The Insulation of Aerial Lines.** The proper design of a transmission line brings in many factors. In designing an insulator for a transmission line, the following points must be considered.

**First:** It must be constructed so that there is sufficient dielectric strength of the material to avoid puncture under the maximum stress which will occur.

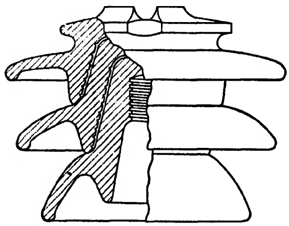


FIG. 22–16. A pin insulator with three petticoats.

**Second:** It must have sufficient surface so that it will not break down between wire and ground, over the surface of the material. This is the reason that insulators are constructed with a deeply corrugated surface, or with petticoats, as shown in Fig. 22–16. Also, these petticoats must be of such shape that, when it rains, there will be enough surface of the insulator left dry to stand the applied potential.

**Third:** The insulator must be strong enough to stand the mechanical stresses which are brought to bear upon it; and these are often high when a heavy transmission wire is being supported.

**Fourth:** There must be no point close to the insulator at which the electric stress is so great that it tears electrons from the atoms, and thus "ionizes" the air, and makes it a conductor of electricity.

Insulators of the general type of Fig. 22–16 are used on lines operating at 30,000 volts and below; but for higher voltages the increased size, weight and cost of this type have made their use prohibitive, and insulators of the strain type, shown in Fig. 23–16, have been developed.

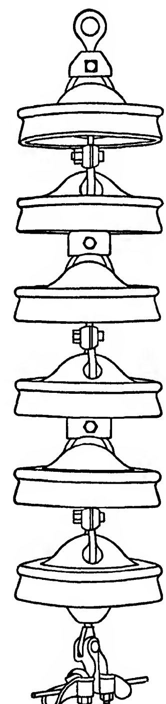


FIG. 23–16. A string of strain insulators.

The strain type has an advantage over the pin type in that a number of insulators are strung together in series and thus divide the total voltage of the line among several units. This is illustrated in Figs. 24a–16 and 24b–16. Note in Fig. 24a–16 that although the units are exactly alike, the voltage does not divide equally among them, but is greater across the end units, being especially high across the unit connected to the conductor.

The reason for this is as follows.*

* For a more complete treatment of this problem see "Electric Power Transmission and Distribution" by L. F. Woodruff (John Wiley & Sons).

The insulating material of each insulator, being between two metal pieces, forms a condenser. But also the air between each piece of metal and the ground forms other condensers. And likewise, the air between the plates of the units and the metal of the conductor forms still other condensers. Thus in Fig. 24*b*–16, the metal between the insulation 1 and 2 forms condenser (*a*) with the air as dielectric and the ground as the other plate. But the same metal forms condenser (*w*) with the air as dielectric and the conductor as the other plate.

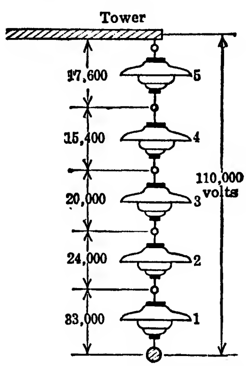



FIG. 24a–16. The distribution of voltage in a string of five strain insulators.

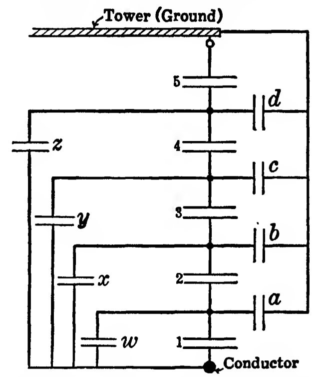



FIG. 24b–16. The equivalent circuit of the string of insulators of Fig. 24a–16.

The charge on condenser (*a*) is fairly large because of the large area of one plate (the ground) and also because of the high voltage between the metal parts and the ground. On the other hand, the charge on condenser (*w*) will be lower because of the small area of both plates and the lower voltage between the metal parts and the conductor. Condensers 1 and *w* are in parallel, and condensers 2 and *a* are in parallel. The parallel combination of 1 and *w* is in series with the parallel combination of 2 and *a*.

The sum of the charges on (*w*) and (1) must be the same as the sum of the charges on (2) and (*a*). That is,

$$Q_1 + Q_w = Q_2 + Q_a,$$

or

$$Q_2 = Q_1 + Q_w - Q_a.$$

But we have seen that $Q_a$ is larger than $Q_w$. Therefore $Q_1 + Q_w - Q_a$ is smaller than $Q_1$. Therefore $Q_2$ is smaller than $Q_1$.

The capacitances of all the units are the same; let us call it $C$. Then the voltage across condenser (1) is

$$E_1 = \frac{Q_1}{C},$$

and the voltage across condenser (2) is

$$E_2 = \frac{Q_2}{C}.$$

But we have seen that $Q_2$ is smaller than $Q_1$, therefore $E_2$ is smaller than $E_1$, as shown in Fig. 24a–16.

Similarly, the voltage across each succeeding unit will be found to be smaller than that of the one below it; until a point is reached where the charge on the air condenser, formed by the metal parts and the conductor, is larger than the charge on the condenser formed by the metal parts and the ground. This occurs when we compare the condensers (4) and (5).

The sum of the charges on condensers (4) and ($z$) are the same as the sum of the charges on condensers (5) and ($d$). This may be written

$$Q_4 + Q_z = Q_5 + Q_d,$$

or

$$Q_5 = Q_4 + Q_z - Q_d.$$

Now, although the capacitance of condenser ($d$) is larger than that of condenser ($z$) because of the large area of one plate (the ground), still the voltage across ($z$) is much larger than the voltage across ($d$). Thus the charge on ($z$) is larger than the charge on ($d$). Therefore

$$Q_4 + Q_z - Q_d \text{ is larger than } Q_4.$$

But

$$Q_4 + Q_2 - Q_d = Q_5.$$

Therefore $Q_5$ is larger than $Q_4$.

The voltage across $Q_5$ is

$$E_5 = \frac{Q_5}{C}.$$

The voltage across $Q_4$ is

$$E_4 = \frac{Q_4}{C}.$$

Thus $E_5$ must be larger than $E_4$.

This shows that the voltage across either end unit is greater than that across the unit next to the end, although the voltage across the unit on the ground end of the string is usually but slightly greater than the voltage across the unit next to it. The voltage across the conductor end, however, is usually about 30 per cent of the entire voltage, regardless of the number of units in the string. This shows the futility of trying to insulate against greater and greater voltage by increasing the number of units in a string. The added units have almost no effect toward decreasing the voltage across the unit on the conductor end of the string. This unequal distribution can be somewhat corrected, by making the capacitance of unit No. 1 enough greater than that of unit No. 2, to make up for the loss of charge through the parallel connection of the condenser formed by the air and the wire. The capacitance of unit No. 2 is similarly made greater than that of No. 3. This is done by merely increasing the size of the metal caps sufficiently to give more area of dielectric between the metal parts which form the plates of the condensers.

## SUMMARY OF CHAPTER XVI

In an INSULATOR or DIELECTRIC, the electrons are so closely bound to the atoms that they cannot be torn away by ordinary voltages. In solid CONDUCTORS, there are free electrons present, which a moderate voltage will detach from the atom and pass on in the circuit — thereby setting up a current.

The function of an INSULATOR is to prevent a leakage of current.

Since there is no perfect insulator, if the voltage across any insulator is raised high enough, there will be a leakage of current. ALL THE ENERGY used in forcing a leakage current through the resistance of an INSULATOR is LOST IN HEAT or $I^2R$ LOSSES which CANNOT BE RECOVERED.

When there is a difference of potential between two insulated conductors, an ELECTRIC FIELD is always set up around them in much the same manner that a magnetic field is set up between two magnetic poles. The insulation, separating these conductors, is a DIELECTRIC; and the resistance or opposition to the flow of a leakage current is elastic in nature. ELECTRICITY or ENERGY is STORED in this FIELD, so that much of it CAN BE RECOVERED.

A Dielectric is always an Insulator. Some insulators are better dielectrics than others.

The ELECTRIC FIELD set up between two insulated conductors is called the DIELECTRIC or ELECTROSTATIC FIELD. It exists in lines of electric stress between two conductors of opposite polarity. These lines seem to repel one another as do magnetic lines. They do not exist inside the conductors and are therefore not closed lines. They emanate from the surface of one conductor at right angles to its surface; and end at the surface of the other conductor, also at right angles to its surface.

POTENTIAL GRADIENT is the drop in potential per unit length along any stress line in a dielectric field. It may be calculated in VOLTS PER CENTIMETER or in VOLTS PER INCH.

DIELECTRIC STRENGTH is the stress necessary to break down or disrupt an insulator or a dielectric. It is also measured in volts per unit length of path. See Table VI.

A CONDENSER consists of a dielectric between two conductors.

While ordinary stresses will not tear the electrons from the atoms of an insulator, they may strain the bonds and cause the electrons to move slightly, as the voltage changes. This property in a condenser is called its CAPACITANCE. When a voltage change of ONE VOLT PER SECOND produces sufficient movement of the electrons to constitute a momentary current of ONE AMPERE in a condenser, the condenser is said to have a CAPACITANCE of one FARAD. A MICROFARAD is one millionth of a farad.

The DIELECTRIC CONSTANT of a material is a measure of its effectiveness, as compared to air, when used as the dielectric in a condenser. See Table V for values.

PLATE CONDENSERS consist of thin sheets of lead, tin or alumi-

num foil, separated by thin sheets of insulating material. The capacitance of a plate condenser is

$$C = \frac{8.84}{10^8 l} KA \text{ microfarads}$$

The QUANTITY OF ELECTRICITY moved, or stored, in a condenser is called its CHARGE and is computed by the equation

$$Q = EC \text{ ampere-seconds.}$$

CAPACITANCE produces effects which are opposite to the effects produced by INDUCTANCE. Capacitance in an electric circuit acts like a SPRING, while inductance acts like the INERTIA OF A WEIGHT.

The ELECTRICAL ENERGY STORED IN A DIELECTRIC FIELD is

$$W = \frac{CE^2}{2} \text{ watt-seconds.}$$

Capacitance may be measured: (1) By direct deflection method, using standard condenser. Deflections of ballistic galvanometer on discharge of standard and unknown condensers, charged to the same potential, are proportional to the respective capacitances. (2) By bridge, using source of a-c power, standard variable resistances, standard condenser and telephone receiver. Equation of balanced bridge is

$$\frac{R_1}{R_2} = \frac{C_2}{C_1},$$

which is NOT the Wheatstone bridge equation for resistances.

LOCATING BREAK IN CABLE. Apparatus arranged as in bridge measurement of capacitance; using good pair of twin wires of known length exactly like the faulty pair.

Equation is

$$\frac{l_1}{l_2} = \frac{R_2}{R_1}.$$

CAPACITANCE OF CONDENSERS IN PARALLEL. Equals the sum of their separate capacitances.

CAPACITANCE OF CONDENSERS IN SERIES. Equals the reciprocal of the sum of the reciprocals of their separate capacitances.

CAPACITANCE OF CABLES. All cables have some capacitance. Submarine cables have a large capacitance, due to water forming one plate. This is because they are separated but a small distance, by the insulation, from the conductor which forms the other plate.

The usual equation for capacitance of a SUBMARINE CABLE is

$$C = \frac{0.0388\ Kl}{\log_{10} \frac{D}{d}} \text{ microfarads.}$$

**The capacitance of an AERIAL LINE is**

$$C = \frac{0.0194\,l}{\log_{10}\dfrac{S}{r}} \text{ microfarads.}$$

**The mechanical force exerted between the plates of a condenser is**

$$f = \frac{1.59\,KAE^2}{l^2 \times 10^{11}} \text{ ounces.}$$

**PIN INSULATORS, constructed with petticoats, are used on aerial lines for pressures up to 30,000 volts.**

**THE ELECTRIC STRESS ON A STRING OF STRAIN INSULATORS is greatest across the unit next the conductor, the voltage across it being usually nearly 30 per cent of the voltage across the total string. Adding more units will not materially affect this.**

## PROBLEMS ON CHAPTER XVI

**Prob. 36–16.** What is the capacitance of a condenser made up of 800 plates of aluminum foil 25 × 20 cm, with a dielectric of paraffined paper 0.015 cm thick?

**Prob. 37–16.** What is the capacitance of a condenser having 800 plates of aluminum foil 25 × 20 cm, if the dielectric consists of mica 0.015 cm thick?

**Prob. 38–16.** What is the capacitance of a condenser made up of 200 sheets of lead foil 20 × 30 inches, if the dielectric consists of lead glass $\frac{1}{32}$ inch thick?

**Prob. 39–16.** If the condensers of Prob. 36–16 and Prob. 37–16 were joined in parallel, what would be their joint capacitance?

**Prob. 40–16.** If condensers of Prob. 36–16 and Prob. 37–16 were joined in series, what would their joint capacitance be?

**Prob. 41–16.** What charge would the condensers joined as in Prob. 39–16 hold, if a voltage of 500 were placed across the terminals?

**Prob. 42–16.** If the voltage across the condensers, connected as in Prob. 40–16, were changed from 900 to 300 volts in 0.005 second, what average current would flow from the condensers?

**Prob. 43–16.** What would be the size of a plate condenser of one farad capacitance, if it could be made of the following materials: mica sheets 0.002-inch thick, and lead foil 0.0025-inch thick, with an area of 50,000 square inches per plate?

**Prob. 44–16.** It is desired to build up a capacitance of 6 microfarads. There are at hand three condensers of 4, 5 and 3 microfarads, respectively. Show by diagram what arrangement of these condensers will produce a capacitance nearest to 6 microfarads. Compute exact capacity of such an arrangement.

**Prob. 45–16.** How would condensers of Prob. 44–16 be arranged to produce the minimum capacitance? What would be the value of this minimum capacitance?

**Prob. 46–16.** An 8-microfarad condenser is charged to a potential of 320 volts and a 12-microfarad condenser to a potential of 480 volts. They are then joined together, positive to positive and negative to negative. What is the voltage across the parallel combination?

**Prob. 47–16.** If the condensers in Prob. 46–16 had been joined in series after being charged, what would the voltage across the combination become?

**Prob. 48–16.** What charge would there be on each condenser in Prob. 46–16 after being joined in parallel?

**Prob. 49–16.** What charge would there be on condensers in Prob. 46–16 after being joined in series?

**Prob. 50–16.** A condenser of 15 microfarads capacitance charged to 500 volts is suddenly discharged through a short wire. What amount of energy in watt-seconds is used in heating the wire? Note — Average voltage on discharge $= \frac{E}{2}$.

**Prob. 51–16.** If wire in Prob. 50–16 has a resistance of 75 ohms, in what time will condenser be discharged through it at average rate?

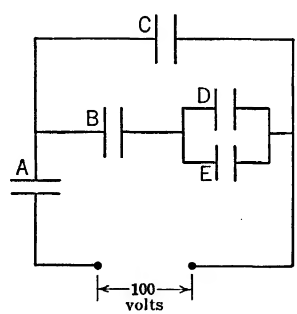



FIG. 25–16. Series-parallel arrangement of condensers connected across a 100-volt supply.

**Prob. 52–16.** The group of condensers in Fig. 25–16 are connected across a 100-volt line. The capacitances are: $A = 2\ \mu f$, $B = 4\ \mu f$, $C = 1\ \mu f$, $D = 3\ \mu f$, $E = 5\ \mu f$.

(*a*) What is the capacitance of the series-parallel arrangement?
(*b*) What is the charge on each condenser?
(*c*) What is the voltage across each condenser?

**Prob. 53–16.** The Sierra and San Francisco Power Co. transmits power from Stanislaus to San Francisco, a distance of 138 miles. The

wires are No. 00 B & S, and are spaced 96 inches apart. Compute the capacitances of two of these wires, if used as line and return.

**Prob. 54–16.** What is the capacitance between two wires of the 154-mile Big Bend-Oakland transmission line? Two wires, No. 000 B & S, are spaced 10 feet apart.

**Prob. 55–16.** What is the capacitance to sheath, per mile of a rubber-covered, single-conductor cable, having a wire of No. 0000 B & S, and insulation $\frac{27}{64}$-inch thick? $K = 3.42$.

**Prob. 56–16.** If impregnated paper is substituted for the rubber in the cable of Prob. 55–16, what will be the capacitance per mile? The relative dielectric constant of impregnated paper may be taken as 2.8.

**Prob. 57–16.** Describe with diagrams how an ungrounded break in an insulated cable could be located, provided the capacitance of the whole cable before the break were known.

**Prob. 58–16.** One method of obtaining a very high d-c voltage is to charge a number of condensers in parallel, and then connect them in series, so that the individual voltages add. A device of this type is called an impulse generator. An impulse generator consists of 50 condensers, each of 0.1-microfarad capacitance, which are charged in parallel to 20,000 volts and are then connected in series. If the series-condenser group is discharged in 0.00002 second, what average current does it deliver during discharge?

# CHAPTER XVII

## ELECTRICAL MEASURING INSTRUMENTS

In the following detailed description of electrical measuring instruments, no attempt is made to cover the field. Only those fundamental types which are in general use for d-c measurements in standardization laboratories, and in the testing and maintenance departments of manufacturing and operating companies, are considered.

**1. Galvanometers. The D'Arsonval Galvanometer.** A galvanometer is a very sensitive electrical instrument used in detecting and measuring extremely small currents.

The most common form of galvanometer is one based upon the principle of a coil turned by a magnetic field. This is called the

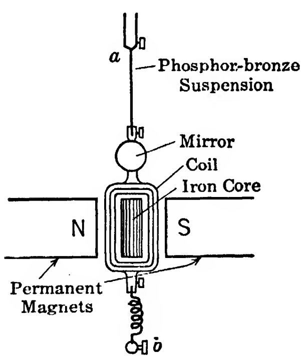



FIG. 1–17. Essential parts of a D'Arsonval galvanometer.

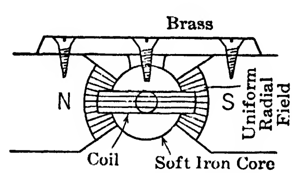



FIG. 2–17. Uniform radial field produced by concave pole faces and cylindrical iron core, D'Arsonval galvanometer

D'Arsonval principle and the instrument is called the D'Arsonval galvanometer. Due to its ruggedness, simplicity and ease of adjustment, it has practically displaced all other types of galvanometers. Figure 1–17 shows the essential parts in its construction. A coil of very fine copper wire is wound, generally on an aluminum bobbin, and suspended between the poles of a strong

permanent magnet by means of a fine phosphor-bronze wire or ribbon.

This phosphor-bronze suspension, *a*, in Fig. 1–17, acts as one lead-in wire to the coil, while a very flexible spiral filament to terminal *b* acts as the other.

The faces of the poles are usually cylindrical in shape. A cylindrical core of soft iron is mounted between the poles and inside the moving coil, as indicated in Fig. 2–17. By this means, the length of the air gap is reduced and the sides of the coil are always in a uniform radial magnetic field. The deflections of the suspended coil will thus be directly proportional to the current in the coil, and the scale divisions will be uniform. Figure 3–17 shows the appearance of one type of D'Arsonval galvanometer.

Owing to its strong permanent magnetic field, a D'Arsonval galvanometer is not appreciably affected by stray magnetic fields.

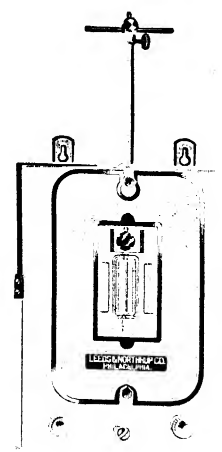


FIG. 3–17. One type of Leeds and Northrup reflecting galvanometer.

When the suspended coil is deflected by a current, the **deflecting force** is the motor action of the current flowing in the coil, as explained in Chapter XI. The turning of the coil produces torsion in the suspension wire or ribbon, which opposes the turning of the coil and tends to restore, or bring back, the coil to its normal position after a deflection. This force is called the **control** or **restoring force.**

It is desirable that the galvanometer coil, after a deflection, should return promptly to its zero or neutral position. A galvanometer coil, freely suspended (as that in Fig. 1–17), after a deflection will continue to swing or oscillate for some time. Any means employed to keep the moving parts from oscillating, or to retard them, is called **damping.** A well-damped instrument or galvanometer will not oscillate and is called **dead-beat.**

A D'Arsonval type galvanometer is most easily and effectively **damped** by winding the moving coil on a very light aluminum bobbin. The bobbin comprises a short-circuit path, in which currents are induced, as it moves with the deflecting coil. The currents in this bobbin create a retarding force on the coil, just as the currents in the armature of a generator oppose the rotation of

the armature. This has been fully explained in Chapters X and XI.

D'Arsonval galvanometers are made so sensitive that one-hundred-millionth part of an ampere will cause a deflection of one scale division. The deflections of this galvanometer, at least through small ranges, are proportional to the current flowing in the coil. And since the resistance of the coil is constant, the voltage must vary as the current. Therefore the deflections are also proportional to the voltage across the coil.

The more sensitive D'Arsonval galvanometers are the **reflecting type,** in which a mirror is mounted directly on the moving element, as in Figs. 1–17 and 3–17.

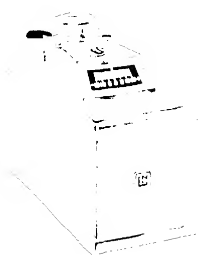


Fig. 4–17. Portable type, D'Arsonval galvanometer. *Leeds & Northrup Co.*

There are two common methods of reading the **deflection** of a reflecting galvanometer. One method is to attach a plane mirror to the moving element, and mount a telescope and scale about a half meter from the mirror. As the coil moves, the reflection of the scale in the mirror can be read through the telescope.

The other method is to mount a concave mirror on the moving system. A small incandescent lamp is placed some distance from the galvanometer, and the light is focused on a ground-glass scale. As the galvanometer coil moves, the reflected light beam travels along the scale.

A portable, more rugged and less sensitive type of D'Arsonval galvanometer is manufactured, in which the movable coil is supported by hardened steel pivots turning in jewelled bearings. The control or restoring force is obtained by spiral springs at top and bottom of the coil. A needle or pointer attached to the moving coil moves over a scale. Such a galvanometer is illustrated in Fig. 4–17.

**2. Galvanometer Shunts.** In order that a galvanometer may be used through a wide range of current measurements, or to reduce the sensitivity of the galvanometer so that the deflections shall not be beyond the scale, shunts may be placed across the moving coil. These shunts may be so proportioned that the galvanometer current is reduced to $\frac{1}{1000}$, $\frac{1}{100}$ or $\frac{1}{10}$, etc., of the current in the circuit.

Suppose that it is desired to measure the current $I_m$ flowing in

the main line in Fig. 5–17, but that the galvanometer $G$ cannot safely carry all this current. A shunt, $S$, might be placed across the terminals of the galvanometer, which would conduct a certain proportion of the current around the galvanometer. But in order to know how much current is flowing in the main line, it is necessary to know what fraction of the total current the galvanometer carries and what fraction the shunt carries.

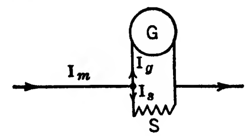


FIG. 5–17. Diagram of a shunted galvanometer.

Let $S$ = resistance of shunt in ohms;
$G$ = " " galvanometer in ohms;
$I_m$ = current in main line;
$I_s$ = " " shunt $S$;
$I_g$ = " " galvanometer $G$.

The circuit through $S$ is in parallel with the circuit through $G$. Then

$$I_m = I_s + I_g. \tag{1}$$

(Current in parallel combination equals the sum of currents in each branch.)

The voltage across the shunt = $I_sS$ $(IR)$.
" " galvanometer = $I_gG$ $(IR)$.

But voltages across parallel circuits are equal.

Therefore $$I_gG = I_sS,$$

or $$I_s = \frac{I_gG}{S}. \tag{2}$$

The relation of the current in galvanometer $(I_g)$ to current in main line $(I_m)$ may be expressed by the fraction $\frac{I_g}{I_m}$.

But from (1)

$$\frac{I_g}{I_m} = \frac{I_g}{I_g + I_s}. \tag{3}$$

Substituting (2) in (3),

$$\frac{I_g}{I_m} = \frac{I_g}{I_g + \frac{I_gG}{S}} = \frac{I_g}{\frac{I_gS + I_gG}{S}} = \frac{I_gS}{I_g(S + G)} = \frac{S}{S + G}.$$

Thus
$$\frac{I_g}{I_m} = \frac{S}{S + G}. \tag{4}$$

The current through the galvanometer holds the same relation to the current in the main line that the resistance of the shunt does to the resistance of shunt plus resistance of galvanometer.

**Example 1.** Assume the resistance of the galvanometer, Fig. 5–17, to be 2500 ohms and the shunt to be 500 ohms. What fraction of the current in the main line goes through the galvanometer?

$$G = 2500,$$
$$S = 500,$$
$$\frac{I_g}{I_m} = \frac{S}{S + G} = \frac{500}{2500 + 500} = \frac{1}{6}.$$

Thus $\frac{1}{6}$ of the current in the main line passes through the galvanometer.

The same equation can be used to compute the amount of resistance with which a galvanometer must be shunted in order that a given fraction of the current in the main line may pass through the galvanometer.

**Example 2.** Suppose it is desired to shunt a galvanometer in above example so that $\frac{1}{10}$ of the current in the main line shall pass through it. What resistance must the shunt be?

$$\frac{I_g}{I_m} = \frac{S}{S + G} = \frac{1}{10};$$
$$\frac{S}{S + 2500} = \frac{1}{10};$$
$$10\,S = S + 2500;$$
$$S = \frac{2500}{9} = 277.8 \text{ ohms}.$$

**Prob. 1–17.** A galvanometer of 4000 ohms resistance has a shunt of 100 ohms resistance. What part of main current flows through galvanometer?

**Prob. 2–17.** It is desired to shunt the galvanometer in Prob. 1–17 so that but $\frac{1}{10}$ the main current shall pass through it. Of what resistance must the shunt be?

**Prob. 3–17.** A galvanometer of 2000 ohms resistance is shunted with a 500-ohm shunt. On a certain circuit, it gives a deflection of 2.4 scale divisions when thus shunted. What deflection would it give on same circuit if unshunted?

**Prob. 4–17.** What resistance shunt would be required to cause a deflection of 1.4 scale divisions of galvanometer in Prob. 3–17 on same circuit?

**3. Ayrton-Mather Universal Shunt.** Figure 6–17 represents a shunt arrangement by means of which several shunt values can be obtained throughout a wide range, depending on the position of the arm $K$. As this arm is swung from right to left, it lowers, by some decimal fraction, the amount of current going through the galvanometer. Since this fraction is always the same, regardless of the resistance of the galvanometer, this arrangement is called a **universal shunt.**

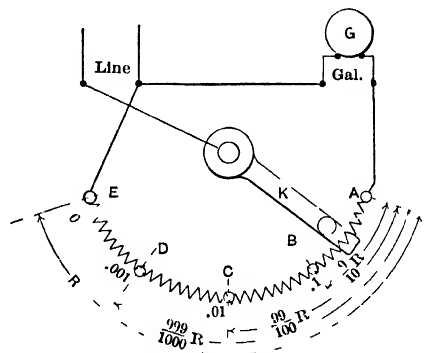


FIG 6–17. The Ayrton-Mather universal shunt

**These fractions do not represent the ratio of the current in galvanometer to the current in the main line.** They merely denote the ratio of the current in galvanometer with arm in given position, to current in galvanometer with arm at $A$.

Thus, when the arm is at $B$, $\frac{1}{10}$ as much current flows through the galvanometer as when it is at $A$, not $\frac{1}{10}$ of the current in the main line. The fraction of the current that flows through the galvanometer when the arm is at $A$ depends upon the resistance of the galvanometer, and is, of course, different in connection with different galvanometers.

But, regardless of the resistance of the galvanometer, when the arm is at $B$, $C$ or $D$, $\frac{1}{10}$, $\frac{1}{100}$ or $\frac{1}{1000}$ as much current flows through the galvanometer as when the arm is at $A$. The term **universal** is therefore a little broad. The great advantage of this shunt is the ease with which the same galvanometer can be used to measure a wide range of currents when equipped with it.

It is necessary, however, to prove that the ratios are true as stated, regardless of the resistance of the galvanometer. Note first that as the arm moves from right to left, it cuts resistance out of the shunt and adds it to the galvanometer circuit.

Let $G$ = resistance of galvanometer circuit,
$R$ = resistance of $EA$.

Then $BA$ is made $\frac{9}{10}R$,
and $CA$ " $\frac{99}{100}R$,
" $DA$ " $\frac{999}{1000}R$.

Let $I_g$ = current in galvanometer,
$I_m$ = current in line,
$S$ = resistance of shunt.

Then $$\frac{I_g}{I_m} = \frac{S}{S+G},$$ as in previous paragraph.

When arm is at $A$,

$$\frac{I_g}{I_m} = \frac{S}{S+G} = \frac{R}{R+G}.$$

When arm is at $B$,

$$\frac{I_g}{I_m} = \frac{S_1}{S_1+G_1}.$$

But $$S_1 = \tfrac{1}{10} R,$$

and $$G_1 = G + \tfrac{9}{10} R.$$

Thus

$$\frac{I_g}{I_m} = \frac{S_1}{S_1+G_1} = \frac{0.1\,R}{0.1\,R + (G + 0.9\,R)} = \frac{1}{10}\left(\frac{R}{R+G}\right).$$

This ratio is $\frac{1}{10}$ of that when the arm is at $A$.

This shunt works especially well on ballistic galvanometers, and is in common use in capacity tests.

**4. Sensitivity of Galvanometers. Working Constant.** As there are several uses to which galvanometers are put, so there are several ways of stating their sensitivity.

If the galvanometer is to measure either current or quantity of electricity, the sensitivity is rated as the fractional part of either an ampere or a coulomb that will produce a deflection of one scale division. Thus, it may be stated of a certain galvanometer that its sensitivity is 0.000004 ampere. By this would be meant that 0.000004 of an ampere would cause a deflection of one scale division. A galvanometer of a sensitivity of 0.0000004 ampere would be 10 times as sensitive as the first instrument, because it would require but $\frac{1}{10}$ as much current to cause the same deflection. The **smaller** the current to produce a deflection of one scale division, the **greater** the sensitivity.

Again, if the galvanometer is to be used to measure insulation or high resistance, as described in Chapter V, the sensitivity may be expressed as the number of megohms which must be connected in series with it across one volt pressure to produce a deflection of one scale division.

Thus, a galvanometer of 8000 megohms sensitivity, means a galvanometer which will give a deflection of one scale division when 8000 megohms resistance are in series with it across one volt. If another instrument has a sensitivity of 16,000 megohms, it would be twice as sensitive as the first. The **greater** the number of megohms through which one volt pressure can produce a deflection of one scale division, the greater the sensitivity.

In **purchasing** a galvanometer, much is learned about its characteristics from a statement of its **sensitivity.**

In **using** a galvanometer, however, it is of greater importance to know the **working constant** of the instrument. The term "working constant" refers merely to a local "set-up" of the instrument and not to standard conditions as in the case of the term "sensitivity." For instance, the sensitivity of a galvanometer may be 1000 megohms, while its working constant for a certain set-up may be 100,000 megohms. That is, the galvanometer may be on a 100-volt line and give a deflection of one scale division when 100,000 megohms are in series with it. Knowing the "working constant," it is possible to compute the resistance that is in series with a galvanometer at any given time, by noting the deflection. Thus if the above galvanometer gave a deflection of 4, it would indicate that only $\frac{100{,}000}{4}$ or 25,000 megohms must be in series with it at that time. For a complete discussion of the method for finding the working constant of a galvanometer, as applied to insulation measurement, see Chapter V.

**5. Ballistic Galvanometer.** A current, which flows for an exceedingly short time only, is often sent through a galvanometer. This is the case with many induced currents and condenser discharges. Since the current in such cases does not flow long enough to be measured, a galvanometer has been devised, the deflections of which are proportional to the **quantity** of electricity, or the **charge,** passing through the coil. These instruments are said to "**throw**" rather than "**deflect,**" and for this reason are called **ballistic galvanometers.** They are made with a heavier moving coil and can be damped magnetically, without destroying the ratio between the "**charge**" and the "**throw.**" Such an instrument is used in capacitance measurements as follows.

A condenser of known capacitance is discharged through the ballistic galvanometer and the "throw" noted. Then a source of unknown capacitance, charged to the same voltage, is discharged

through the galvanometer and the throw noted. The capacitance of the two sources are to each other as their respective galvanometer throws. See Chapter XVI.

**6. Thermoelectric Effects.** Before going further into the structural details of electrical measuring instruments, it is necessary to consider briefly some phenomena which may affect the choice of materials employed.

It has been discovered that if two metals are employed in constructing an electric circuit, and one juncture of the metals is heated to a higher temperature than the other juncture, an electric pressure is set up tending to cause a current to flow. The metals may be welded, soldered, or merely held together by mechanical pressure; the emf depends entirely upon the materials selected and the difference in temperature of the two junctures, varying greatly with the different combinations of materials; and being almost directly proportional to the difference in temperature of the joints. Consider the electric circuit in Fig. 7–17; *BA* is a bismuth bar, joined at *A* to *AC*, a bar of antimony; two metals often chosen because of the high value of the thermal emf per degree difference of temperature of junctures. When heated at point *A*, a current will flow in the direction *BAC*, because that juncture is at a higher temperature than the other ends *B* and *C*. If, however, the points *B* and *C* (really the other juncture) were heated, the current would flow in the opposite direction. Such an arrangement is called a **thermocouple.** The current will continue to flow as long as one juncture is maintained at a higher temperature than the other.

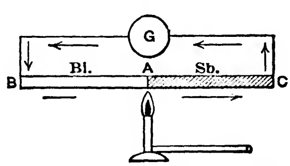



Fig. 7–17. A thermo-couple.

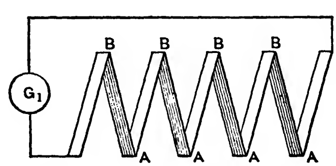



Fig. 8–17. A thermo-pile.

**7. Thermo-Bolometer. Pyrometer.** No successful commercial use has yet been made of this thermal emf for supplying electric power; though enough current can be generated by means of several joints in series, alternate ones of which are heated by a gas flame, to supply electroplating vats. The materials used in such a device in a short time seem to undergo a molecular change in their structure which greatly diminishes their thermoelectric properties.

The thermoelectric emf of a pair of bismuth and antimony bars is only about 0.1 millivolt for every degree centigrade that one juncture is above the other. Thermopiles, therefore, have to be built up of many such pairs in series, as in Fig. 8–17; and one set of junctures must be maintained at a temperature considerably above the other set, in order to procure a usable amount of pressure.

Use is made of this thermal emf in making instruments to measure very minute differences in temperature, and also very high or very low temperatures.

A thermopile, such as that in Fig. 8–17, may be used as a sensitive instrument for detecting small differences of temperature. If the temperature of the joints $A$ is but a small fraction of a degree above or below that of the joints $B$, an emf will be set up which can be detected by the deflections of a sensitive galvanometer, $G_1$. Since the emf set up is nearly proportional to the difference in temperature of the joints, the deflections of the galvanometer can be made to measure this difference in temperature.

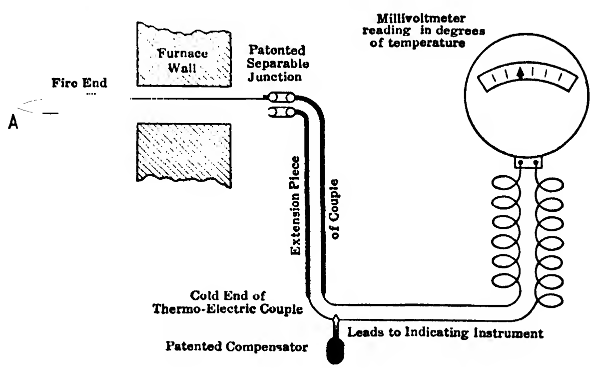



FIG. 9–17. A Bristol pyrometer.

By inserting in the moving coil of a sensitive D'Arsonval galvanometer, a bismuth-antimony thermocouple, as small a rise or fall in temperature as one-hundred-millionth of a degree centigrade can be measured. Such an instrument is called a **bolometer.**

For measuring high temperatures, a thermocouple, of platinum and rhodium is made in such a form that one end, $A$, Fig. 9–17, may be inserted into furnaces of molten metals, etc., and be raised to the temperature of the furnace or metal, while the other end remains at room temperature. The thermal emf thus set up causes a deflection in a millivoltmeter calibrated to read temperature of end $A$ of thermocouple.

The electrical connections of the Bristol Pyrometer, which employs special alloys for the thermocouple, are shown in Fig. 9–17. Note the device for compensating for any change in temperature which may take place at cold end of couple.

**8. Peltier and Thomson Effects.** It has also been discovered that the thermoelectric effect is reversible. That is, if we send a current through a thermocouple, it will either heat or cool the joint, according to the direction of the current. This is called the **Peltier** effect and must be carefully distinguished from the $I^2R$ effect, which never **cools** a wire, nor is it reversible.

Sir William Thomson (Lord Kelvin) discovered that in most conductors composed of pure metals, if one part were raised to a higher temperature than another part, a thermal emf would be set up between these two points. This effect is also reversible.

In consequence of these discoveries of the thermoelectric effects of different metals and combinations of metals, it is seen that too great care cannot be exercised in the selection of proper materials for the construction of accurate electrical measuring instruments. It would not do to have within the instrument itself a source of emf which would affect its indications, if the temperature of parts of it were changed from any cause.

**9. Requirements for a Satisfactory Instrument.** A satisfactory instrument to measure either current or voltage (or both) should meet the following requirements:

(*a*) Its indications should not be affected by friction.

(*b*) Its indications should not be affected by stray magnetic fields.

(*c*) It should require little energy to operate it.

(*d*) It should respond quickly to changes in quantities under measurement in order that readings may be rapidly made, and, therefore, should be "dead-beat."

(*e*) It often must be portable, and, therefore, should be ruggedly built.

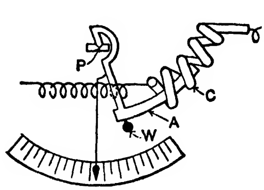


FIG. 10–17. An old type iron-plunger ammeter.

**10. Moving Iron Plunger Instruments.** Early instruments were of a crude and cumbersome **moving iron plunger type,** shown in Fig. 10–17.

In ammeters, coil $C$ was of comparatively few turns of heavy low-resistance wire; while in voltmeters, the coil consisted of a large number of turns of fine wire, connected in series, with a comparatively high resistance.

The current flowing through the coil $C$ sucks the soft-iron plunger $A$, pivoted at $P$, up into the coil. This causes the pointer to move over a scale, which is calibrated by sending a known current through the coil, if an ammeter; or by putting a known voltage across the coil, if it is a voltmeter. The deflections are nearly proportional to the square of the current in the coil; since the deflecting force depends upon the strength of the field, multiplied by the strength of the induced magnetism; both of which are nearly proportional to the current in the coil. The control is effected by means of the weight, $W$, called a "gravity control"; damping is by means of eddy currents set up in the plunger, $A$. Neither the damping nor the control was good, and the instrument needle fluctuated on rapidly changing voltage, or current, as the case might be. Due to the comparatively large mass of the moving parts, the friction on the pivot and the hysteresis of the iron plunger, this instrument was sluggish and inaccurate.

Modern adaptations of the old moving iron plunger instrument are the **magnetic vane and inclined coil** types. These instruments operate fairly satisfactorily on both d-c and a-c circuits, although they must be calibrated separately for each class of service. These instruments are influenced by stray magnetic fields and have not a high degree of accuracy.

The Weston-type instruments, described later, are much superior, and are considered standard for use as voltmeters and ammeters for d-c measurement.

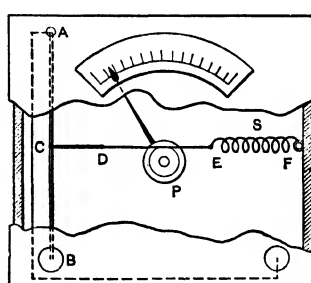


FIG. 11–17. A hot-wire meter.

**11. Hot-Wire Ammeters.** When an electric current passes through a wire, heat is generated. If this heat is allowed to raise the temperature of the wire, expansion will take place. Hot-wire ammeters make use of this expansion of wire, by the heat generated, to measure the electric current which generates the heat.

When a current is sent through wire $AB$, Fig. 11–17, the heat causes it to sag. This sag is taken up in the arrangement $CF$. The part $CD$ is a wire; $DE$ is a silk thread wound around the pulley $P$; $EF$ is a spring. As the thread moves around the pulley, it causes it to turn, and the pointer attached moves over the scale. The deflections here are almost proportional to the square of the

current, since the heating effect is proportional to the current squared; so the scale is not uniform and must be calibrated. The control is effected by means of the elasticity of the wire $AB$ and spring $S$. The instrument, however, requires frequent resetting to zero, is very slow in action and, unless special attachments are used, is affected by changes in temperature of the room. It measures both alternating and direct currents, and can be used with shunts to increase its range. It can also be used as a voltmeter by placing a high resistance in series with the hot wire. Instruments of this type have come into use again in the measurement of alternating currents of the high frequencies used in wireless telegraphy and telephony.

**12. Permanent-Magnet, Weston-Type Instruments.** For measuring direct current and voltage, the permanent-magnet or Weston-type instrument, is in universal use. It operates on the principle of the D'Arsonval galvanometer; but is designed so that it is portable, and meets all the requirements of Art. 9. The fundamental principles of this instrument have already been discussed in Chapter VII.

The moving coil, instead of being supported by a delicate suspension wire as in a galvanometer, is fastened to hardened steel pivots, which are as accurately fitted into jewelled bearings as the balance wheel of a watch. The control is effected by means of two spiral springs which are wound in opposite directions and act against each other. They also serve as lead-in wires to the moving coil. A pointer is attached to this coil, and travels over a calibrated scale with the movement of the coil. Figure 12-17 is an interior view, showing the essential parts of the instrument.

The scale divisions of the Weston-type instruments are uniform throughout, which is a decided advantage. This is brought about by producing an absolutely uniform radial field in which the coil swings. Soft-iron pole pieces are fitted to the poles of a horseshoe-shaped permanent magnet. A soft-iron core, encircled by the moving coil, is held between these pole pieces by a brass strip. This produces a radial field (at right angles to the arc in which the coil moves), and a very short air gap of uniform length. In whatever position the coil may be, there is always acting upon it the same number of lines at right angles to its motion. The deflections are, therefore, exactly proportional to the current flowing in the coil.

The moving coil is made of fine silk-covered copper wire wound

on a very light aluminum bobbin, which supports not only the coil, but also the hardened steel pivots, the spiral springs and the pointer, shown in Fig. 13–17.

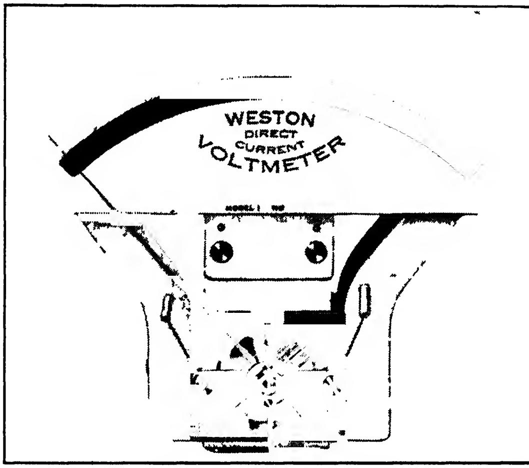



Fig. 12–17. An interior view of a Weston meter showing details of construction.

Fig. 13–17. The moving coil assembly of a Weston meter.

The damping of the instrument, which makes it " dead-beat," is obtained by the currents induced in the closed path through the aluminum bobbin, as it moves through the magnetic field.

The pointer, which is tubular in form and very light, is balanced by an adjustable counterweight, also shown in Fig. 13–17. This reduces the friction of the supporting pivots on the jewelled bearings.

It is readily seen that the instrument is essentially a D'Arsonval galvanometer. In fact, the portable galvanometer of Fig. 4–17 is practically of the same construction, except that it employs a smaller permanent magnet and the sensitivity is increased by the use of very weak spiral springs.

In the Weston-type instrument, then, the moving element may be used either as an ammeter or as a voltmeter, depending upon whether the resistance of the coil is high or low. For use as an ammeter, the moving coil is provided with a shunt which carries by far the larger portion of the current. For use as a voltmeter, the coil has a resistance connected in series with it, so that it will take but a very small current when put across the line.

**13. Ammeters.** The Weston-type instrument actually is a millivoltmeter, and a few thousandths of a volt across the moving coil will cause a maximum deflection of the pointer. In most portable instruments, approximately 50 millivolts, or 0.05 volt, across the coil will give a full-scale deflection. Furthermore, the fine wire of which the coil is wound will carry only from 0.01 to 0.05 ampere.

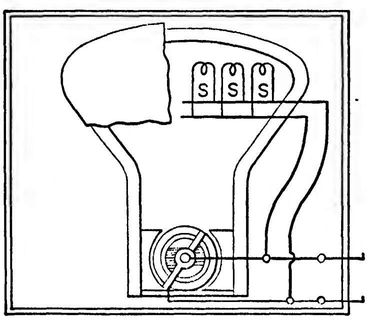


FIG. 14–17. An ammeter with an internal shunt, S, S.

When the moving element is designed for use as an ammeter, the coil is generally wound of fewer turns of larger wire and is, therefore, of low resistance. Thus the voltage drop across the instrument will be low, and it will not add appreciable resistance to the circuit. The current in the moving coil of an ammeter is thus somewhat greater than if the coil is designed for use as a voltmeter, but is never more than about 0.05 ampere.

Therefore, an ammeter designed to measure even moderately large currents must be supplied with a shunt across the moving coil, as shown in Fig. 14–17, in order that most of the current may be diverted. The ammeter, thus, is just a divided circuit consisting of a millivoltmeter and a shunt; and the millivoltmeter measures the drop over the shunt. Assume that a certain ammeter requires a drop of about 0.045 volt across the instrument for a full-scale deflection of the pointer, when the resistance of the shunt is 0.0094 ohm, and that of the moving coil is 2.25 ohms. According to

Ohm's law, the current in the shunt is $\frac{0.045}{0.0094}$ or 4.78 amperes, while that in the coil is $\frac{0.045}{2.25}$ or 0.02 ampere.

Of course the instrument is calibrated so that the scale reads, not millivolts, but the amperes flowing through the instrument, both moving coil and shunt.

If the resistances of the two circuits remain the same, the current in the moving coil and the scale reading vary in direct proportion to the voltage drop across the shunt, which in turn is directly proportional to the current in the shunt. In other words, the

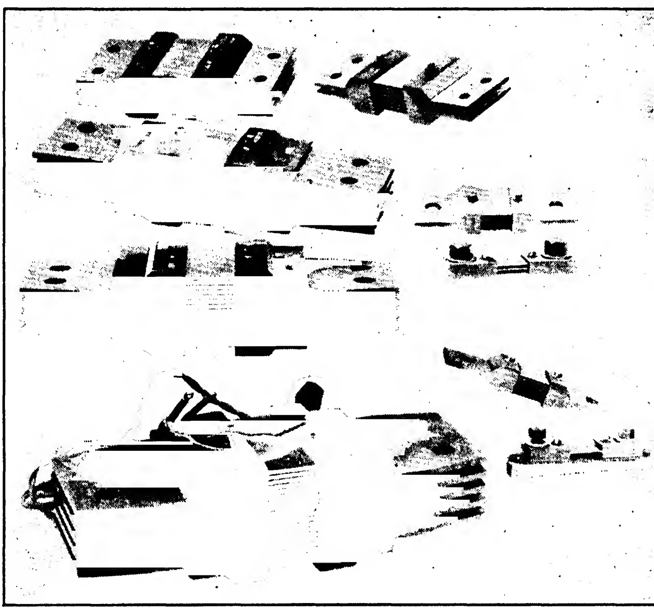


FIG. 15-17. Shunts for use with millivoltmeters. *Weston Electrical Instrument Co.*

ratio of the currents in the coil and in the shunt must remain the same, to insure accuracy of the instrument. It is, therefore, important that the shunt be made of a material that has low temperature coefficient of resistance. It must also have low thermoelectric properties, when soldered to copper. Such a substance is manganin, invented by Dr. Edward Weston. It is an alloy consisting of copper, nickel and ferromanganese.

Shunts are low-resistance strips soldered to heavy copper terminals, as shown in Fig. 15–17. The copper terminals help to dissipate the heat from the manganin strips, since a shunt operates at higher temperature than the coil. An instrument can be adjusted to its shunt by varying a low adjustable resistance, connected inside the instrument in series with the moving coil.

In the smaller ranges, up to 50 amperes, where the instrument has only one scale, the shunts are generally placed inside the instrument, as in Fig. 14–17. From 50 to 100 amperes, the shunts may be either external or internal. Above 100 amperes, external shunts are generally used. In the latter case, the instrument itself is generally marked as a "millivoltmeter," and is equipped with small rubber-covered binding posts. The instrument, with special connecting leads, is calibrated to be used with a particular shunt, as indicated in Fig. 16–17. These leads should always be used with the instrument. An ammeter with its calibrated leads may be designed to have a number of widely different ranges, by equipping it with several external shunts of different resistances.

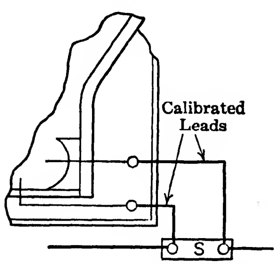


FIG. 16–17. An ammeter with external shunt, S.

**Example 3.** What resistance must a shunt $S$ have in order that it may be used with a 0.045-volt millivoltmeter to measure 100 amperes? Resistance of moving-coil circuit and connecting leads is 1.00 ohm.

**Solution.**

$$\text{Maximum current through coil} = \frac{0.045}{1.00} = 0.045 \text{ ampere.}$$

Maximum current through shunt = 100 − 0.045 = 99.955 amperes. Resistance of shunt must be, by Ohm's law,

$$\frac{0.045}{99.955} = 0.0004502 \text{ ohm.}$$

Note here, that the current in the moving coil is so small that it can be neglected, and the line current assumed to be the current in the shunt, thus:

$$\frac{\text{current in coil}}{\text{line current}} = \frac{\text{resistance of shunt}}{\text{resistance of coil}} \quad \text{or} \quad \frac{0.045}{100} = \frac{x}{1}.$$

$$x = \text{resistance of shunt} = \frac{0.045}{100} = 0.00045 \text{ ohm.}$$

**Example 4.** What resistance must a shunt have in order that it may be used with the millivoltmeter in Example 3 to measure 1000 amperes?

**Solution.**

$$\text{Current through coil} = \frac{0.045}{1.00} = 0.045 \text{ ampere.}$$

Current through shunt = 1000 amperes.

$$\textbf{Resistance of shunt} = \frac{0.045}{1000} \text{ or } 0.000045 \text{ ohm.}$$

**Prob. 5-17.** What resistance must a shunt have in order that it may be used with a millivoltmeter of 50 millivolts range to measure 60 amperes? Resistance of moving coil and leads is 2.05 ohms.

**Prob. 6-17.** If leads were used which increased the "coil-and-lead" resistance to 2.15 ohms, using the same shunt, what error would be introduced at full-scale reading in Prob. 5-17?

**Prob. 7-17.** Assume that a copper wire, annealed, is used for the shunt in Prob. 5-17, and its temperature rises 15° C during an observation; what error is introduced at full-scale reading?

**Prob. 8-17.** What would the true reading be when the instrument in Prob. 7-17 reads 40 amperes? Assume same temperature rise.

**14. Voltmeters. Multipliers.** It was stated in Art. 12 that the moving element of the Weston-type instrument could be used either as an ammeter or as a voltmeter. Thus the millivoltmeter described above can be used as a voltmeter by placing a high resistance in series with the moving coil.

Since a voltmeter is placed directly across a line to measure voltage, it is desirable that it take as little current as possible. Therefore, the moving coil of a voltmeter is generally wound with more turns of finer wire, having a higher resistance than that of the ammeter. It then requires less current (to develop the same torque) for a full-scale deflection. However, the resistance of the moving coil, with more turns, is actually comparatively low. Therefore, in any voltmeter, it is necessary that a high resistance be connected in series with the moving coil. This additional resistance is placed inside the instrument case. Many Weston-type voltmeters require approximately 0.01 ampere for a full-scale deflection, which is about 100 ohms per volt (maximum scale reading).

**Example 5.** The resistance of the moving coil of an instrument which is to be used as a 150-volt voltmeter is 4 ohms, Fig. 17-17. If the instrument is to take 0.01 ampere for a full-scale deflection, what resistance must be connected in series with the coil?

**Solution.** Total resistance of the instrument $= \dfrac{150}{0.01} = 15{,}000$ ohms between terminals $a$ and $b$, Fig. 17–17.

Resistance connected in series with moving coil $= 15{,}000 - 4 = 14{,}996$ ohms.

It is often desirable that the same instrument also give a full-scale deflection at a lower voltage; that is, be equipped with one or more additional scales, as in the instrument of Fig. 18–17, which has three scales marked for 150, 15 and 3 volts, respectively.

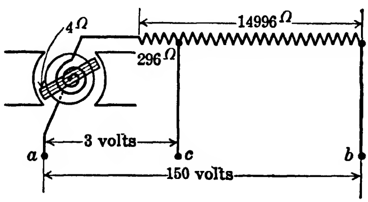



FIG. 17–17. One method of bringing out taps from the series resistance in a double scale voltmeter.

Thus, if the instrument in Example 5 is also to give a full-scale deflection, when connected across 3 volts, the resistance between terminals $a$ and $c$, Fig. 17–17, must be, $R = \dfrac{3}{0.01}$ or 300 ohms; and the value of the resistance connected in series with the moving coil equals $300 - 4$ or 296 ohms.

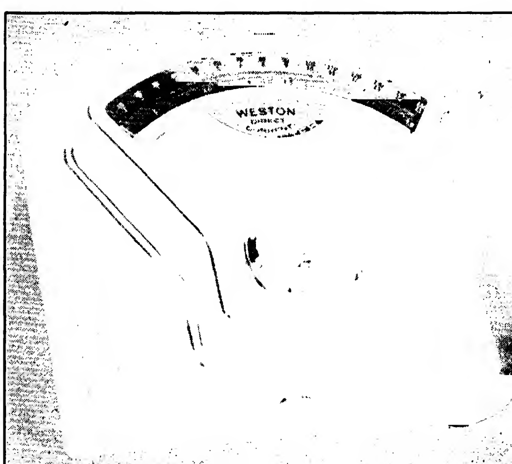



FIG. 18–17. Weston voltmeter equipped with three scales. This instrument gives a full scale deflection on circuits of 3 volts, 15 volts or 150 volts, depending upon which pair of binding posts is used.

The range of a voltmeter, having its series resistance enclosed within the case, can be increased by means of an **additional external**

resistance, called a "multiplier," connected in series with the instrument.

**Example 6.** It is desired to increase the range of the 150-volt voltmeter of Example 5 to (*a*) 300 volts; (*b*) 750 volts. The instrument takes 0.01 ampere for full-scale deflection and has a resistance of 15,000 ohms. What external resistance must be connected in series with the instrument?

**Solution.** (*a*) Necessary resistance to obtain 0.01 ampere when 300 volts is connected across the instrument $= \dfrac{300}{0.01} = 30{,}000$ ohms.

External resistance to be added $= 30{,}000 - 15{,}000$ or 15,000 ohms.

(*b*) Necessary resistance when 750 volts is connected across the instrument $= \dfrac{750}{0.01} = 75{,}000$ ohms.

External resistance to be added $= 75{,}000 - 15{,}000 = 60{,}000$ ohms.

Note, from Example 6, that when 15,000 ohms is added to the resistance of the instrument circuit, 300 volts would give a full-scale deflection of the pointer which would indicate 150 volts on the scale. The indications of the instrument with this additional 15,000 ohms in circuit, now, must be multiplied by 2. So this external resistance is called a **multiplier.** With the 60,000 ohms in series with the instrument, 750 volts would give a full-scale deflection; or the pointer would indicate 150 volts or $\frac{1}{5}$ of 750 volts. Therefore, the indications of the instrument must be multiplied by 5 to obtain correct values.

Multiplier resistances are generally enclosed in perforated metal boxes, and the terminals brought out to binding posts on which is marked the multiplying constant of the connection.

**Prob. 9–17.** The moving coil of a Weston millivoltmeter of 50 millivolts capacity has 5 ohms resistance. What additional resistance must be connected in series with it, in order that it may have a capacity of 600 volts?

**Prob. 10–17.** A voltmeter has a range of 125 volts and a resistance of 14,000 ohms. What resistance must a multiplier be in order that the voltmeter may have a range of 300 volts?

**Prob. 11–17.** A 3-volt voltmeter has a resistance of 300 ohms, and the range of the instrument is to be increased by a multiplying resistance. Suitable taps from the resistance are to be brought out to binding posts, so that the instrument will have ranges of 15, 150, 300 and 650 volts. What will be the resistance between each pair of binding posts? Show diagram.

**Prob. 12–17.** Assume the multiplier in Prob. 10–17 to be made of pure copper. What error is introduced by a rise in temperature of 10° C, if instrument is reading 250 volts? The instrument reads correctly at 20° C.

**Prob. 13–17.** If multiplier in Prob. 10–17 were made of German silver, what error is introduced by a rise of 10° C, when instrument reads 250 volts? Temperature coefficient of resistance, for German silver, at 20° C is 0.00036.

**15. Indicating Wattmeter. Electrodynamometer.** An electrodynamometer instrument consists primarily of two coils: one stationary, the other pivoted in the field set up by the stationary coil. The deflecting force is directly proportional to the field set-up, and therefore to the current in the fixed coil (since there is no iron in the magnetic circuit). It is also directly proportional to the current in the pivoted coil. The total deflecting force in the instrument is thus proportional to the product of the current in the fixed coil, multiplied by that in the pivoted coil.

For use as an ammeter or voltmeter, there are two fixed coils and one pivoted coil. The three coils would be connected in series and carry the same current. The deflection, therefore, would be proportional to the square of the current. As a voltmeter, the coils are of high resistance. Since they are connected across the circuit, the current is proportional to the voltage and, therefore, the deflections are proportional to the square of the voltage.

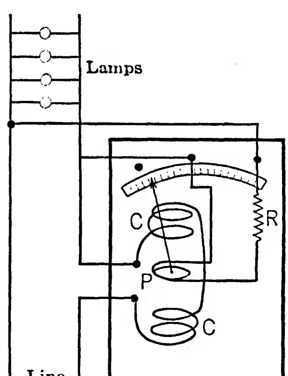


FIG. 19–17. A wattmeter with no compensating coil.

The dynamometer principle is applied in the construction and operation of the wattmeter. The **wattmeter** is an instrument which measures **power directly.** It consists of two fixed coils, $CC$, Fig. 19–17, which are wound of a few turns of wire capable of carrying the total current of the circuit. These coils take the place of the permanent magnet of the D'Arsonval type instrument.

The pivoted coil, $P$, consisting of a few turns of fine wire mounted on an aluminum bobbin, is held in position by spiral springs, which also act as lead-in wires, as in the Weston-type voltmeters. A high resistance is connected in series with this coil, which is connected across the circuit as a voltmeter.

Thus the current in the fixed coils, *CC*, is proportional to the current taken by the lamps, or load, Fig. 19–17; and that in the pivoted coil is proportional to the voltage across the lamps. Since the deflecting force is proportional to the product of the two currents, or to the product of the volts and amperes of the circuit, the deflection of the instrument is, therefore, proportional to the power taken by the lamps in the load circuit.

The wattmeter measures power in both direct-current and alternating-current circuits. However, it is seldom used in direct-current circuits, because of the accuracy of power measurement by the voltmeter-ammeter method. Furthermore, it is affected by stray magnetic fields and must, in certain cases, be equipped with a magnetic shield. It is universally used for power measurement in alternating-current circuits.

**16. Compensation in a Wattmeter.** By referring to the diagram of Fig. 19–17, it will be seen that the wattmeter will read too high when connected as shown, because the current flowing in the stationary coils is not only that through the lamps, but also the current taken by the moving coil. Thus, the current is too large. A correction can easily be made for this, by subtracting from the reading the power consumed by the moving coil. This value should be found as follows:

If $R_m$ be the resistance of moving coil and $E$ the voltage across the lamps, and therefore, also across the coil, the power consumed by the coil would be $\dfrac{E^2}{R_m}$.

If, however, we should connect the voltage coil of the wattmeter across both the lamp and the current coils, then the current flowing through the stationary coils would be that of the lamps only. But the voltage across the moving coil would be the voltage across both the lamp and the stationary coils. The wattmeter would still read too high, because this voltage is higher than the lamp voltage. The power measured would be the power consumed by the lamps plus the power consumed by the stationary coils. The correction can be made by subtracting from the reading the power consumed by the stationary coils. This correction is found as follows:

Let $I$ be current through stationary coils, and $R_s$ be resistance of stationary coils. Then power consumed by stationary coils = $I^2R_s$.

In some wattmeters, neither of these corrections has to be made,

because of a special compensating device shown in Fig. 20–17. The voltmeter, or moving-coil, terminals are connected as in Fig. 19–17, but a compensating coil, $M$, is placed in series with the moving coil. The field of this coil opposes the field of the stationary coils, $CC$, and weakens it by an amount proportional to the current in the moving coil. The field then is exactly proportional to the current flowing through the lamps alone, and the instrument now indicates the watts consumed by the lamps alone.

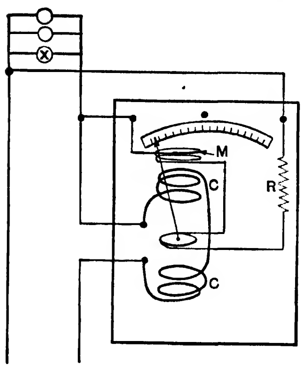


FIG. 20–17. A wattmeter with compensating coil.

In some cases, it is desirable not to use this compensating coil, but to apply corrections computed as above. For this reason, some Weston wattmeters have an extra binding post, marked "Ind" (for Independent), which cuts out this coil. In almost all commercial tests, however, the regular connections are used, which include the compensating coil.

**17. Thomson Integrating Wattmeter or Watthour Meter.** The watthour meter is an instrument for measuring **electrical energy.** It has been stated in Chapter III that electrical **energy** is paid for by the kilowatthour, one kilowatthour being the quantity of electrical energy used when power is consumed for one hour at the rate of one kilowatt. It is therefore important, both from the consumer's standpoint and from that of the operating company, that such an instrument be accurate.

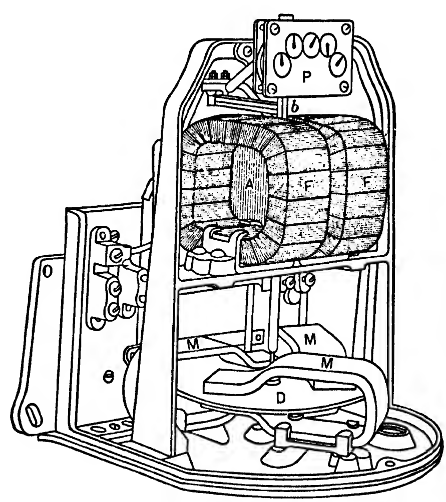


FIG. 21–17. A Thomson watthour meter.

The Thomson meter, shown in Fig. 21–17, is primarily a small

motor, the armature of which revolves at a speed proportional to the rate at which electrical energy is passing through it. The armature is geared to recording dials, which register the number of kilowatthours of energy which have passed through the meter.

The line is connected to the left-hand terminals, *PP*, Fig. 22–17. The upper terminal is connected to the two stationary coils, *FF*, in series, wound with wire sufficiently heavy to carry the maximum load current, which should never be much greater than the rated current of the meter. The coils, *FF*, are also connected to the upper load terminal *L* on the right-hand side of the meter. The other side of the circuit, or line, runs directly through the meter and connects the two lower terminals, *P* and *L*. The magnetic

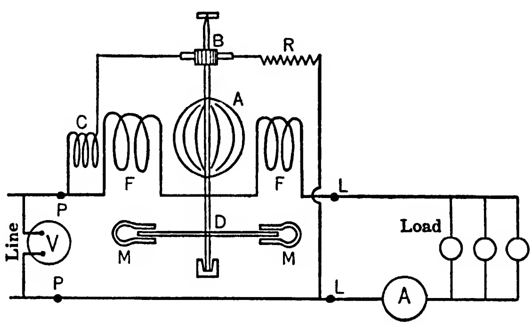



Fig. 22–17. Circuits and connections of a Thomson watthour meter.

field of the instrument is supplied by the current in the coils, *FF*. There being no iron in this field, the field strength varies directly with the current flowing in the main line, or the load.

A shunt circuit is tapped from the upper terminal *P* and connected, by means of the commutator *B*, through the armature *A*, the coil *C* and the resistance *R*, to the other side of the line. Friction is reduced to a minimum by mounting the armature shaft in jewelled bearings.

Since there is no iron in the magnetic circuit, the field due to the coils, *FF*, is comparatively weak, and no appreciable counter emf is set up by the rotation of the armature. Therefore, the current in the shunt circuit of the armature is proportional to the voltage across the line.

The torque, then, must be proportional to the product of the voltage multiplied by the current, or the watts in the line.

Under the above conditions, the armature would race if there were no retarding force applied to the armature. This retarding

force must be proportional to the speed, so that it will decrease as the speed decreases. Such a force is secured by attaching an aluminum disk, *D*, Figs. 21–17 and 22–17, to the armature shaft, and causing it to rotate between the poles of permanent magnets, *M*, as the armature rotates. This motion sets up eddy currents in the aluminum disk, which are proportional to the speed of the disk. The retarding torque of these currents must then be proportional to the speed of the disk, as explained in Chapter XI.

Thus the speed of the armature will at all times be proportional to the torque. To illustrate this, suppose that we assume the driving torque to be increased suddenly; the speed would naturally rise, but as the speed rises, the opposing torque, due to increased eddy currents, also rises, until it is equal to the driving torque; and the armature speed becomes constant. The driving torque and the retarding torque are thus always equal.

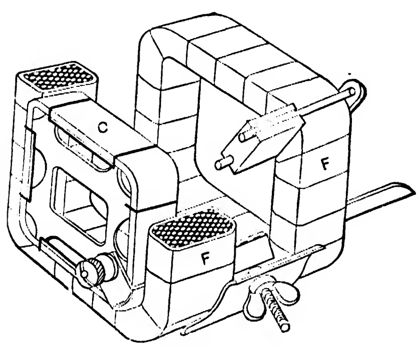



Fig. 23–17. The coil C compensates for the friction of the meter.

The above discussion is correct only when we consider the armature to rotate without any friction. Since there is always some friction present, some arrangement must be made to compensate for it. This is done by means of the small coil, *C*, Fig. 22–17, shown in more detail in Fig. 23–17. This coil is connected in series with the armature and placed so as to strengthen the field of the coils, *FF*. The position of this coil is adjustable, so that it can be moved toward, or away from, the armature as the need may be. It is placed at such a distance that the field due to it is not strong enough to cause the armature to rotate, when no current is flowing through the main field coils, *FF*; but still the lightest current through these coils will be sufficient to start the motor. This "compensating coil," then, just overcomes the friction of the armature.

If the compensating winding is too near the main coils, and the motor runs when no current is being used, it is said to "creep." This causes the dials to register more energy than was used by the consumer.

The same type instrument is used in an Edison three-wire system, by connecting one of the stationary coils, *FF*, in each outside line. The armature, or shunt circuit, may be connected across the two outside lines, or between one outside and the neutral. In the latter case, in order that the torque may be the same, the resistance of the armature circuit is reduced one-half, so that the current in this circuit will be the same on half voltage.

The question is often asked: If this meter is a shunt motor, why is it that the armature speed **increases** as the field **increases,** when it is a well-known fact that the armature speed of a shunt motor **decreases** as the field **increases?** The answer to this question is simple, when we consider the reason why a shunt motor decreases in speed as the field strength increases. The increase in the field strength of a shunt motor is **not** the direct cause of the decrease in speed. The large decrease in the **armature current,** due to the increased counter emf, is the reason for this falling off in speed. In the case of the watthour meter, there is no counter emf of any appreciable value, as compared with the impressed emf. Thus any increase in the counter emf does not appreciably decrease the **armature current.** An increase in the field strength, therefore, accompanied by no decrease in the armature current, causes a larger torque, and, therefore, increased speed.

**18. Adjustment of the Thomson Watthour Meter.** To check the meter for accuracy, it is loaded and the power supplied to the load is measured by a calibrated voltmeter and ammeter as in Fig. 22–17. The number of revolutions of the disk, *D*, are counted over a period of time — generally in seconds.

Now for most meters as actually constructed,

$$\text{watts} \times \text{hours} = K \times \text{number of revolutions of disk}, \quad (5)$$

where $K$ is generally the number of watthours registered on the dial per revolution of the disk. This is called the " constant " of the meter, and it is usually marked on the aluminum disk.

If the disk makes $N$ revolutions in $t$ seconds, Equation (5) becomes

$$\text{watts} \times \frac{t}{3600} = K \times N; \quad (6)$$

where $\frac{t}{3600}$ = fractional part of an hour (3600 = number of seconds in an hour).

Then the **average watts** registered by the watthour meter during the time $t$ (which should be about 1 minute) are

$$\text{watts} = K \times N \times \frac{3600}{t}. \quad (7)$$

The actual load on the watthour meter in watts is calculated from the average of the voltmeter and ammeter readings, which should be taken at intervals during the test.

The ratio of the **registered** watts to the actual watts × 100 gives the percentage of **accuracy** of the meter. The meter should check with actual load to about 1 per cent.

If the meter gives too high an indication at, or near, full load, the armature is running fast, and the magnets *M* are moved farther from the center of the disk, increasing the retarding action. If the indication is low, the meter is running slow and the retarding action is decreased by moving the magnets nearer the center of the disk.

The meter should now be checked at light load (5 to 10 per cent), in the manner described above, to within 1 per cent of correct value; and coil *C* adjusted for "creeping." Then a second check should be made at, or near, full load to be certain the light-load adjustment has not affected the indications at full load.

**19. The Potentiometer.** The instrument most used for making very precise measurements of voltage is the potentiometer. The

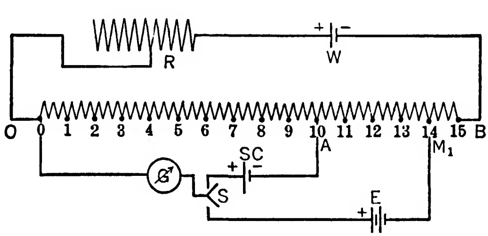



FIG. 24–17. Illustration of the potentiometer principle.

potentiometer principle consists of balancing an unknown voltage, or emf, against a known potential difference, or voltage drop. The known voltage is checked against some standardized value of emf, such as that of the Weston Standard Cell. See Chapter XIV. The principle is illustrated as follows:

In Fig. 24–17, *OB* is a conductor of uniform resistance per unit length, stretched along a scale graduated in uniform divisions. This is called the **measuring circuit.** The storage battery, *W*, supplies a steady current to this conductor, the positive terminal of the battery being connected to *O* through the adjustable resistance *R*. When current flows in the conductor from *O* to *B*, the point *O* is at a higher potential than *B*. The potential difference

between $O$ and $B$ is directly proportional to the resistance of the conductor. The resistance being uniform, the fall in potential is uniform per unit length.

Assume the conductor $OB$ is divided into 15 equal 10-ohm sections, for example, and that the resistance $R$ is adjusted until just 0.01 ampere flows in $OB$. Under this condition, the voltage drop in each section will be $0.01 \times 10$ or 0.1 volt, or the total potential difference between $O$ and $B$ will be 1.5 volts.

The current in the circuit $OB$ (in this case 0.01 ampere) is called the **working current.**

A Weston standard cell, $SC$, is also connected to the measuring circuit, $OB$, so that its emf is in opposition to that of the storage battery, $W$. The positive terminal of $SC$ is connected to $O$ through a galvanometer and the switch $S$; while the negative terminal is connected to some other point, such as $A$, at the end of the tenth 10-ohm section on $OB$ in the figure.

If the emf of the standard cell $SC$ be assumed to be exactly 1 volt, its emf will be just equal to the potential difference between $O$ and $A$, and when the switch $S$ is thrown **up,** the galvanometer will indicate that no current is flowing in the standard-cell circuit; or that its emf just balances the voltage drop from $O$ to $A$.

If the working current in $OB$ be decreased below 0.01 ampere, the potential difference from $O$ to $A$ will be **less** than the emf of the standard cell, and current will flow out of the standard cell $SC$. If, on the other hand, the working current be increased above 0.01 ampere, the potential difference from $O$ to $A$ will be **greater** than the emf of the standard cell; a current will flow the other way in $SC$.

Thus, the working current can be adjusted to give a **known potential difference** in each section of the measuring circuit $OB$, which is determined when the drop from $O$ to $A$ is just equal to the emf of the standard cell. This is called **balancing the potentiometer.** Also, with the proper working current (0.01 ampere here), each 10-ohm section is equivalent to 0.1 volt drop along $OB$.

Now, assume that it is desired to check an unknown emf whose value is known to be not more than 1.5 volts, as $E$ in Fig. 24–17. The positive terminal of the unknown emf, $E$, is connected through the galvanometer and the switch $S$ to $O$, and the negative terminal to the movable contact $M_1$. With the switch $S$ thrown **down,** the position of contact point $M_1$ is adjusted till the galvanometer reads zero. Since the potentiometer has been balanced, the drop

in each section of *OB* is 0.1 volt. If the galvanometer in series with the unknown emf, *E*, shows a balance, or reads zero, with the contact $M_1$ on point 14 of *OB*, for instance, the potential difference between *O* and $M_1$ is $0.1 \times 14$ or 1.4 volts. Thus the value of the unknown emf is just 1.4 volts.

Instead of numbering the contact points on *OB*, as in Fig. 24–17, they are usually marked on a scale in terms of the potential difference in volts from point *O*. In this case, the contact points would be numbered in sequence from point *O* as 0.1, 0.2, 0.3 volt, etc.; and the value of the unknown emf can be read directly from the instrument.

It will be noted that voltage or emf values are checked when no current flows in the galvanometer. This is called a **null** method of measurement, and is very accurate.

**20. Leeds and Northrup Type K Potentiometer.** A diagram of the electrical circuits in the Leeds and Northrup Type K Potentiometer are shown in Fig. 25–17. These circuits are very similar to those of Fig. 24–17. The measuring or potentiometer circuit, corresponding to *OB*, in Fig. 24–17, consists of three sections, *OA*, *AD* and *DB*, marked, respectively, *C*, *F* and *P*, in Fig. 25–17.

Section *C* is a slide-wire dial to which one terminal of the standard cell is attached at *T*; *F* is a dial section made up of fifteen 5-ohm resistors in series, brought out to 15 contact points; while *P* is a circular slide-wire section of 5.5 ohms resistance.

The battery, *W*, is connected to this measuring circuit through the regulating resistance *R*. This is made up of two small dial resistors $R_1$ and $R_2$ for fine adjustment, in series with resistors 1 *r*, 2 *r* and 4 *r*, which may be short-circuited by plugs.

The greater part of the measuring circuit consists of the dial resistance *F*. The working current must be adjusted until the drop in each of the 15 sections of this dial is 0.1 volt. Since the resistance corresponding to 0.1-volt drop is 5 ohms, the working current must be $\frac{0.1}{5}$ or 0.02 ampere.

This makes $5.5 \times 0.02$ or 0.11-volt drop in slide-wire *P* when the instrument is balanced. This slide-wire consists of 11 turns of resistance wire mounted on a vertical bakelite cylinder. Each turn represents 0.01 volt and the entire length of wire is divided into 1100 divisions by means of a vertical scale.

We have seen that one terminal of the standard cell is connected to the contactor *T* of the dial *C*. The other terminal is connected,

through the galvanometer, to the 0.5-volt contact point of dial *F*. The potential drop from the 1.5-volt contact to the 0.5-volt contact is 1 volt. The actual voltage of a Weston standard cell is not 1 volt as assumed in Art. 19, but is about 1.0186 volts (see Chapter XIV). The emf of all Weston standard cells is not the same, but it is always greater than 1 volt. To obtain the extra potential drop, the resistance in dial *C* is added; so that *T* can be set at a point that will produce a difference of potential between

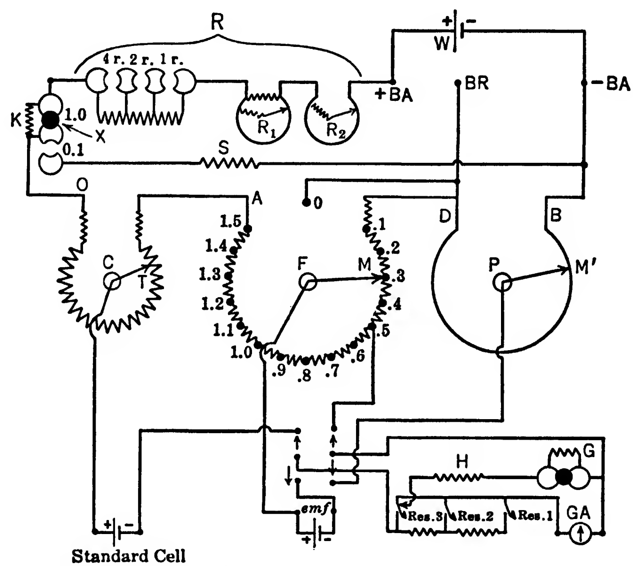



FIG. 25–17. Circuit diagram of the Leeds and Northrup Type K potentiometer.

point *T* and the 0.5-volt point on dial *F*, exactly equal to the emf of the particular standard cell being used. A scale of volts is marked on the dial *C* and the resistance of the slide-wire is such that contact *T* can be set at the proper point for any Weston standard cell.

*M* and *M'* are movable contacts which can be adjusted to balance an unknown emf. Since the maximum voltage drop in dial *F* is 1.5 volts, and that in slide-wire *P* is 0.11 volts, the maximum possible difference in potential between *M* and *M'* (with the normal working current of 0.02 ampere) is 1.61 volts. This is the maximum voltage any potentiometer is designed to measure. For higher voltages, a " volt-box " is used, which will be described later.

In measuring an unknown emf, after the instrument is balanced, the double-pole, double-throw switch is thrown down. This connects the galvanometer and the unknown emf in series between the points $M$ and $M'$. The position of contacts $M$ and $M'$ are adjusted until the galvanometer reads zero; the value of the unknown emf is read directly from the scales or dials $F$ and $P$.

There are two resistances in the galvanometer circuit to protect the galvanometer against excessive deflections when the opposing electromotive forces are not closely balanced. When the key marked **Res. 1** is closed, the galvanometer circuit is closed through a high resistance; key **Res. 2** closes the circuit through a lower resistance; and key **Res. 3** closes the galvanometer circuit with no series protective resistance. The resistors $G$ and $H$ are for damping the galvanometer, when **Res. 3** key is opened.

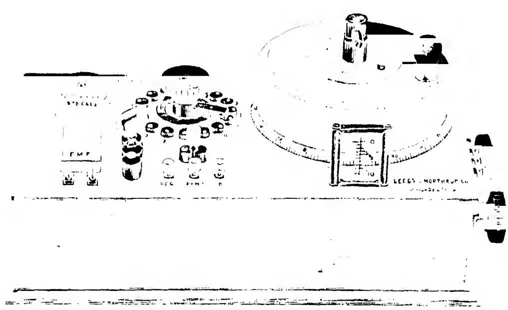


FIG. 26–17. Potentiometer without standard cell, galvanometer, or other accessories. *Leeds & Northrup Co.*

When the plug $X$ is changed, it puts the resistance $S$ in parallel with the measuring circuit and reduces the working current to $\frac{1}{10}$ its former value. Resistance $K$ is automatically thrown in circuit at the same time, keeping the total resistance in the circuit of the battery $W$ the same. This reduces the range of the instrument from a maximum of 1.61 volts to 0.161 volt, and makes possible the measurement of very much smaller voltages, or currents.

The binding post $BR$ is used only when it is desired to check the resistance of the measuring circuit.

The type K potentiometer, without the standard cell, the galvanometer and other accessories, and with the cover removed, is shown in Fig. 26–17.

This instrument is called a low-resistance potentiometer. It has only two dials, $F$ and $P$, for voltage adjustment; but it lends

itself to rapid work in the standardization laboratory, and the accuracy of its results is sufficient for most work. Several other types of potentiometers are manufactured, which have more dials for close balance and are known as high-resistance instruments. Potentiometers are also particularly designed for temperature measurements, etc.

**21. Voltmeter Calibration by Potentiometer. The Volt-Box.** It was stated in the previous article that the maximum potential difference which can be measured by the adjustment of the movable contacts *M* and *M'*, Fig. 25–17, is about 1.6 volts. In order to use the potentiometer to check higher voltages, such as the calibration of a voltmeter, etc., it is necessary to use a voltage multiplier called a "**volt-box.**"

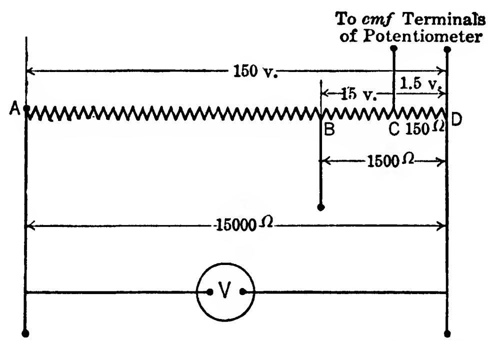



Fig. 27–17. Connections of a volt box for use with a potentiometer, for the calibration of a voltmeter.

A volt-box consists merely of a very high resistance from which taps are brought out for use on particular voltage ranges. The resistance in a volt-box is generally several hundred ohms per volt, so that it takes only a minute current from the line. Figure 27–17 shows a diagram of a volt-box, arranged for maximum voltages of 1.5, 15 and 150 volts, having a resistance of 100 ohms per volt. Leads are connected from the 1.5-volt taps *CD* to the potentiometer terminals, marked *emf*. When a higher voltage is impressed across other terminals of the volt-box, such as *AD*, Fig. 27–17, and the potentiometer is balanced, no current flows out at *C*. The same current, then, flows in *CD* as in *AC*, and therefore:

$$\frac{E \text{ across } CD}{E \text{ across } AD} = \frac{150^{\Omega}}{15{,}000^{\Omega}}$$

or, the voltage across *CD* is just $\frac{1}{100}$ of the voltage across *AD*.

The voltage readings on the scales of the potentiometer, then, must be multiplied by 100 to obtain the correct voltage across *AD*.

When a 150-volt voltmeter is to be calibrated, it is connected in parallel with the circuit *AD* of the volt-box across a source of emf whose value can be varied. If, for instance, the potentiometer is balanced at an indication of 1.425 volts, while the voltmeter reads 143 volts, it shows that the voltage on the circuit *AD* and on the voltmeter is 100 × 1.425 or 142.5 volts; and that the voltmeter is in error and reading 0.50 volt high at this point of the scale.

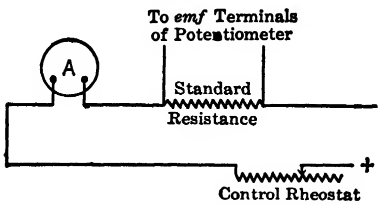



FIG. 28–17. Use of a standard resistance with a potentiometer for the calibration of an ammeter.

**22. Ammeter Calibration by Potentiometer. Standard Resistances.** The potentiometer can be used for accurate current measurements, such as the calibration of ammeters, by the employment of standard resistors, or shunts.

An ammeter to be calibrated is connected as in Fig. 28–17. Current is passed through the ammeter and the standard or calibrated resistor. The voltage drop over the resistor is then measured with the potentiometer. Thus, if the potentiometer connected across a 0.1-ohm resistor, as in the figure, indicates 1.2 volts, the actual current in the resistor and ammeter is $\frac{1.2}{0.1}$ or 12 amperes.

Since the maximum voltage that can be measured by a potentiometer is about 1.6 volts, the resistance of standard resistors is so proportioned that the voltage drop with maximum rated current is seldom more than 1.5 volts. Actually this drop is often less than 1 volt for maximum current. Standard resistors may vary from 10 ohms for a 0.1-ampere shunt to 0.00001 ohm for 3000-ampere shunts.

The potentiometer may be used in conjunction with both a volt-box and a standard resistor to calibrate an indicating wattmeter.

**23. The Megger.** The "megger" is the trade name given to a rugged, direct-reading, portable "**ohmmeter,**" designed for rapid measurement of high resistances, such as insulation resistance. Its readings are independent of the voltage of the current supplied for testing.

Figure 29–17 shows an external view of the instrument. It consists fundamentally of a small d-c generator, or magneto, generally hand-driven through gearing, which supplies current to the moving system of an " ohmmeter." This moving system gives a direct indication of the resistance in the circuit.

A diagram of the internal construction is shown in Fig. 30–17. The two permanent bar magnets $M$ and $M$ furnish the magnetic field for the moving element, and also for the d-c generator $D$.

The instrument proper consists of the two moving coils $A$ and $B$, mounted rigidly at about 90° to each other, on the same moving system to which a pointer is attached. The moving system turns in spring-supported jewelled bearings. The lead-in wires to the

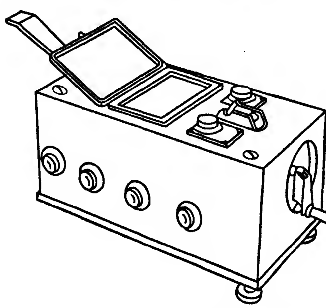


FIG. 29–17. A standard megger for testing high resistances.

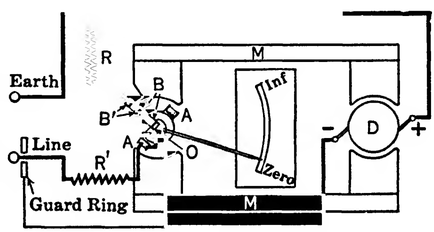



FIG. 30–17. Circuits in the standard megger. (*Courtesy James G. Biddle Co., Phila.*)

coils, instead of being held by spiral springs, as in the Weston-type ammeter, consist of very flexible conducting filaments (ligaments), shown in Fig. 31–17, having the least possible torsion. Thus, when there is no current in the coils, the moving element has no " control." It is not restrained in any way, and the pointer " floats " over the scale.

It is to be noted, from Fig. 30–17, that a C-shaped iron ring or core is mounted between the pole pieces, instead of the customary iron cylinder; and that coil $A$, similar to the coil in a Weston-type ammeter, spans the entire core; while one side of coil $B$, which is narrower than $A$, moves inside the ring or core.

Coil $A$ is called the **current coil.** It is connected in series with the resistance $R'$, between the negative side of the generator and the **line terminal** of the megger. The resistance to be measured is connected between the **earth** and **line** terminals, and is thus in series with coil $A$ and resistance $R'$ across the generator terminals.

Any current in coil $A$ sets up a clockwise torque, tending to move the pointer to the **zero** position of the scale.

Coil $B$ is called the **potential** or **control coil,** and is connected in series with the resistance $R$ across the generator terminals. A current in this coil sets up a counter-clockwise torque. The coil tends to assume a position where the least flux is threading it, which is opposite the gap in the iron ring, and where the pointer will indicate "infinity." The torque set up by the current in the two coils is thus in opposite directions.

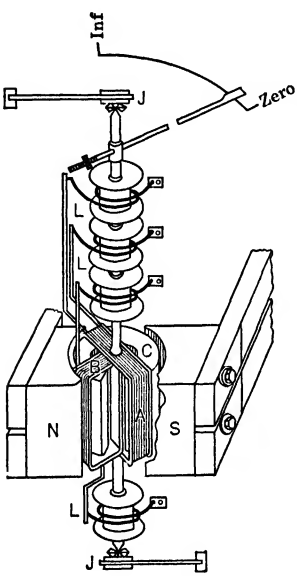


Fig. 31–17. Details of moving elements in the megger. (*Courtesy James G. Biddle Co., Phila.*)

In the operation of the instrument, if the **line** and **earth** terminals are open-circuited, or if they are connected to a circuit of practically perfect insulation, no current flows in coil $A$ and the instrument indicates **infinity.**

If a resistance is connected between the **earth** and **line** terminals, so that some current flows in the circuit of coil $A$, a torque is set up tending to move the pointer toward **zero.** The moving system will then take such a position that the forces, or torques, balance each other; and the pointer stands at some intermediate position between **zero** and **infinity.**

If the **earth** and **line** terminals are short-circuited, or if the insulation resistance is zero, the current and, therefore, the torque of coil $A$ will be such that the pointer will stand at **zero.** In this case, no harm is done, for the ballast resistance $R'$ limits the current to a safe value.

By inserting **known resistances** across the terminals of the instrument, and marking the corresponding positions of the pointer, a scale calibrated in resistance for direct reading can be obtained.

Since the two coils are connected to the **same** source of current, any change in generator voltage affects both coils alike, and thus the indications of the instrument are independent of the supply voltage. The usual generator voltage is 500 volts, although for high-resistance meggers, voltages as high as 2500 volts are used.

The guard ring, in Fig. 30–17, intercepts any current which may tend to leak across the face of the instrument, or which may flow, due to surface leakage of the insulation. It is carried back to the negative generator terminal without passing through coil $A$ and affecting the indications of the pointer.

A compensating coil $B'$, Fig. 30–17, which is connected in series with coil $B$ and reacts on one of the pole tips, is added to the moving system of some instruments. This gives better scale proportions and makes the instrument less susceptible to stray magnetic fields.

The megger is also adapted as a direct-reading instrument for measurement of ground resistances.

## SUMMARY OF CHAPTER XVII

**D'ARSONVAL GALVANOMETER. Permanent-magnet type. Operates on the principle of a pivoted coil moving in a uniform magnetic field. Has practically displaced other types because of the following advantages:**

**(a) Is sensitive;**
**(b) Is rugged;**
**(c) Is " dead-beat ";**
**(d) Has positive control;**
**(e) Is easily adjusted.**

**With shunts, galvanometers can be used to measure current; with series resistance, to measure voltage.**

**EQUATION FOR SHUNTS.**

$$\frac{\text{Current through galvanometer}}{\text{Current through main circuit}} = \frac{S}{S + G}$$

**SENSITIVITY. (a) Number of amperes or coulombs to produce a deflection of one scale division. (b) Number of megohms which may be placed in series with galvanometer across one volt pressure and have the instrument give a deflection of one scale division.**

**A BALLISTIC GALVANOMETER. " Throws " are proportional to quantity of electricity discharged through it.**

**THERMOELECTRIC PRESSURE. Is set up when one juncture of two unlike metals is at a higher temperature than the other. Possessed by different combinations of metals in a greatly varying degree. Effect used in pyrometers to measure high temperatures. Such combinations of metals must be avoided in ordinary voltage-, current- and power-measuring instruments.**

**TYPES OF AMMETERS AND VOLTMETERS.**

**(1) MOVING-IRON INSTRUMENTS.**

**(a) Are simple and cheap.**
**(b) Can be used on both d-c and a-c circuits, though must be calibrated for each separately.**
**(c) Are not particularly accurate.**

(2) HOT-WIRE.
   (a) Are simple and cheap.
   (b) Can be used interchangeably on both d-c and a-c circuits.
   (c) Slow acting but dead-beat.
   (d) Uncertain control, but fairly accurate.

(3) PERMANENT-MAGNET, D'ARSONVAL TYPE.
   (a) Extremely accurate.
   (b) Dead-beat.
   (c) Scale uniform.
   (d) Can be used on d-c circuits only.
   (e) Positive control.
   (f) Standard type for d-c measurements.

D'ARSONVAL TYPE AMMETER. Is really a millivoltmeter, and is used with a shunt. While the instrument measures millivolts drop over the shunt for various currents, the scale is calibrated in amperes flowing through both the moving coil and shunt. For capacities up to 50 amperes, the shunts are generally placed inside the instrument case. Above 100 amperes, external shunts, together with calibrated leads to the millivoltmeter, are generally used. The instrument itself may be marked as a millivoltmeter.

VOLTMETER. A high resistance inside the instrument case is connected in series with the moving coil, and the turns on this coil are sufficient to give a full-scale deflection in most instruments with a current of a few thousandths of an ampere. Resistance of such instruments is generally in the neighborhood of 100 ohms per volt of scale reading.

INDICATING WATTMETER. A dynamometer-type instrument which measures power directly. Consists of stationary coils, which carry the current of the circuit, and a pivoted coil connected across the circuit in series with high resistance. Deflection of pivoted coil is proportional to the product of the current in the coils, which is also proportional to the power in the circuit.

ARRANGEMENT OF INSTRUMENTS FOR POWER MEASUREMENT. In using wattmeters or ammeters and voltmeters for accurate power measurement, they should be connected to the circuit in such a way as to cause as small an error in readings as possible. In very accurate work, the errors due to power consumed by instruments must be corrected.

WATTHOUR METER — THOMSON. A small shunt motor, the speed of which is proportional to rate of power consumption. Armature current proportional to voltage on circuit; field proportional to current in circuit. Opposing torque, supplied by eddy currents in disk, which revolves between permanent magnetic poles. Measures energy — not power.

POTENTIOMETER. A device for measuring voltage or emf; takes no current from source, the pressure of which it measures. Emf to be measured is balanced against a known fall of potential along a wire. Potentiometers directly measure unknown emf's only as high as about 1.6 volts. By use of a VOLT-BOX, higher emf's may be

measured by balancing a definite fraction of the unknown emf against this fall of potential. Can be used as a standard in the calibration of voltmeters.

When used to measure the voltage drop over STANDARD RESISTANCES, the potentiometer can also be used in the calibration of ammeters.

The MEGGER is a rugged, portable, direct-reading instrument for rapidly measuring INSULATION RESISTANCE.

It is also adapted as a direct-reading instrument for measuring the resistance of GROUND CONNECTIONS.

# APPENDIX

## TABLE I

### Resistivity and Temperature Coefficient

| Material | Resistivity in ohms per mil-foot at 20° C | Temperature coefficient per degree C per ohm at 20° C |
|---|---|---|
| Advance Metal | 326 | 0.00002 |
| Aluminum | 17.0 | 0.00388 |
| Antimony | 251 | 0.0039 |
| Bismuth | 718 | 0.0046 |
| Brass | 39 | 0.0010 |
| Calorite | 720 | ........ |
| Constantin | 300 | 0.000005 |
| Copper | | |
| Annealed Intnl. Standard | 10.4 | 0.00393 |
| Hard Drawn | 10.7 | 0.00382 |
| Pure | 10.15 | 0.0041 |
| German Silver | 181 | 0.00036 |
| Gold | 133 to 136 | 0.0037 |
| *Iala* | 295 | 0.000005 |
| Iron, Commercial | 66.4 to 81.4 | 0.0055 |
| Hard cast | 592 | ........ |
| Lead | 12.1 to 12.9 | 0.0039 |
| Manganin | 250 to 450 | 0.00003 |
| Monel Metal | 257 | 0.00198 |
| Mercury | 580 | 0.0007 |
| Nichrome | 675 | 0.00017 |
| Nichrome V | 650 | 0.00013 |
| Nickel | 72.3 to 84.4 | 0.006 |
| Platinum | 58.5 to 100 | 0.0038 |
| Platinum-iridium | 148 | 0.0012 |
| Silver | 9.89 to 11.2 | 0.0040 |
| Steel, Hard | 28.5 | 0.0016 |
| Soft | 105 | 0.0042 |
| Rail | 83.5 to 130 | ........ |
| Superior | 521 | 0.00081 |
| Tantalum | 93.8 | 0.0027 |
| Tin | 63 to 75.5 | 0.0037 |
| Tungsten | 35.8 | 0.0039 |
| Zinc | 36.4 to 39.6 | 0.0040 |

TABLE II

RESISTANCE OF INTERNATIONAL STANDARD ANNEALED COPPER
American Wire Gauge (B & S)

| B & S gauge, No. | Diameter in mils, $d$ | Area in circular mils, $d^2$ | Ohms per 1000 ft at 20° C or 68° F | Pounds per 1000 ft | B & S gauge, No. | Diameter in mils, $d$ | Area in circular mils, $d^2$ | Ohms per 1000 ft at 20° C or 68° F | Pounds per 1000 ft |
|---|---|---|---|---|---|---|---|---|---|
| 0000 | 460.00 | 211,600 | 0.04901 | 640.5 | 21 | 28.462 | 810.10 | 12.80 | 2.452 |
| 000 | 409.64 | 167,810 | 0.06180 | 508.0 | 22 | 25.347 | 642.40 | 16.14 | 1.945 |
| 00 | 364.80 | 133,080 | 0.07793 | 402.8 | 23 | 22.571 | 509.45 | 20.36 | 1.542 |
| 0 | 324.86 | 105,530 | 0.09827 | 319.5 | 24 | 20.100 | 404.01 | 25.67 | 1.223 |
| | | | | | 25 | 17.900 | 320.40 | 32.37 | 0.9699 |
| 1 | 289.30 | 83,694 | 0.1239 | 253.3 | 26 | 15.940 | 254.10 | 40.81 | 0.7692 |
| 2 | 257.63 | 66,373 | 0.1563 | 200.9 | 27 | 14.195 | 201.50 | 51.47 | 0.6100 |
| 3 | 229.42 | 52,634 | 0.1970 | 159.3 | 28 | 12.641 | 159.79 | 64.90 | 0.4837 |
| 4 | 204.31 | 41,742 | 0.2485 | 126.4 | 29 | 11.257 | 126.72 | 81.83 | 0.3836 |
| 5 | 181.94 | 33,102 | 0.3133 | 100.2 | 30 | 10.025 | 100.50 | 103.2 | 0.3042 |
| 6 | 162.02 | 26,250 | 0.3951 | 79.46 | 31 | 8.928 | 79.70 | 130.1 | 0.2413 |
| 7 | 144.28 | 20,816 | 0.4982 | 63.02 | 32 | 7.950 | 63.21 | 164.1 | 0.1913 |
| 8 | 128.49 | 16,509 | 0.6282 | 49.98 | 33 | 7.080 | 50.13 | 206.9 | 0.1517 |
| 9 | 114.43 | 13,094 | 0.7921 | 39.63 | 34 | 6.305 | 39.75 | 260.9 | 0.1203 |
| 10 | 101.89 | 10,381 | 0.9989 | 31.43 | 35 | 5.615 | 31.52 | 329.0 | 0.0954 |
| 11 | 90.742 | 8,234.0 | 1.260 | 24.93 | 36 | 5.000 | 25.00 | 414.8 | 0.0757 |
| 12 | 80.808 | 6,529.9 | 1.588 | 19.77 | 37 | 4.453 | 19.82 | 523.1 | 0.0600 |
| 13 | 71.961 | 5,178.4 | 2.003 | 15.68 | 38 | 3.965 | 15.72 | 659.6 | 0.0476 |
| 14 | 64.084 | 4,106.8 | 2.525 | 12.43 | 39 | 3.531 | 12.47 | 831.8 | 0.0377 |
| 15 | 57.068 | 3,256.7 | 3.184 | 9.858 | 40 | 3.145 | 9.89 | 1049 | 0.0299 |
| 16 | 50.820 | 2,582.9 | 4.016 | 7.818 | | | | | |
| 17 | 45.257 | 2,048.2 | 5.064 | 6.200 | | | | | |
| 18 | 40.303 | 1,624.3 | 6.385 | 4.917 | | | | | |
| 19 | 35.890 | 1,288.1 | 8.051 | 3.899 | | | | | |
| 20 | 31.961 | 1,021.5 | 10.15 | 3.092 | | | | | |

## TABLE III

### Resistance of International Standard Annealed Copper Cables (Stranded)

| Circular Mils | Ohms per 1000 ft at 20° C | Circular Mils | Ohms per 1000 ft at 20° C |
|---|---|---|---|
| 200,000 | 0.0520 | 1,100,000 | 0.00946 |
| 250,000 | 0.0416 | 1,200,000 | 0.00867 |
| 300,000 | 0.03467 | 1,300,000 | 0.0080 |
| 350,000 | 0.02871 | 1,400,000 | 0.00743 |
| 400,000 | 0.0260 | 1,500,000 | 0.00693 |
| 500,000 | 0.0208 | 1,600,000 | 0.00650 |
| 600,000 | 0.01733 | 1,700,000 | 0.00612 |
| 700,000 | 0.01486 | 1,800,000 | 0.00578 |
| 800,000 | 0.0130 | 1,900,000 | 0.00547 |
| 900,000 | 0.01156 | 2,000,000 | 0.0052 |
| 1,000,000 | 0.0104 | ......... | ........ |

## TABLE IV

### Allowable Carrying Capacities of Wires

*National Electrical Code,* 1931

The following table is for copper wires of ninety-eight per cent conductivity, and must be followed in placing interior conductors.

For insulated aluminum wire, the safe carrying capacity is *eighty-four per cent* of that given for copper wire with the same kind of insulation.

| B & S gauge | Diameter of solid wires in mils | Area in circular mils | Rubber insulation. Amperes | Varnished cloth insulation. Amperes | Other insulation. Amperes |
|---|---|---|---|---|---|
| 18 | 40.3 | 1,624 | 3 | | 6 |
| 16 | 50.8 | 2,583 | 6 | | 10 |
| 14 | 64.1 | 4,107 | 15 | 18 | 20 |
| 12 | 80.8 | 6,530 | 20 | 25 | 30 |
| 10 | 101.9 | 10,380 | 25 | 30 | 35 |
| 8 | 128.5 | 16,510 | 35 | 40 | 50 |
| 6 | 162 | 26,250 | 50 | 60 | 70 |
| 5 | 181.9 | 33,100 | 55 | 65 | 80 |
| 4 | 204.3 | 41,740 | 70 | 85 | 90 |
| 3 | 229.4 | 52,630 | 80 | 95 | 100 |
| 2 | 257.6 | 66,370 | 90 | 110 | 125 |
| 1 | 289.3 | 83,690 | 100 | 120 | 150 |
| 0 | 325 | 105,500 | 125 | 150 | 200 |
| 00 | 364.8 | 133,100 | 150 | 180 | 225 |
| 000 | 409.6 | 167,800 | 175 | 210 | 275 |
| | | 200,000 | 200 | 240 | 300 |
| 0000 | 460 | 211,600 | 225 | 270 | 325 |
| | | 250,000 | 250 | 300 | 350 |
| | | 300,000 | 275 | 330 | 400 |
| | | 350,000 | 300 | 360 | 450 |
| | | 400,000 | 325 | 390 | 500 |
| | | 500,000 | 400 | 480 | 600 |
| | | 600,000 | 450 | 540 | 680 |
| | | 700,000 | 500 | 600 | 760 |
| | | 800,000 | 550 | 660 | 840 |
| | | 900,000 | 600 | 720 | 920 |
| | | 1,000,000 | 650 | 780 | 1000 |
| | | 1,100,000 | 690 | 830 | 1080 |
| | | 1,200,000 | 730 | 880 | 1150 |
| | | 1,300,000 | 770 | 920 | 1220 |
| | | 1,400,000 | 810 | 970 | 1290 |
| | | 1,500,000 | 850 | 1020 | 1360 |
| | | 1,600,000 | 890 | 1070 | 1430 |
| | | 1,700,000 | 930 | 1120 | 1490 |
| | | 1,800,000 | 970 | 1160 | 1550 |
| | | 1,900,000 | 1010 | 1210 | 1610 |
| | | 2,000,000 | 1050 | 1260 | 1670 |

## TABLE V

### Relative Dielectric Constants (Average Values)

| Material | Dielectric Constant | Material | Dielectric Constant |
|---|---|---|---|
| Air | 1 | Acetone | 26.6 |
| Ebonite | 2.7 | Alcohol (0° C) | ..... |
| Glass | ..... | Amyl | 17.4 |
| Flint | 9.9 | Ethyl | 28.4 |
| Hard crown | 7.0 | Methyl | 35 |
| Lead | 6.6 | Ammonia | 22 |
| Gutta Percha | 4.1 | Benzine | 2.3 |
| Mica | 5.8 | Glycerine | 56.2 |
| Paraffin | 2.1 | Petroleum | 2.1 |
| Shellac | 3.1 | Water (pure) | 81 |

## TABLE VI

### Dielectric Strength (Average Values)

| Material | Dielectric Strength in Volts per Centimeter | Material | Dielectric Strength in Volts per Centimeter |
|---|---|---|---|
| Air | 30,000 | Gutta Percha | 140,000 |
| Bakelite | 210,000 | Mica | 500,000 |
| Ebonite | 700,000 | Paraffin | 290,000 |
| Glass | 350,000 | Transil oil | 100,000 |

## TABLE VII

### Useful Numbers

$\pi = 3.1416 = \dfrac{\text{circumference}}{\text{diameter}}$. Surface of cyl. $= 2\,\pi r l + 2\,\pi r^2$.

$\pi^2 = 9.8696$; $\dfrac{1}{\pi} = 0.3183$. Volume of cyl. $= \pi r^2 l$.

Area of circle $= \pi r^2 = \dfrac{\pi d^2}{4} = 0.7854\ d^2$. Surface of sphere $= 4\,\pi r^2$.

Volume of sphere $= \dfrac{\pi d^3}{6} = \dfrac{4\,\pi r^3}{3}$.

### Metric-English Equivalents

| | | | | | |
|---|---|---|---|---|---|
| 1 cm | = | 0.39 in. | 1 in. | = | 2.54 cm |
| 1 m | = | 39.37 in. | 1 ft | = | 30.48 cm |
| 1 m | = | 3.23 ft | 1 ft | = | 0.305 m |
| 1 km | = | 0.6 mile | 1 mile | = | 1.60 km |
| 1 g | = | 0.035 oz (avoir.) | 1 oz | = | 28.35 gm |
| 1 kg | = | 2.204 lb (avoir.) | 1 lb | = | 453.6 gm |
| 1 sq cm | = | 0.155 sq in. | 1 sq in. | = | 6.45 sq cm |
| 1 cu cm | = | 0.061 cu in. | 1 cu in. | = | 16.39 cu cm |

### Units of Force, Work, Power, etc.

| | | |
|---|---|---|
| 1 dyne | = | 0.00102 g |
| 1 ft-lb | = | $1.356 \times 10^7$ ergs |
| 1 joule | = | $10^7$ ergs |
| 1 horsepower | = | 33,000 ft-lb/min |
| 1 horsepower | = | 550 ft-lb/sec |
| 1 horsepower | = | $7.46 \times 10^9$ ergs/sec |
| 1 horsepower | = | 746 watts |
| 1 watt | = | 0.00134 horsepower |
| 1 watt | = | $10^7$ ergs/sec = 1 joule/sec |

### Mechanical Equivalents of Heat

| | | |
|---|---|---|
| 1 g of water heated 1° C | = | $4.2 \times 10^7$ ergs |
| 1 lb of water heated 1° C | = | 1400 ft-lb |
| 1 lb of water heated 1° F | = | 780 ft-lb |

The combustion of 1 lb of coal produces about 14,000 Btu.

# INDEX

## A



## B



## C